NEUTRINO MASS MODELS AND CP VIOLATION

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Abstract. Theoretical ideas on the origin of (a) neutrino masses (b) neutrino mass hierarchies and (c) leptonic mixing angles are reviewed. Topics discussed include (1) symmetries of neutrino mass matrix and their origin (2) ways to understand the observed patterns of leptonic mixing angles and (3)unified description of neutrino masses and mixing angles in grand unified theories.

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1. INTRODUCTION

Results from solar and atmospheric neutrino oscillations have revealed [1] that (*a*) there exists a sub-eV scale associated with neutrino masses. Such a scale cannot be accommodated in the standard model (SM) characterized by a scale of ~ 100 GeV. (*b*) neutrinos mix among themselves and the mixing pattern is qualitatively different from the observed pattern in mixing among quarks. Both these features suggest substantially different mechanism for neutrino mass generation which can be obtained by postulating new particles and new interactions. Neutrinos are therefore regarded as providing a window into physics beyond the standard electroweak model. This review is aimed at summarizing both old and mainly recent attempts which are aimed at understanding (*i*) smallness of neutrino masses (*ii*) hierarchies among these masses and (*iii*) origin of neutrino mixing pattern and its reconciliation with the quark mixing patterns in unified frameworks. Different ideas leading to above features also lead to predictions of two of the unknowns in neutrino physics namely, the reactor mixing angle θ_{13} and the CP violating phase δ. We will summarize these predictions. Rather than elaborating on any specific model, we would emphasize the basic mechanisms and ideas behind them using models only for illustrations.

2. WHAT WE KNOW AND DO NOT KNOW ABOUT NEUTRINOS?

The present information on neutrino masses m_{ν_i} $(i = 1,2,3)$ can be summarized [2] as

$$\begin{aligned}
\Delta_\odot \equiv m^2_{\nu_2} - m^2_{\nu_1} &\approx 7.6\times 10^{-5}~~\text{eV}^2 ~,\\
|\Delta_{atm}| \equiv |m^2_{\nu_3} - m^2_{\nu_1}| &\approx 2.4\times 10^{-3}~~\text{eV}^2 ~,\\
0.3~~\text{eV} \leq \Sigma_{i=1,2,3} m_{\nu_i} &\leq 2~\text{eV}~. \qquad (1)
\end{aligned}$$

The first two scales result from the global fits to the solar and atmospheric neutrino data. The last line is the bound obtained by combining various cosmological observation [3]. While $\Delta_\odot$ is known to be positive, Δ_{atm} can have either sign. Physically this means that both normal ($m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$) and inverted ($m_{\nu_1} \approx m_{\nu_2} \gg m_{\nu_3}$) neutrino mass hierarchies are allowed by the present results. The third possibility would be all neutrinos being quasi degenerate with normal or inverted hierarchies. The last line of eq.(1) allows a common mass m_0 in the $0.1-0.7$ eV range.

The unitary matrix U relating neutrino mass to flavour eigenstates can be transformed into the standard form which depends on three mixing angles and three phases. θ_{12} (θ_{23}) dominantly control the amplitudes of solar (atmospheric) neutrino oscillations and the reactor angle θ_{13} dominantly controls the survival probability of the electron (anti) neutrinos in short baseline reactor experiments. U also contains three phases δ,α,β. δ measures the amount of CP violation in neutrino oscillations and α,β are CP violating phases associated with the lepton number violating processes such as the neutrinoless double beta decay. 3σ limits on mixing angles can be nicely summarized by [4]

$$\begin{aligned}
|\sin^2\theta_{12} - \frac{1}{3}| &\leq 0.04 ~,\\
|\sin^2\theta_{23} - \frac{1}{2}| &\leq 0.12 ~,\\
|\sin^2\theta_{13}| &\leq 0.04 ~. \qquad (2)
\end{aligned}$$

All three mixing angles are close to "magic values" $\sin^2\theta_{ij} = \frac{1}{3}, \frac{1}{2}, 0$ in case of the solar θ_{12} atmospheric θ_{23} and reactor θ_{13} mixing angles respectively. Future solar, atmospheric and reactor experiments are aimed at improving the precision on these values. The magic values of these angles can be used as a hint to look for the underlying theory of neutrino masses. They may be pointing either to some special leptonic symmetries and small departures from the exact values may be result of the breaking of such symmetries. Conversely, the specific values of mixing angles close to these magic values may be a dynamical accident of some underlying mechanism which generates leptonic mixing. At present, both these are open possibilities. In order to distinguish these possibilities we need to know the predicted departures from symmetry values in the first option and the nature of dynamical mechanism in the second. We will address these issues in section (6). Before doing this, we review basic mechanisms for neutrino mass generation.

3. MECHANISMS FOR NEUTRINO MASS GENERATION

Different mechanisms for neutrino mass generations [1] aim at explaining (1) smallness of neutrino masses and (2) origin of lepton number violation if it is violated. If not then one has bigger problem of explaining very small Dirac mass six orders of magnitude smaller than the mass of the lightest charged fermion -the electron. These mechanisms are quite well-known and have been discussed [1]. We collect various information here for the sake completeness.

Since the direct coupling of a left-handed neutrino with itself violates $SU(2)_L$ gauge invariance, such a coupling can be generated indirectly through coupling it with either (1) a singlet fermion (2) triplet Higgs or (3) triplet fermion. These are respectively known in the literature as type-I, type-II and type-III seesaw mechanisms.

3.1. Type-I+II mechanism

This is obtained by coupling ν_L with a right handed (RH) neutrino ν_R. The combined mass matrix has the form

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \quad (3)$$

m_L term transforms as a triplet of $SU(2)_L$ and arise from coupling with the corresponding Higgs. m_D denotes the lepton number conserving Dirac coupling between ν_L and ν_R and M_R correspond to the lepton number violating Majorana mass term for the latter. In the limit of $m_L \ll m_D \ll M_R$ one obtains the seesaw formula

$$M_\nu \approx m_L - m_D M_R^{-1} m_D^T \; . \quad (4)$$

The second term is the standard (type-I) term and denotes the seesaw relation between the physical neutrino mass and the high scale M_R. The first term (type-II contribution) can be a priori an independent scale. But in left right symmetric theories one has a relation [1]

$$m_L M_R \sim M_W^2$$

This represent second type of seesaw mechanism between the scales of the vacuum expectation value (vev) of the left handed and right handed triplets.

3.2. Type-III mechanism

This mechanism requires extension of standard model by a fermion triplet with zero hypercharge. Neutral component of this fermion mix with the left handed neutrino through a doublet Higgs vev very much as in the type-I mechanism. But unlike the RH neutrinos in the latter case, the preferred scale associated with the triplet fermion masses is argued to be close to the electroweak scale rather than to the GUT scale. Such light triplets are motivated in [5] from the requirement of successful gauge coupling unification in a non-supersymmetric theories. They are welcome from the point of view of observability at LHC but they require some amount of fine tuning in the Dirac mass in order to obtain very light neutrino. The most natural scenario of the type-III seesaw in this regards is supersymmetric models with broken R parity. This theory naturally has light triplet-the gaugino which helps in achieving the gauge coupling unification. The breaking of R parity through sneutrino vev generates a coupling

$$\nu_L \lambda \, \langle \tilde{\nu} \rangle$$

between neutrino ν_L and neutralino λ. If R parity violation occurs through bilinear term then one avoids fine tuning in Dirac mass required in non-supersymmetric models with ad-hoc triplet. In supergravity theories with universal boundary condition, sneutrino vev gets a contribution only at low energy from the b and τ Yukawa couplings. This results in its suppression. It is indeed possible to build successful models of neutrino masses and this approach has been extensively studied in the literature [1].

3.3. Variants of seesaw, double seesaw,inverse seesaw, linear seesaw

The seesaw mechanisms named as above are extensions of the conventional seesaw mechanisms requiring singlets S in addition to the normal RH neutrinos. The mass matrix in the basis $(\nu_L, N_R^c, S)^T)$ has the following form [6]

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix} \; . \quad (5)$$

Note that introduction of S with the above structure leads to lepton number conservation in the limit $\mu \to 0$. Hence neutrinos are massless in this limit and turning on a small

μ generates the neutrino mass matrix

$$m_D M^{-1} \mu M^{T-1} m_D^T \tag{6}$$

The structure of this mass matrix is quite different from the type-I seesaw model. In particular if $m_D \propto M$ then the mixing is solely determined by the structure of μ. This versions of seesaw model allows the scale M to be accessible at collider energies and leads to various observable consequences, lepton flavour violation, sizable non-unitarity etc which have been explored, see [6] for a review. The above formula for neutrino masses holds even in the limit $\mu >> M$. In this case, one gets light neutrino masses through double seesaw. First the RH neutrinos get mass $M_R \sim M\mu^{-1}M^T$. This then leads to eq.(6) through the usual seesaw.

4. SYMMETRIES OF $\mathscr{M}_{\nu f}$

The neutrino mass matrix $\mathscr{M}_{\nu f}$ in the flavour basis is the object of main interest since it can be constructed from experiments:

$$\mathscr{M}_{\nu f} = U^* D_\nu U^\dagger \ , \tag{7}$$

where U is the leptonic mixing matrix and D_ν is a diagonal matrix with real and positive masses. Conversely, knowledge of $\mathscr{M}_{\nu f}$ can be used to obtain neutrino masses and leptonic mixing. Symmetries of $\mathscr{M}_{\nu f}$ play important role in determining this. These can be of two types. One arising due to some specific structure of the mixing matrix alone. The other also related to the values of the neutrino masses as well. The former "mass independent symmetries" are easy to identify. Recent studies [7] have brought out the fact that any $\mathscr{M}_{\nu f}$ is invariant under a $Z_2 \times Z_2$ symmetry independent of its detailed structure. This is easy to demonstrate. One can write

$$(\mathscr{M}_{\nu f})_{ij} = (D_\nu)_{kk} U^*_{ik} U^*_{jk} \ . \tag{8}$$

Now define three operators $S(l)\ ;\ l = 1,2,3$:

$$S_{ij}(l) = 2U^*_{il}U^*_{jl} - \delta_{ij} \ . \tag{9}$$

These operators define symmetries of $\mathscr{M}_{\nu f}$:

$$S^T(l)\mathscr{M}_{\nu f}S(l) = \mathscr{M}_{\nu f}$$

They satisfy

$$S^2(l) = I$$

and each defines a Z_2 symmetry. Moreover,

$$S(1) + S(2) + S(3) = -I$$

and only two are independent. Thus any $\mathscr{M}_{\nu f}$ is invariant under a $Z_2 \times Z_2$ symmetry[1]. $S(l)$ represent a class of $Z_2 \times Z_2$ symmetries each element corresponding to specific mixing pattern. In the most generality, it cannot shed light on the underlying dynamics. But special cases are of significance. We know that to a good approximation $U_{i3} = (0, 1/\sqrt{2}, 1/\sqrt{2}) \ \ , i = 1,2,3$ form an eigenvector of U. Then corresponding $S(3)$ is given from (9) by

$$S(3) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} . \tag{10}$$

This is nothing but the very well-studied μ-τ symmetry. In addition, if solar mixing angle is close to the magic value then $U_{i2} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) \ \ , i = 1,2,3$ is another eigenvector and the corresponding S is

$$S(2) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} . \tag{11}$$

Simultaneous invariance under $S(2), S(3)$ lead to the tri-bimaximal pattern of the neutrino mixing. To the extent that the observed mixing angles are close to the magic values the $Z_2 \times Z_2$ symmetry generated by eqs.(10,11) must at least be an approximate symmetry of $\mathscr{M}_{\nu f}$ in any successful description of the leptonic mixing. This symmetry may be either (a) result of some different symmetry imposed on Lagrangian or (b) may be an approximately broken symmetry of the Lagrangian itself or (c) may be dynamical in origin and appear as an approximate accidental symmetry. Examples in category (a) are the D_4 and A_4 symmetries. The former after spontaneous breaking leads to effective μ-τ symmetry for $\mathscr{M}_{\nu f}$ and the latter leads to the tri-bimaximal mixing pattern. Both these are studied extensively and we refer to a recent review [9] and references there. It is possible to start from an approximate Z_2 symmetry itself at the Lagrangian level and realize (b). This is shown to be feasible in case of the μ-τ symmetry [10]. The corresponding analysis of the full $Z_2 \times Z_2$ symmetry in case possible does not exist. Finally there are well motivated models which lead to approximate μ-τ symmetry or the full $Z_2 \times Z_2$ symmetry as a dynamical symmetry of $\mathscr{M}_{\nu f}$. We will discuss these now.

5. DYNAMICAL MECHANISMS FOR LARGE MIXING

Large leptonic mixing angles may arise due to underlying structure/dynamics and may not be linked to flavour symmetries like D_4, A_4 etc. Several scenarios and their explicit realizations are known in this category. Seesaw mechanism by itself may be the cause of large mixing [11]. This happens if contribution of one RH neutrino

[1] This symmetry is a simple conseuqnece of the fact that neutrino mass terms are invariant when sign of any of the neutrino mass eigenstate is reversed [8].

dominates or if M_R is nearly singular [12]. Grand unified theories in some versions automatically lead to large solar and atmospheric mixing angles for leptons and small mixing for quarks. The old known example is $SU(5)$ theory with "lopsided " structure. Because of the $SU(5)$ relation $M_d = M_l^T$ between the down quark and charged lepton mass matrices, the mixing among the RH quarks get linked to the mixing among the left handed charged leptons. This mixing could be large and can simultaneously exist with small mixing among the left handed quarks. This becomes possible if M_d has lopsided structure. Implications of these have been studied in the context of $SO(10)$ models [1].

An attractive possibility of understanding large mixing angles occurs in the context of the $SO(10)$ models based on the type-II seesaw mechanism [13]. The neutrino mass matrix in this model get linked to $M_d - M_l$ and almost equality of the τ and b-quark Yukawa couplings automatically lead to a large atmospheric mixing angle. The same model also leads to large solar mixing angle [14]. This model and its variations have been studied in detail in a number of papers [15] for a summary.

One more realization of dynamical generation of large leptonic mixing angles is provided in a recent study of fermion masses in non-supersymmetric model [16]. Detailed fits to fermion masses lead to a unified description of all fermion mass matrices with a form

$$M_f \approx \begin{pmatrix} c_{11}\lambda^4 & c_{12}\lambda^3 & c_{13}\lambda^2 \\ c_{21}\lambda^3 & c_{22}\lambda^2 & c_{23}\lambda \\ c_{31}\lambda^2 & c_{32}\lambda & c_{33} \end{pmatrix} ,$$

where $c_{ij}(1)$ and $\lambda \sim$ Cabibbo angle. This structure realized dynamically here in detailed fits to fermion masses and mixing in an $SO(10)$ theory has been argued to lead to correct descriptions of fermion mass hierarchies and mixing angles [12].

6. EXPECTATIONS ON θ_{13} SYMMETRY VERSUS DYNAMICAL MECHANISMS

The values of θ_{13} and CP violating phase δ are still unknown and much experimental efforts are geared at determining them. At the semi-quantitative level one can argue that preferred "theoretical value" of θ_{13} is large not far away from the present limit and δ is also expected to be large. Let us start with the zeroth order result of vanishing θ_{13} and the solar scale. Typical and well-motivated structures of neutrino mass matrices in this limit are:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & c^2 & sc \\ 0 & cs & s^2 \end{pmatrix} \quad ; \quad \begin{pmatrix} 0 & s & c \\ c & 0 & 0 \\ c & 0 & 0 \end{pmatrix} . \tag{12}$$

The first correspond to the normal hierarchy and the second to the inverted hierarchy. Consider now a small perturbation to the first:

$$\begin{pmatrix} 0 & 0 & \lambda \\ 0 & c^2 & sc \\ \lambda & sc & s^2 \end{pmatrix} . \tag{13}$$

This perturbation generates (a) solar scale and (b) θ_{13}. Both are related to each other by

$$|\theta_{13}| \approx \frac{\tan 2\theta_\odot}{2\tan\theta_A} \left(\frac{\Delta_\odot}{\Delta_A \cos 2\theta_\odot} \right)^{\frac{1}{2}} \approx 0.13 \tag{14}$$

Eq.(13) is realized in specific model based on $SO(10)$ [14]. The above relation is more general than its derivation presented here. It is a simple consequence of texture zeros at at "11" and "12" entries. As long as these elements are sub-dominant compered to others, one would end up having a large θ_{13}. This is born out by many explicit models and a list of various models can be found in [17].

The models which lead to vanishing θ_{13} would posses some underlying symmetry,e.g. D_4. Such symmetries may be broken and theoretically it is important to distinguish two cases broken symmetries versus dynamical models without any symmetry. This can be done by systematically investigating effects of perturbation on the structure implied by symmetry. This has been done in case of μ-τ symmetry in [18] and in case of tri-bimaximal mixing for example in [19]. It turns out that the induced value of θ_{13} in a broken symmetry scenario depends on neutrino mass hierarchies. In case of the broken μ-τ symmetry one finds
Normal Hierarchy:

$$\begin{aligned} \sin\theta_{13} &\approx c_{12}s_{12}\sqrt{\frac{\Delta_{sun}}{\Delta_A}}(\varepsilon+\varepsilon'/2) \approx 0.1(\varepsilon+\varepsilon'/2) , \\ \cos 2\theta_{23} &\approx \varepsilon' . \end{aligned} \tag{15}$$

Inverted hierarchy:

$$\begin{aligned} \sin\theta_{13} &\approx \frac{1}{4}\sin 2\theta_{12}\frac{\Delta_{sun}}{\Delta_A}(\varepsilon-\varepsilon'/2) \approx 0.01(\varepsilon-\varepsilon'/2) , \\ \cos 2\theta_{23} &\approx \varepsilon' . \end{aligned} \tag{16}$$

where $\varepsilon, \varepsilon'$ are symmetry breaking parameters introduced in [18]. Special case, $\varepsilon = \varepsilon'/2$ is realized when

$\theta_{13} = 0$ at a high scale and is induced at 1-loop radiatively by the tau Yukawa coupling.

The CP violating phase δ can be rotated if $\theta_{13} = 0$. The Majorana phases α, β of neutrino masses represent only sources of CP violation in this case. Now small perturbations inducing non-zero θ_{13} also leads to δ related to to α, β. For example, in case of the μ-τ symmetry one finds [18] that $\tan\delta$ can be large independent of the strength of perturbation $\varepsilon.\varepsilon'$ and the neutrino mass hierarchies. Similarly, non-zero and large δ also gets induced radiatively if θ_{13} is zero at the high scale[20]. There exists a special symmetry which leads to CP violating phase $\delta = \frac{\pi}{2}$ [21]. δ also gets predicted in several grand unified models through the fit to other fermion masses and in several cases it tends to be large[22]

7. NEUTRINO MASS HIERARCHIES FROM SYMMETRIES

Nature of neutrino mass hierarchy is yet another unknown in neutrino physics. Normal mass hierarchy is natural if neutrino mass structure is related in some way to the charged fermion masses. This happens in seesaw models based on GUTs. Such models invariably lead to normal hierarchy and if experimental findings in future are otherwise, one needs some special symmetries to obtain inverted or quasi-degenerate spectrum. Well-known example of the inverted hierarchy is the $L_e - L_\mu - L_\tau$ symmetry. One can identify more general class of symmetries which lead to the inverted spectrum. Assume type-I seesaw mechanism and invariance under $\nu_L \to S\nu_L$ in the flavour basis. This implies $m_D = Sm_D$. As long as $detS \neq 1$ one obtains a massless neutrino after the seesaw mechanism. If S corresponds to the generalized μ-τ symmetry [18] then this massless state can be identified as the lightest state with $\theta_{13} = 0$ and one gets the inverted spectrum, see [23] for details.

Quasi degenerate spectrum can also be obtained from underlying symmetry. The standard examples are based on the type-II seesaw models [24] but even type-I seesaw model can also lead to such spectrum with appropriate symmetry [25]. One strong theoretical motivation for the quasi degenerate neutrinos is understanding the leptonic mixing structure. The most general neutrino mass matrix with degenerate neutrino is a unitary symmetric matrix. Such a matrix can be diagonalized by a matrix with unconstrained θ_{12}, θ_{23} and vanishing θ_{13}. This is just the right mixing pattern one encounters. As long as the perturbations which lift degeneracy of masses do not appreciably change these angles, one has automatic explanation for the mixing pattern [26]. Interestingly, one can give realistic $SO(10)$ models in which large leptonic mixing and degenerate spectrum of neutrinos co-exist with small quark mixing and hierarchical masses[27]

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