Four Zero Neutrino Yukawa Textures, Mu-Tau Symmetry and Leptogenesis

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B.Adhikary, A. Ghosal and PR-**JHEP**, 0910, 040, 2009 and arXiv:1009.2635

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Outline

- Background
- Present Scheme
- Laboratory Phenomenology
- 18 possibilities of baryogenesis via SUSY leptogenesis
- Discussion

Background

Magic of four zero Yukawa textures
Great phenomenological success in quark sector $\longrightarrow$ u-type, d-type mass matrices
Fritzsch and Xing; Prog.Part.Nucl.Phys., 45, 2001
Lepton Sector: common weak basis: real mass diagonal charged lepton l_α and heavy right chiral neutrino N_i. Light Neutrino mass matrix: see-saw

$$
\begin{aligned}
m_\nu &= -\frac{v_u^2}{2} Y_\nu \mathrm{diag}(M_1^{-1},\ M_2^{-1},\ M_3^{-1}) Y_\nu^T \\
&= U \mathrm{diag}(m_1,\ m_2,\ m_3) U^T
\end{aligned}
$$

where $v_u = v \sin\beta$ in MSSM, $v = 246$ GeV. m_ν is complex symmetric and we choose following parametrization of U

$$
U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -s_{13}e^{-i\delta_D} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_D} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_D} & -s_{23}c_{13} \\ -s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta_D} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta_D} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} e^{i\alpha_M} & 0 & 0 \\ 0 & e^{i\beta_M} & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

$\mu\tau$ symmetry,

$$(Y_\nu)_{12} = (Y_\nu)_{13},\ (Y_\nu)_{21} = (Y_\nu)_{31},\ (Y_\nu)_{23} = (Y_\nu)_{32},\ (Y_\nu)_{22} = (Y_\nu)_{33},$$
$$M_2 = M_3.$$

Custodial $\mu\tau$ symmetry in $m_\nu \Rightarrow (m_\nu)_{12} = (m_\nu)_{13},\quad (m_\nu)_{22} = (m_\nu)_{33}$

Automatic consequence: $\theta_{23} = \frac{\pi}{4},\ \ \theta_{13} = 0$

72 allowed four zero Yukawa textures: under$\mu\tau$ symmetry $\Rightarrow 4$ allowed ones

Two categories: A and B, one m_ν for each

Scheme

Category A

$$m_{DA}^{(1)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & b_1 & 0 \\ 0 & 0 & b_1 \end{pmatrix}, \qquad m_{DA}^{(2)} = \begin{pmatrix} a_1 & a_2 & a_2 \\ 0 & 0 & b_1 \\ 0 & b_1 & 0 \end{pmatrix}$$

Neutrino mass matrix:

$$m_{\nu A} = m_A \begin{pmatrix} k_1^2 e^{2i\bar{\alpha}} + 2k_2^2 & k_2 & k_2 \\ k_2 & 1 & 0 \\ k_2 & 0 & 1 \end{pmatrix}.$$

where

$$m_A = -\frac{b_1^2}{M_2}, \qquad k_1 = \sqrt{\frac{M_2}{M_1}} \times |\frac{a_1}{b_1}| \qquad k_2 = |\frac{a_2}{b_1}|$$

$$\bar{\alpha} = \arg\frac{a_1}{a_2}$$

$a_1,\ a_2,\ b_1$ are complex and $k_1,\ k_2,\ \bar{\alpha}$ are real

Only inverted mass ordering is allowed in Category A for data on R and $\tan 2\theta_{12}$

Category B

$$m_{DB}^{(1)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_1 & 0 & b_2 \end{pmatrix}, \qquad m_{DB}^{(2)} = \begin{pmatrix} a_1 & 0 & 0 \\ b_1 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

Neutrino mass matrix:

$$m_{\nu B} = m_B \begin{pmatrix} l_1^2 & l_1 l_2 e^{i\bar\beta} & l_1 l_2 e^{i\bar\beta} \\ l_1 l_2 e^{i\bar\beta} & l_2^2 e^{2i\bar\beta} + 1 & l_2^2 e^{2i\bar\beta} \\ l_1 l_2 e^{i\bar\beta} & l_2^2 e^{2i\bar\beta} & l_2^2 e^{2i\bar\beta} + 1 \end{pmatrix}.$$

where

$$m_B = -\frac{b_2^2}{M_2}, \qquad l_1 = |\frac{a_1}{b_2}| \times \sqrt{\frac{M_2}{M_1}} \qquad l_2 = |\frac{b_1}{b_2}| \times \frac{M_2}{M_1}$$

$$\bar\beta = \arg\frac{b_1}{b_2}$$

$a_1,\ b_1,\ b_2$ are complex and $l_1,\ l_2,\ \bar\beta$ are real

Only normal mass ordering is allowed in Category B, for data on R and $\tan 2\theta_{12}$

Parametric Functions for Category A

$$X_{1A} = 2\sqrt{2}k_2\sqrt{(2k_2^2+1)^2 + 2k_1^2(2k_2^2+1)\cos 2\bar{\alpha} + k_1^4}$$
$$X_{2A} = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\bar{\alpha}$$
$$X_A = \sqrt{X_{1A}^2 + X_{2A}^2}$$
$$X_{3A} = 1 - 4k_2^4 - k_1^4 - 4k_1^2k_2^2\cos 2\bar{\alpha} - 4k_2^2$$
$$X_{4A} = k_1^4 + 4k_2^4 + 4k_1^2k_2^2\cos 2\bar{\alpha}$$

and for Category B

$$X_{1B} = 2\sqrt{2}k_2\sqrt{(2l_2^2+l_1^2)^2 + 2(2l_2^2+l_1^2)\cos 2\bar{\beta} + 1}$$
$$X_{2B} = 4l_2^4 - l_1^4 + 4l_2^2\cos 2\bar{\beta} + 1$$
$$X_B = \sqrt{X_{1B}^2 + X_{2B}^2}$$
$$X_{3B} = 1 - (2l_2^2+l_1^2)^2 - 4l_2^2\cos 2\bar{\beta}$$
$$X_{4B} = l_1^4.$$

$$\tan 2\theta_{12} = \frac{X_1}{X_2} \qquad R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{2X}{X_3 - X}$$

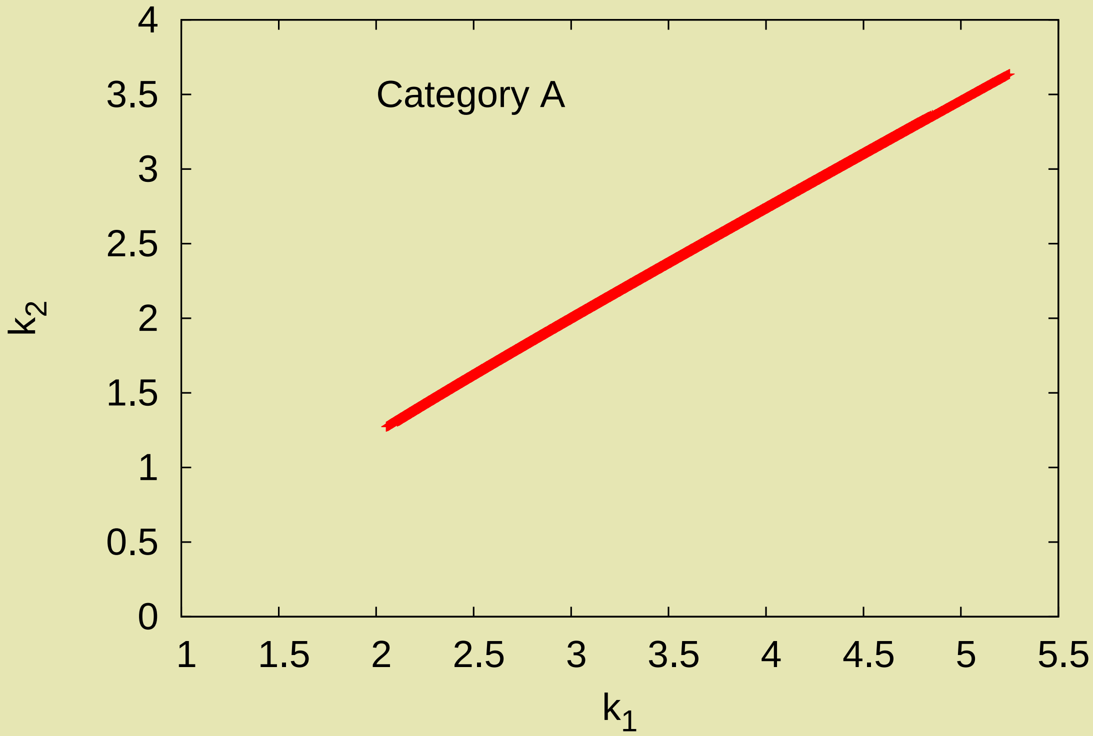
$$m_3 = |\Delta m_{21}^2 X^{-1}|^{1/2}, \quad m_{2,1} = |\Delta m_{21}^2 (X^{-1} - \frac{1}{2}X_3X^{-1} \pm \frac{1}{2})|^{1/2},$$

$$m_{\beta\beta} = |(m_\nu)_{ee}| = |\Delta m_{21}^2 X_4 X^{-1}|^{1/2}$$

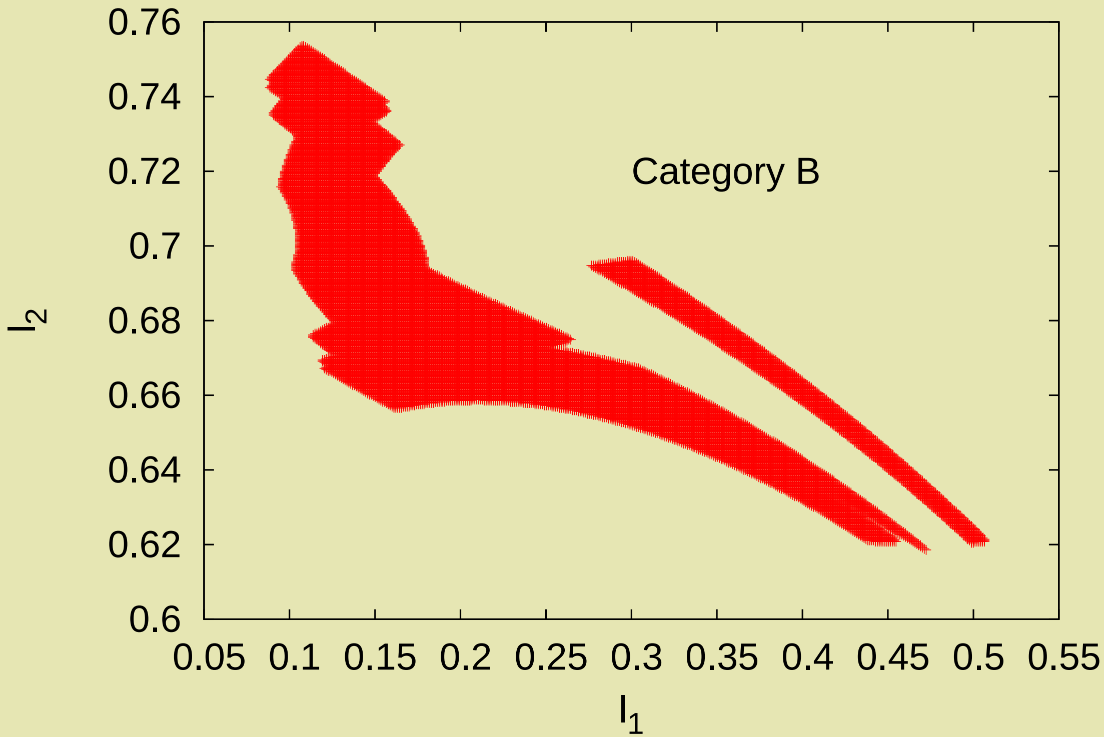
Input data (M.C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1008:117, 2010)

Quantity	Experimental 3σ range
θ_{12}	$31.5°$ to $37.6°$
$R_{inverted}$	-4.13×10^{-2} to -2.53×10^{-2}
R_{normal}	2.46×10^{-2} to 3.92×10^{-2}
Δm^2_{21}	$6.90 \times 10^{-5}\ eV^2$ to $8.20 \times 10^{-5}\ eV^2$
Δm^2_{32} (inverted)	$-2.73 \times 10^{-3}\ eV^2$ to $-1.99 \times 10^{-3}\ eV^2$
Δm^2_{32} (normal)	$2.09 \times 10^{-3}\ eV^2$ to $2.83 \times 10^{-3}\ eV^2$

Allowed k_1 and k_2 parameter space with allowed $0 \leq \cos\bar{\alpha} \leq 0.0175$ for the 3σ experimental ranges of R and θ_{12}



Allowed l_1 and l_2 parameter space with allowed $0 \leq \cos\bar{\beta} \leq 0.0523$ for the 3σ experimental ranges of R and θ_{12}



Mass ranges in our scheme

Category A	Category B
$\Sigma m_i = 0.092$ to 1.35 eV	$\Sigma m_i = 0.052$ to 0.123 eV
$m_{\beta\beta} = 0.005$ to 0.450 eV	$m_{\beta\beta} = 0.0003$ to 0.0156 eV

Latest cosmological bound $\Sigma m_i < 0.28$ eV removes a large part of the allowed parameter space of Category A

S. A. Thomas, F. B. Abdalla, O. Lahav, PRL, 105, 031301, (2010)

Possibilities of baryogenesis via SUSY leptogenesis

Lepton asymmetry generarted from $\Gamma(N_i \rightarrow \phi\bar{l}_\alpha) - \Gamma(N_i \rightarrow \phi^\dagger l_\alpha)$ at scale M_{lowest}=min$(M_1,\ M_2,\ M_3)$.

3 possible mass hierarchical cases for N_i:
(1) NHN: $M_{lowest} = M_1 \ll M_2 = M_3$, (2) IHN: $M_{lowest} = M_2 = M_3 \ll M_1$, (3) QDN: $M_{lowest} \simeq M_1 \simeq M_2 = M_3$,

3 possible regimes:
(1) Unflavored leptogenesis:$M_{lowest}(1+\tan^2\beta)^{-1} > 10^{12}$ GeV, (2) Fully flavored leptogenesis:$M_{lowest}(1+\tan^2\beta)^{-1} < 10^9$ GeV, (3) τ flavored leptogenesis:10^9 GeV$<$ $M_{lowest}(1+\tan^2\beta)^{-1} < 10^{12}$ GeV,
Two Categories: **A** and **B**. Thus $3 \times 3 \times 2 = 18$ different possibilities
Sphaleron conversion $\longrightarrow$

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -2.45\frac{n_l - n_{\bar{l}}}{n_\gamma}.$$

Standard calculation using flavor dependent Boltzman equations.

Results: $\bar{\alpha}$, $\bar{\beta}$ in fourth quadrant (< 0) for NHN and QDN cases and in the first quadrant ($0 >$) for IHN in all regimes.

Unflavored leptogenesis

	Category A			Category B		
	NHN	IHN	QDN	NHN	IHN	QDN
$\bar{\alpha}, \bar{\beta}$	$\bar{\alpha} < 0$ 89.9^o	$\bar{\alpha} > 0$ 89.99^o	$\bar{\alpha} < 0$ $89.1^o - 89.9^o$	$\bar{\beta} < 0$ $89.1^o - 89.9^o$	$\bar{\beta} > 0$ $89.0^o - 89.9^o$	$\bar{\beta} < 0$ $89.0^o - 89.9^o$
x	$10 - 10^3$	$0.01 - 0.1$	$2.0 - 9.1$	$10 - 10^3$	$0.1 - 0.9$	$6.08 - 9.10$
$\tan\beta$	$2 - 60$	$2 - 5$	$2 - 60$	$2 - 8$	Not Allowed	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	1.69×10^4 — 7.62×10^6	2.60×10^4 — 5.0×10^3	4.60×10^3 — 3.60×10^6	4.88×10^3 — 8.5×10^4	Not Allowed	3.60×10^3 — 6.80×10^3

Fully flavored leptogenesis

	Category A			Category B		
	NHN	IHN	QDN	NHN	IHN	QDN
$\bar{\alpha}, \bar{\beta}$	$\bar{\alpha} < 0$ $89.4^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.8^o$	$\bar{\alpha} < 0$ $89.9^o - 89.9^o$	$\bar{\beta} < 0$ $89.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ 89.9^o
x	$10 - 10^3$	$0.01 - 0.1$	$1.1 - 10$	$10 - 10^3$	$0.01 - 0.1$	$1.1 - 10$
$\tan\beta$	$25 - 60$	$22 - 60$	$2 - 60$	$28 - 39$	$24 - 31$	$34 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	6.70×10^1 — 3.6×10^3	4.90×10^2 — 3.6×10^3	2.30×10^1 — 3.60×10^3	7.80×10^1 — 1.55×10^2	5.70×10^2 — 9.6×10^2	1.16×10^3 — 3.60×10^3

τ flavored leptogenesis

	Category A			Category B		
	NHN	IHN	QDN	NHN	IHN	QDN
$\bar{\alpha}, \bar{\beta}$	$\bar{\alpha} < 0$ $89^o - 89.9^o$	$\bar{\alpha} > 0$ $89.0^o - 89.9^o$	$\bar{\alpha} < 0$ $89.0^o - 89.9^o$	$\bar{\beta} < 0$ $89.0^o - 89.9^o$	$\bar{\beta} > 0$ $87.0^o - 89.9^o$	$\bar{\beta} < 0$ 89.9^o
x	$10 - 10^3$	$0.01 - 0.1$	$1.1 - 10$	$10 - 10^3$	$0.01 - 0.1$	$1.1 - 2$
$\tan\beta$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$	$2 - 60$
$\frac{M_{lowest}}{10^9 GeV}$	1.70×10^3 — 4.0×10^4	4.0×10^2 — 1.03×10^4	1.0×10^2 — 1.97×10^4	8.4×10^2 — 2.30×10^4	1.94×10^3 — 2.30×10^4	5.0×10^2 — 5.0×10^3

Discussion

Have considered radiative breaking of $\mu\tau$ symmetry. Assume $\mu\tau$ symmetry at $\Lambda \sim 10^{12}$ GeV (highest M_i) One loop RG running from Λ to $\lambda \sim 10^3$ **GeV** will break it.

In MSSM using $m_\tau^2 \gg m_{e,\mu}^2$ deviation Δ_τ:

$$\Delta_\tau \simeq \frac{m_\tau^2}{8\pi^2 v^2}(\tan^2\beta + 1)\ln\left(\frac{\Lambda}{\lambda}\right)$$

Now

$$m_\nu^\lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\Delta_\tau) \end{pmatrix} m_\nu^\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (1-\Delta_\tau) \end{pmatrix}$$

Have worked out $m_{1,2,3}$, $\Sigma\, m_i$ and $m_{\beta\beta}$ as well as η. Their changes are marginal.

Results for 3σ variations of R and θ_{12}:

Category A

- $0^\circ \le \theta^\lambda_{13} \le 2.7^\circ$
- $\theta^\lambda_{23} \le 45^\circ$
- Inverted mass ordering retained

Category B

- $0^\circ \le \theta^\lambda_{13} \le 0.85^\circ$
- $\theta^\lambda_{23} \ge 45^\circ$
- Normal mass ordering retained

But measurable low energy CP violation

$$J_{CP} \simeq -\frac{8\Delta_\tau W}{X(X_3^2 - X^2)}$$

where

$$W_A = k_1^2 k_2^4 \sin 2\bar{\alpha} \qquad W_B = l_1^2 l_2^4 \sin 2\bar{\beta}$$

$$J_{CP}^A \simeq 2.0 \times 10^{-3} \qquad J_{CP}^B \simeq 3.0 \times 10^{-4}$$

The same phase $\bar{\alpha}$, $\bar{\beta}$ contributes to J_{CP}, $0\nu\beta\beta$ decay and leptogenesis