A new idea to search for charged lepton flavor violation using a muonic atom

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Abstract. We propose a new process of $\mu^- e^- \to e^- e^-$ in a muonic atom for a quest of charged lepton flavor violation. This is based on the work[1]

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Charged lepton flavor violation (cLFV) is known to be one of the important rare processes to search for new physics beyond the Standard Model (SM). Various theoretical models predict sizable rates of cLFV processes, which are just below the present experimental upper limits. The on-going and future experiments for cLFV searches would reach sensitivities in the range of predictions by many theoretical models. At this moment, the cLFV searches with muons present the best limits owing to a large number of muons available for measurements [2]. Typical cLFV processes with muons include $\mu^+ \to e^+\gamma$, $\mu^+ \to e^+e^+e^-$ and μ^- - e^- conversion in a muonic atom ($\mu^- N \to e^- N$). However, after the discovery of cLFV process in future, many other different cLFV processes should be studied to shed lights upon understanding of the nature of the cLFV interactions and develop insights into new physics responsible for cLFV.

In this talk, we would like to propose a new cLFV reaction process of a bound μ^- in a muonic atom, which is

$$\mu^- e^- \to e^- e^-, \tag{1}$$

where μ^- and e^- in the initial state of Eq.(1) are the muon and the atomic 1S electron(s) bound in a Coulomb field of the nucleus in a muonic atom respectively.

This $\mu^- e^- \to e^- e^-$ process in a muonic atom has various significant advantages. First of all, this process could have not only photonic dipole interaction but also four-Fermi contact interaction, as in the processes of $\mu^+ \to e^+e^-e^-$ and $\mu^- N \to e^- N$, whereas $\mu^+ \to e^+\gamma$ has only the former. This would potentially allows us to investigate the full structure of new physics beyond the SM. Secondly, this process has a two-body final state, in which the two signal electrons are emitted almost back to back and each of them has an energy of about a half the muon mass, $m_\mu/2$. This would provide a cleaner experimental signature as well as a larger final-state phase space than $\mu^+ \to e^+e^+e^-$ decay. Also, in comparison with the $\mu^+ \to e^+\gamma$ search, the measurement of this process would be relatively easier since no photon detection is involved. Thirdly, one can consider a similar reaction process with a muonium, such as $\mu^+ e^- \to e^+ e^-$. However, the rate of this $\mu^+ e^- \to e^+ e^-$ process can not be large because of small overlap between the μ^+ and e^- wave functions.

We describe the process of $\mu^- e^- \to e^- e^-$ in a muonic atom of Eq. (1) by an effective Lagrangian at the energy scale of the muon mass m_μ. Following Ref. [2], we define

$$\begin{aligned}
\mathscr{L}_{\mu^- e^- \to e^- e^-} = &-\frac{4G_F}{\sqrt{2}}\Big[m_\mu A_R\,\overline{\mu_R}\sigma^{\mu\nu}e_L F_{\mu\nu} \\
&+ m_\mu A_L\,\overline{\mu_L}\sigma^{\mu\nu}e_R F_{\mu\nu} \\
&+ g_1(\overline{\mu_R}e_L)(\overline{e_R}e_L) + g_2(\overline{\mu_L}e_R)(\overline{e_L}e_R) \\
&+ g_3(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_R}\gamma_\mu e_R) + g_4(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_L}\gamma_\mu e_L) \\
&+ g_5(\overline{\mu_R}\gamma^\mu e_R)(\overline{e_L}\gamma_\mu e_L) + g_6(\overline{\mu_L}\gamma^\mu e_L)(\overline{e_R}\gamma_\mu e_R) \\
&+ (\mathrm{H.c.})\Big]\,.
\end{aligned} \tag{2}$$

We estimate the branching ratio for the process of Eq. (1), which takes place in a muonic atom with an atomic number Z. The initial state has the muon and the electron in their 1S ground state of the atomic orbits. For simplicity, we ignore the three-momenta of the initial bound muon and electron. The final state has the two electrons, which, in the lowest order, can be treated as monochromatic plane waves that propagate with opposite momentum vectors. Each of the two electrons in the final state takes energy of about $m_\mu/2$, when the bound effects at the 1S state and the Coulomb interaction from the nucleus can be neglected.

We begin with the first case where the four-Fermi interaction is dominant and A_R and A_L are negligibly small, making no contribution of the photonic interactions. The remaining four-Fermi interaction allows the processes such as $\mu^- e^- \to e^- e^-$ and $\mu^+ \to e^+ e^+ e^-$. The cross section of the process of Eq. (1) is calculated to be

$$\sigma v_{\rm rel} = \frac{1}{m_\mu^2} \frac{(G_{\rm F}^2 m_\mu^2)^2}{16\pi} G, \tag{3}$$

where $G \equiv G_{12} + 16G_{34} + 4G_{56} + 8G'_{14} + 8G'_{23} - 8G'_{56}$ with $G_{ij} \equiv |g_i|^2 + |g_j|^2$ and $G'_{ij} \equiv {\rm Re}\,(g_i^* g_j)$. The transition rate is then given by

$$\begin{aligned} \Gamma(\mu^- e^- \to e^- e^-) &= 2\sigma v_{\rm rel} \left| \psi_{\rm 1S}^{(e)}(0; Z-1) \right|^2 \\ &= m_\mu \frac{1}{8\pi} (Z-1)^3 \alpha^3 (G_{\rm F}^2 m_\mu^2)^2 \left(\frac{m_{\rm e}}{m_\mu} \right)^3 G. \end{aligned} \tag{4}$$

Here we took into account the facts that the 1S state can accommodate two electrons, and that the nuclear charge is shielded by the negative muon. We used the non-relativistic wave functions so that $\psi_{\rm 1S}^{(e)}(0; Z-1) = [(Z-1)\alpha m_{\rm e}]^{3/2}/\sqrt{\pi}$. The rate of Eq. (4) is enhanced for a larger atomic number, Z, by a factor of $(Z-1)^3$, giving a notable advantage for heavy nuclei. This enhancement comes from the factor of $|\psi_{\rm 1S}^{(e)}(0; Z-1)|^2$, and the large positive charge of a heavy nucleus strongly attracts the 1S wave functions of the leptons toward the nucleus position, rendering the overlap of the two wave functions large, and enhances the transition of the process of Eq. (1). We normalize the rate of Eq. (4) by the lifetime of a muonic atom, $\tilde{\tau}_\mu$, to define the branching ratio of this process as

$$\begin{aligned} {\rm Br}(\mu^- e^- \to e^- e^-) &\equiv \tilde{\tau}_\mu \Gamma(\mu^- e^- \to e^- e^-) \\ &= 24\pi (Z-1)^3 \alpha^3 \left(\frac{m_{\rm e}}{m_\mu} \right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} G, \end{aligned} \tag{5}$$

The value of $\tilde{\tau}_\mu$ ranges from $\tilde{\tau}_\mu = 2.19 \times 10^{-6}\,{\rm s}$ for ^{1}H to $\tilde{\tau}_\mu = (7-8) \times 10^{-8}\,{\rm s}$ for ^{238}U as listed in Ref. [3]. This is shorter than the lifetime of free muons, $\tau_\mu = 2.197 \times 10^{-6}\,{\rm s}$ [4], which is equal to $192\pi^3/(G_{\rm F}^2 m_\mu^5)$ at the lowest order. The obtained branching ratio of Eq. (5) is to be compared with that of $\mu^+ \to e^+ e^+ e^-$. This branching ratio is given by [5]

$${\rm Br}(\mu^+ \to e^+ e^+ e^-) = \frac{1}{8} (G_{12} + 16G_{34} + 8G_{56}). \tag{6}$$

The contribution from the interference among the four-Fermi interactions are not present in Eq.(6), whereas it is found to be present in Eq. (5) as the terms of G'_{ij}'s. The search for the process of Eq. (1) will thereby serve complementarily with that for $\mu^+ \to e^+ e^+ e^-$. By assuming $G/(G_{12} + 16G_{34} + 8G_{56}) \sim O(1)$ and denoting ${\rm Br}(\mu^+ \to e^+ e^+ e^-) < B_{\rm max}$ as

$$\begin{aligned} &{\rm Br}(\mu^- e^- \to e^- e^-) \\ &\qquad < 192\pi (Z-1)^3 \alpha^3 \left(\frac{m_{\rm e}}{m_\mu} \right)^3 \frac{\tilde{\tau}_\mu}{\tau_\mu} B_{\rm max}. \end{aligned} \tag{7}$$

Figure 1 shows these upper limits as a function of an atomic number Z by the dotted curves. These upper limits are obtained by taking the present experimental limit of $B_{\rm max} = 1.0 \times 10^{-12}$ from the SINDRUM experiment [6]. The light-shaded region in Fig. 1 is excluded by the current SINDRUM limit. The reciprocal of the shown limit gives an estimation of the number of muons that is required to detect events of the process of Eq. (1). Let us take an example of the gold atom ($Z = 79$): our estimation requires a collection of $(4.21 \times 10^{-19})^{-1} = 2.38 \times 10^{18}$ muon events to improve the current limit of $B_{\rm max} = 1.0 \times 10^{-12}$. Assuming the detection efficiency of $O(10\%)$, the required number of muons amounts to a few times 10^{19}. Compared to this number is $O(10^{18}\text{–}10^{19})$ of muons, which is the goal of the highly intense muon beams planned in the near future in search for the cLFV [7, 8, 9]. We thereby find that the current limit could be reachable within the capability of these muon sources.

Let us now turn to the second case where the photonic interaction is present and it dominates over the four-Fermi interactions. The cross section, rate, and branching ratio of the process of Eq. (1) are calculated to be

$$\sigma v_{\rm rel} = \frac{4\alpha (G_{\rm F} m_\mu^2)^2}{m_{\rm e}^2} \left(|A_{\rm L}|^2 + |A_{\rm R}|^2 \right), \tag{8}$$

$$\begin{aligned} \Gamma(\mu^- e^- \to e^- e^-) &= 2\sigma v_{\rm rel} \left| \psi_{\rm 1S}^{(e)}(0; Z-1) \right|^2 \\ &= m_{\rm e} \frac{8}{\pi} (Z-1)^3 \alpha^4 (G_{\rm F} m_\mu^2)^2 \left(|A_{\rm R}|^2 + |A_{\rm L}|^2 \right), \end{aligned} \tag{9}$$

and

$$\begin{aligned} &{\rm Br}(\mu^- e^- \to e^- e^-) \\ &\quad = 1536\pi^2 (Z-1)^3 \alpha^4 \left(|A_{\rm R}|^2 + |A_{\rm L}|^2 \right) \frac{m_{\rm e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu}, \end{aligned} \tag{10}$$

respectively. We then have , combining with the upper limit ${\rm Br}(\mu^+ \to e^+ e^+ e^-) < B_{\rm max}$

$$\begin{aligned} &{\rm Br}(\mu^- e^- \to e^- e^-) \\ &\quad < 12\pi (Z-1)^3 \alpha^3 \frac{m_{\rm e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu} \left[\log \left(\frac{m_\mu}{m_{\rm e}} \right)^2 - \frac{11}{4} \right]^{-1} B_{\rm max}. \end{aligned} \tag{11}$$

This upper limit is overlaid in Fig. 1 by a dash-dotted curve, according to the aforementioned SINDRUM limit of $B_{\rm max} = 1.0 \times 10^{-12}$.

The presence of the photonic interactions gives rise to another cLFV process $\mu^+ \to e^+ \gamma$ as well, and search for this process also put a limit to ${\rm Br}(\mu^- e^- \to e^- e^-)$. Then the limit on ${\rm Br}(\mu^- e^- \to e^- e^-)$ is estimated from ${\rm Br}(\mu^+ \to e^+ \gamma) < B_{\rm max}$ as

$$\begin{aligned} {\rm Br}(\mu^- e^- \to e^- e^-) &< \frac{{\rm Br}(\mu^- e^- \to e^- e^-)}{{\rm Br}(\mu^+ \to e^+ \gamma)} B_{\rm max} \\ &= 4(Z-1)^3 \alpha^4 \frac{m_{\rm e}}{m_\mu} \frac{\tilde{\tau}_\mu}{\tau_\mu} B_{\rm max}. \end{aligned} \tag{12}$$

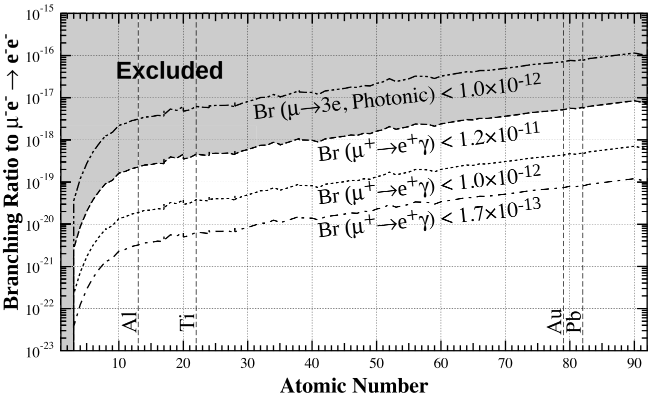

FIGURE 1. Limits on the branching ratio to the process of Eq. (1) imposed by the limits on $\text{Br}(\mu^+ \to e^+\gamma)$ and on $\text{Br}(\mu^+ \to e^+e^+e^-)$. Models without photonic interactions are excluded in the light-shaded region. Similarly, models with photonic interactions are excluded in the dark-shaded region by the current experimental limits.

A solid curve in Fig. 1 presents the upper limits given in Eq.(12) with $B_{\text{max}} = 1.2 \times 10^{-11}$, which is the current upper limit from the MEGA experiment[10]. A dashed curve in Figure 1 also shows the limits with the $B_{\text{max}} = 1.7 \times 10^{-13}$, which is the goal value of the MEG experiment [11]. Even the current limit on the branching ratio to $\mu^+ \to e^+\gamma$ overwhelms the limits on $\text{Br}(\mu^-e^- \to e^-e^-)$ when the photonic interaction is dominant. Accordingly, the present excluded region is above the current MEGA limit. Let us estimate, as we did earlier, the required number of muons to detect events of the process of Eq. (1), taking an example of the gold atom ($Z = 79$): an accumulation of $(1.044 \times 10^{-17})^{-1} = 9.58 \times 10^{16}$ muon events is necessary to surpass the sensitivity of MEGA, and $(1.480 \times 10^{-19})^{-1} = 6.76 \times 10^{18}$ to exceed that of the MEG goal. The required number of muons are estimated to be 10^{18} and a few of 10^{19}, respectively, assuming the $O(10\%)$ of the detection efficiency again. These are not capable now but would be possible in future with the planned highly intense muon sources.

The expected magnitudes of the branching ratio of the process of Eq. (1), which is driven by both the four-Fermi interaction and the photonic interaction, are found to be not significantly large even with the enhancement of $(Z-1)^3$. Thus, this would not be the first process to cultivate the discovery frontier of the cLFV searches. However, thanks to the enhancement of $(Z-1)^3$, this process can be accessible in future by next-generation high-intensity muon beams to produce muons of an order of $O(10^{18}\text{–}10^{19})$ per year.

In summary, the new cLFV process $\mu^-e^- \to e^-e^-$ in a muonic atom is proposed. This process has the rate enhancement of $(Z-1)^3$ over the $\mu^+e^- \to e^+e^-$ owing to the Coulomb interaction from the nucleus in a muonic atom. This process has a final state of two electrons, which would be experimentally very clear signature. The upper limits of the branching ratio of the orders of $O(10^{-17}\text{–}10^{-18})$ are estimated separately for the photonic and the four Fermi interactions from the other cLFV experimental results. Once this process is observed, CP violation might be studied by comparing this process with $\mu^+ \to e^+e^+e^-$.

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