Neutrino mass models and CP violation

Anjan S. Joshipura
Physical Research Laboratory,
Ahmedabad, INDIA

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Neutrino mass hierarchies, mixing angles and CP violation: Theoretical Overview

Flavour puzzle

Neutrino mass and mixing pattern may hold key to understanding the entire flavour puzzle of SM

Magic values:

$$\sin^2\theta_{12} = \frac{1}{3} \quad ; \quad \sin^2\theta_{23} = \frac{1}{2} \quad ; \quad \sin^2\theta_{13} = 0$$

OR

$$\theta_{12} = \theta_{23} = \frac{\pi}{4} \quad + \text{Grand unified “corrections“}$$

- Neutrino flavour puzzle has several pieces.
  - Two of the neutrino mixing angles are large
  - Neutrino mass hierarchies are milder compared to quarks

- Why neutrinos appear to be different from other fermions ?

  Differences between these two sectors may arise due to fundamentally different ways in which their masses are generated.

- Why within the neutrino world itself one sees dissimilarity in mixing angles- two large and one small ?

Origins of magic values

It is likely that these magic values are very close to the actual values. In this case

Lepton sector may need special symmetries

Alternatively, if the actual values are different from the magic values then

- This may be due to broken discrete symmetries
- This may be due to some dynamical mechanisms which generate close to magic values

PLAN

- Mechanisms for neutrino mass generation
- Symmetries of $\mathcal{M}_{\nu f}$
- Discrete flavour symmetries
- Dynamical mechanisms for generating neutrino mixing angles
- Generating neutrino mass hierarchies
- Ways to “predict” CP violating phase

Neutrino Mass Generation: Mechanisms

Different mechanisms aim at explaining

- Smallness of neutrino mass
- Origin of Lepton number violation if it is violated
- If not then origin of a Dirac neutrino with a mass at least six orders of magnitude smaller than the lightest charged fermion

Since the direct coupling of ν_L with itself is not possible one can generate it effectively through indirect coupling with

- Singlet fermion (Type-I seesaw)
- Triplet Higgs (Type-II seesaw)
- Triplet Fermion (Type-III seesaw)

Standard seesaw mechanism

Obtained by coupling ν_L with ν_R:

$$M_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix} \qquad \Rightarrow M_\nu \approx m_L - m_D M_R^{-1} m_D^T$$

In left right theories

$$m_L M_R \sim M_W^2$$

and largeness of M_R explains smallness of M_ν.

Type-III mechanism

Most well-motivated example of type-III mechanism is

Supersymmetry with R-parity violation

$$\nu_L \lambda < \tilde{\nu} >$$

Smallness of neutrino masses is linked to suppression of the sneutrino vev and one can get small neutrino masses even with weak seesaw scale
One can consider non-susy models with a triplet fermion.

- Light triplet fermions help gauge coupling unification
- By adding 24-plet of SU(5) one can obtain both type-I and III mechanisms together
- No specific reason for the lightness of fermions and no specific mechanism for suppression of the triplet fermion coupling

Variants of seesaw: Double seesaw, Inverse seesaw

Add more singlets S in addition to ν_R

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M \\ 0 & M & \mu \end{pmatrix}$$

- Lepton number is conserved in the limit $\mu = 0$. Hence smallness of neutrino mass linked to small μ [Schechter and Valle; Valle]
- allows low seesaw scale since

$$M_\nu \sim m_D M^{-1} \mu M^{T^{-1}} m_D^T$$

THE OBJECT OF MAIN INTEREST: $\mathcal{M}_{\nu f}$

- $\mathcal{M}_{\nu f}$ can be reconstructed from experiments since

$$\mathcal{M}_{\nu f} \equiv U^*_{PMNS} \text{Diag.}(m_1, m_2, m_3) U^\dagger_{PMNS}$$

- $\mathcal{M}_{\nu f}$ is known only partially at present. The gap can be filled by

- Educated guesses on its possible structure
- Studying symmetries of the partially known $\mathcal{M}_{\nu f}$
- Starting with a well-defined theoretical framework e.g. GUTs and use it to 'predict' complete $\mathcal{M}_{\nu f}$

Symmetries of $\mathcal{M}_{\nu f}$

$\mathcal{M}_{\nu f}$ is invariant under two types of symmetries:

For any given mixing patterns,$\mathcal{M}_{\nu f}$ is always invariant under a $Z_2 \times Z_2$ symmetry. This symmetry is a “kinemetical“ symmetry. [Lam; Grimus Lavoura]

$\mathcal{M}_{\nu f}$ can be invariant under more dynamical symmetries which can be used to restrict other properties like CP violation and neutrino mass hierarchies

$Z_2 \times Z_2$ symmetries

Let $|\psi_i>$ $(i = 1, 2, 3)$ be columns of the PMNS matrix. Then

$$\mathcal{M}_{\nu f} = m_i |\psi_i >< \psi_i|$$

Let

$$S_i = 2|\psi_i >< \psi_i| - 1 \quad ; S_i^2 = 1 \quad \text{and} \quad S_1 + S_2 + S_3 = 3$$

S_i define two independent Z_2 symmetries satisfying

$$S_i^T \mathcal{M}_{\nu f} S_i = \mathcal{M}_{\nu f}$$

Thus given any specific mixing matrix one can always construct $Z_2 \times Z_2$ symmetries which leave $\mathcal{M}_{\nu f}$ invariant.

Special $Z_2 \times Z_2$ symmetry

We know to a good approximation

$$\theta_{13} = 0, \theta_{23} = \frac{\pi}{4}$$

. This $\Rightarrow$

$$\psi_3 = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

is one of the column of the U_{PMNS}. This $\Rightarrow$

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

S_3 is the well-known $\mu - \tau$ symmetry.

We know that to a good approximation $\sin^2\theta_{12} = \frac{1}{3}$ also. This leads to another Z_2:

$$\psi_2 = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$$

corresponding to

$$S_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

and

$$S_2^T \mathcal{M}_{\nu f} S_2 = \mathcal{M}_{\nu f}$$

S_2, S_3 define special $Z_2 \times Z_2$ symmetry under which any successful $\mathcal{M}_{\nu f}$ should be invariant at least approximately.

From symmetry of $\mathcal{M}_{\nu f}$ to theory for $\mathcal{M}_{\nu f}$

It is possible that $Z_2 \times Z_2$ invariance of $\mathcal{M}_{\nu f}$ is

- an (approximate) symmetry imposed at the Lagrangian level
- a consequence of some more fundamental symmetry imposed on the theory
- may arise from some dynamical mechanism which leads to

$\theta_{23} \approx \frac{\pi}{4}, \theta_{13} \approx 0, \sin\theta_{12} \approx \frac{1}{\sqrt{3}}$.

Approximate μ-τ symmetry at the Lagrangian level

Exact μ-τ symmetry at the Lagrangian level implies $\theta_{23} = 0$

BUT

Even a tiny breaking of the symmetry can lead to large mixing angle thanks to the seesaw mechanism [ASJ]

$$M_l = \frac{m_\tau}{2} \begin{pmatrix} 1 & 1+\lambda_l \\ 1+\lambda_l & 1 \end{pmatrix} \quad m_D = \frac{m_{3D}}{2} \begin{pmatrix} 1-\epsilon_D & 1+\lambda_D \\ 1+\lambda_D & 1+\epsilon_D \end{pmatrix}$$

$$M_R = \frac{M_3}{2} \begin{pmatrix} 1 & 1+\lambda_R \\ 1+\lambda_R & 1 \end{pmatrix}$$

$$\lambda_f \approx 2\frac{m_{2f}}{m_{3f}} \quad f = l, D, R$$

and only source of the μ-τ breaking is

$$\epsilon_D \sim \frac{m_{2D}}{m_{3D}} \ll 1$$

. This implies after seesaw

$$M_\nu \approx \begin{pmatrix} B(1-\epsilon_\nu) & C \\ C & B(1+\epsilon_\nu) \end{pmatrix}$$

with

$$\epsilon_\nu \approx -\frac{2\epsilon_D\lambda_D}{\epsilon_D^2+\lambda_D^2} \approx 1$$

As a result, mixing is suppressed in M_ν; M_l leads to nearly maximal $\rightarrow \theta_{23} \approx \frac{\pi}{4}$

- The above example illustrates direct but approximate realization of the μ-τ symmetry at the Lagrangian level

- There are indirect ways to obtain exact μ-τ symmetry for $\mathcal{M}_{\nu f}$. Start with entirely different symmetry imposed on the Lagrangian namely D_4. Its judicious breaking then leads to the exact μ-τ symmetry [Grimus Lavoura]

ADDING ANOTHER Z_2: TRI-BIMAXIMAL MIXING

- Assume that the charged lepton mass matrix is invariant under a Z_3 symmetry

$$S_L^\dagger M_l S_R = M_l$$

defined by

$$e_{L,R} \rightarrow S_{L,R} e_{L,R}$$

with

$$S_L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad S_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad \omega^3 = 1$$

This implies

$$M_l = U D_l$$

,

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

- Also assume that

$$\nu_L \rightarrow S_{\nu L} \nu_L$$

with

$$S_{\nu L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- The Z_3 symmetry of ϵ_L and and Z_2 of ν_L do not commute but they are sub-groups of a group A_4 whose breaking leads to these discrete subgroups after the SM symmetry breaking and to the tri-bimaximal mixing.

DYNAMICAL MECHANISMS

We discuss dynamical mechanisms and models which lead to

- Large θ_{23} and θ_{12} but different from their magic values
- Non-zero and generically large θ_{13}

Many of them do not use flavour symmetry but finally lead to approximate $Z_2 \times Z_2$ symmetry!

θ_{13}

- Generic expectation relatively large $\theta_{13} \sim 0.05$ unless one has specific symmetry broken in a specific manner

- θ_{13} obtains contribution from two sources:

$$\sin\theta_{13} \sim s_{13\nu} - s_{12l}\sin\theta_A$$

In the absence of cancellations,

$$\sin\theta_{13} \sim \frac{1}{\sqrt{2}} s_{12l} \approx \mathcal{O}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{m_e}{m_\mu}}\right) \sim \mathcal{O}(0.05)$$

- Start with a specific texture

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2 & sc \\ 0 & sc & c^2 \end{pmatrix} \qquad \begin{pmatrix} 0 & c & s \\ c & 0 & 0 \\ s & 0 & 0 \end{pmatrix}$$

Perturbation$\Rightarrow$

$$\begin{pmatrix} 0 & 0 & \lambda \\ 0 & s^2 & sc \\ \lambda & sc & c^2 \end{pmatrix}$$

This leads to

$$|\theta_{13}| \approx \frac{\tan 2\theta_\odot}{2\tan\theta_A}\left(\frac{\Delta_\odot}{\Delta_A\cos 2\theta_\odot}\right)^{\frac{1}{2}} \approx 0.13$$

• The above structure gets realized in several scenarios,e.g. single RH nu dominance [King], $SO(10)$ model with type-II seesaw [Goh, Mohapatra and Ng]

Broken discrete symmetry versus dynamical mechanisms

Breaking of $Z_2 \times Z_2$ symmetry leads to non-zero θ_{13} and departures from magic values. Special perturbations exist which can distinguish broken discrete symmetries from the models which have small θ_{13} without specific flavour symmetries.

- One can consider arbitrary but small breaking of the μ-τ symmetry or the tri-bimaximal structure and study its consequences[Grimus *et al*; Mohapatra; Albright and Rodejohann]

Normal Hierarchy:

$$\sin\theta_{13} \approx c_{12}s_{12}\sqrt{\frac{\Delta_{sun}}{\Delta_A}} \approx 0.1(\epsilon + \epsilon'/2) \qquad \cos 2\theta_{23} \approx \epsilon'$$

Inverted hierarchy:

$$\sin\theta_{13} \approx \frac{1}{4}\sin 2\theta_{12}(\epsilon - \epsilon'/2) \approx 0.1(\epsilon - \epsilon'/2) \qquad \cos 2\theta_{23} \approx (\epsilon + \epsilon'/2)$$

Radiatively generated θ_{13}

In this case,

$$\epsilon = \epsilon'/2$$

This $\Rightarrow \theta_{13} = 0$ for inverted hierarchy at 1-loop level [Ray,Rodejohann and schmidt].
Very small contribution is generated at two loop and typical values $(1-5) \times 10^{-4}$ generated by the Planck scale effect [Berezensky, Narayan and Vissani] may be regarded as a hallmark of such schemes.

- Typical symmetries which lead to tri-bimaximal mixing also dictate the structure of its breaking by the non-renormalizable interactions. They lead in a specific instance [Altarelli and Feruglio] to

$$\sin^2\theta_{12} = \frac{1}{3} + \mathcal{O}(\epsilon) \quad ; \quad \sin^2\theta_{23} = \frac{1}{2} + \mathcal{O}(\epsilon) \quad ; \quad \sin\theta_{13} = \mathcal{O}(\epsilon)$$

ϵ is restricted to be ~ 0.04 implying relatively small θ_{13}.

DYNAMICAL MECHANISMS

- Seesaw mechanisms
- Grand Unified Theories
- Radiative enhancement
- Quasi Degeneracy of neutrinos

Grand Unified Theories

$SU(5)$ prediction $\qquad M_d = M_l^T \Rightarrow$

"lopsided texture"

$$M_l \approx \begin{pmatrix} 0 & 0 & C_1 \\ 0 & \epsilon & C_2 \\ \epsilon_1 & \epsilon_2 & 1 \end{pmatrix}$$

Minimal $SO(10)$ with fermions coupling to $10+\overline{126}$ of Higgs+type-II seesaw mechanism [Bajc,Senjanovic,Vissani; Aulakh; Garg and Aulakh]

$$M_\nu = M_d - M_l \approx \begin{pmatrix} m_s + m_b\lambda^2 - m_\mu & (m_b - m_s)\lambda \\ (m_b - m_s)\lambda & m_b - m_\tau \end{pmatrix}$$

$b - \tau$ unification leads to large mixing.

- This mechanism also explains why θ_{13} and $\frac{\Delta_{sol}}{\Delta_A}$ are small.
- Predicts θ_{13} near the present limit
- Requires intermediate scale and is uncomfortable with the gauge coupling unification
- Many variants with additional 120 and type-I seesaw mechanism are analyzed.

Large mixing through type-I seesaw

Minimal non-supersymmetric model with 10+126 Higgs leads to excellent fits for fermion masseses and mixing angles.
All fermionic mass matrices exhibit identical structure [K. M. Patel and ASJ]

$$M_f \approx \begin{pmatrix} c_{11}\lambda^4 & c_{12}\lambda^3 & c_{13}\lambda^2 \\ c_{12}\lambda^3 & c_{22}\lambda^2 & c_{23}\lambda \\ c_{13}\lambda^2 & c_{23}\lambda & c_{33} \end{pmatrix}$$

- $c_{ij} \sim \mathcal{O}(1)$, $\lambda \sim$ Cabibbo angle
- Nearly singular M_R after the seesaw mechanism leads to large solar and atmospheric mixing angles [Smirnov]
- One predicts large $\theta_{13} \sim 0.1$

Large mixing through quasi degeneracy

- Hierarchical in fermion masses → small mixing $\tan\theta = \sqrt{\frac{m_1}{m_2}}$
- Quasi degeneracy → implies large mixing

$$M_\nu = \begin{pmatrix} \epsilon_1 & 1 \\ 1 & \epsilon_2 \end{pmatrix} \qquad M_\nu = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Quasi Degeneracy also explains smallness of θ_{13}[Branco et al]

Quasi Degenerate structure

$$\mathcal{M}_{\nu f} = m_0 U = \text{ Symmetric unitary matrix}$$

An Example

$$M_l = U_{lL} D_l U_{lR} \quad \text{and '} M_\nu = m_0 I$$

. This leads to above $\mathcal{M}_{\nu f}$ with $U = U_{lL}^T U_{lL}$ $\mathcal{M}_{\nu f}$ is diagonalized by

$$V = U_{23}(\theta_{23}) U_{12}(\theta_{12})$$

leading to $\theta_{13} = 0$ and unconstrained θ_{12}, θ_{23} at the tree level. If perturbations which lift degeneracy do not change the mixing angles appreciably then correct mixing patterns are dictated by QDG. This happens in several schemes leading to QDG neutrinos in type-I seesaw model [Patel,Vempati, ASJ; Patel, ASJ]

Inverted hierarchy in type-I seesaw

Assume a symmetry [Rodejohann and ASJ]

$$\nu_L \rightarrow S\nu_L \qquad\qquad DetS \neq 1$$

This implies $m_D = Sm_D$; $S^T M_\nu S = M_\nu$ and $Det m_D = 0$
If in addition, $(0, s, c)^T$ is an eigenvector corresponding to zero eigenvalue then one gets inverted hierarchy. The most general matrix satisfying these requirements have scaling form [Mohapatra and Rodejohann]:

$$M_\nu = \begin{pmatrix} a & b & rb \\ b & c & rc \\ rb & rc & r^2 c \end{pmatrix}$$

Type-I seesaw model with quasi degeneracy

Type-I neutrino mass matrix is given by

$$M_\nu = m_D M_R^{-1} m_D^T$$

• Degeneracy follows if $m_D \sim M_R \sim I$ Examples are $O(3)$ or more economic A_4 symmetry [Ma and Rajasekaran; Babu Ma and Valle]

• More general way of obtaining quasi degeneracy follows from application of Minimal Flavour Violation hypothesis to leptonic sector. Specific symmetry of Yukawa interaction y_L, y_D when used to determine structure of M_R[Patel, Vempati and ASJ] implies

$$M_R \sim m_D^T m_D$$

and hence the degeneracy. Departures from degeneracy dictated by symmetry:

$$\mathcal{M}_{\nu f} \approx m_0(I - p y_l y_l^T)$$

• Realized through ideas of "Dirac Screening" [Schmidt, Lindner and Smirnov] and in specific $SO(10)$ model which provide a unified description of all fermion masses along with quasi degenerate neutrinos

Predicting CP violating phase δ

Generic expectation:

Large δ!

- Presence of CP violation in theory
- Smallness of θ_{13}

Models with vanishing θ_{13} and non-zero Majorana phases when perturbed by a small amount ϵ lead to large δ independent of the strength of perturbation. Examples:

- broken μ-τ symmetry

$$\tan\delta \approx \frac{m_1 m_2 \sin 2(\rho-\sigma) - m_3 m_1 \sin 2\rho + m_2 m_3 \sin 2\sigma}{m_1 c_{12}^2 - m_2^2 s_{12}^2 - m_1 m_2 \cos 2\theta_{12} \cos 2(\rho-\sigma) + m_3 m_1 \cos 2\rho - m_2 m_3 \cos 2\sigma}$$

• δ is large irrespective of the neutrino mass hierarchies if ρ, σ are large and not fine-tunned.

• Models with radiatively generated θ_{13} also lead to δ related to the original Majorana phases [Dighe *et al*; Rindani, Singh and ASJ]

Large δ from symmetry

Harrison-Scott structure:

$|U_{\alpha 2}| = |U_{\alpha 3}| \;\; \alpha = e, \mu, \tau$

- This leads to $\theta_{23} = \frac{\pi}{4}$ [Grimus and Lavoura]
- Follows as a consequence of generalized μ-τ symmetry

$$S_{\mu\tau}^T \mathcal{M}_{\nu f} S_{\mu\tau} = \mathcal{M}_{\nu f}^*$$

- $\Rightarrow s_{13} \cos\delta = 0 \qquad \delta = \frac{\pi}{2}$ if $\theta_{13} \neq 0$.

Realized in an example based on A_4 [Babu, Ma, Valle]

There exist other models/scenario which lead to correlation between θ_{13} and $\cos\delta$:

$$\sin\theta_{13}\cos\delta = C$$

C is a known constant. Examples

- Lopsided $SO(10)$: $C \approx \frac{1}{4}\sin\theta_{12}$ [Barr and Khan]
- $|U_{\mu 2}|^2 = |U_{\tau 2}|^2$ $C = \frac{c_{12}^2 - s_{12}^2 s_{13}^2}{\tan 2\theta_{23}\sin 2\theta_{12}}$

$\Delta(27)$ symmetry[Grimus ,Lavoura]

More examples in [(Albright, Dueck,Rodejohann]

- Different $SO(10)$ models lead to predictions of "best fit" values for δ,e.g.

$\delta \sim 227^0$ [Chen-Mohantappa]

$\delta \sim 54^0$ SO(10)+QDG neutrinos

and several other models......

SUMMARY:

• The presently available inmformation on neutrino mixing angles point to "magic values" of mixing angles

• They may arise from special leptonic symmetries

• Breaking of this symmetry may leave its traces:

Small $\theta_{13} \leq 0.02$ for normal or inverted hierarchy

Relatively large θ_{13} for quasi degenrate spectrum

• Mixing angles may arise due to underlying dynamics: Grand Unified theories and seesaw mechanism can lead to this dynamics

• Generic expectations in large class of models: θ_{13} near its present value and large $\tan\delta$