Nuclear effects in neutrino oscillation experiments

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Abstract. We have studied the nuclear medium effects in the neutrino(antineutrino) induced interactions in nuclei which are relevant for present neutrino oscillation experiments in the few GeV energy region. The study is specially focused on calculating the cross sections and the event rates for atmospheric and accelerator neutrino experiments. The nuclear effects are found to be important for the quasielastic lepton production and the charged current incoherent & coherent pion production processes.

Introduction

The phenomenon of neutrino oscillation in few GeV energy region was first seen in experiments with atmospheric neutrinos. The physics of the neutrino oscillation is expected to be understood with the help of neutrino experiments being done with atmospheric and accelerator neutrinos. Analysis of these experiments requires a good knowledge of various neutrino nuclear reactions in this energy region where the quasielastic reactions producing leptons as well as inelastic reactions producing leptons and pions are the most important processes. The MiniBooNE collaboration has reported the results for the ratio R(E)=$\sigma^{CC1\pi^+}/\sigma^{CCQE}$ as a function of neutrino energy[1] and the total charged current quasielastic(CCQE) cross section [2] with and without charged current quasilike(CCQE-like) contributions coming from the inelastic channel. These measurements are meant to improve the limit on the $CC1\pi^+$ production cross section as well as to put a better meaurement of the axial dipole mass M_A. These experiments have been performed using nuclear targets where nuclear effects play an important role in the energy region of a few GeV. The Super-Kamiokande collaboration is also performing an atmospheric neutrino experiment and analyzes the events reported in the Ref.[3]. In this paper, we have used the results for the CCQE and the Charged Current Inelastic(Incoherent+Coherent) processes from the study made earlier and reported in Refs.[4]-[8] and compared them with the data reported by the MiniBooNE collaboration [1]-[2] and also with the experimental data of the atmospheric neutrino experiment[3] performed by SuperK collaboration.

Quasielastic Reaction

The cross section for quasielastic charged lepton production is calculated in the local density approximation[5]-[6]. Inside the nucleus there are nuclear effects like Pauli blocking, Fermi motion, renormalization of the weak transition strength etc. that play an important role in neutrino interactions with a nucleon inside a nuclear target. The effects of Fermi motion and Pauli blocking are taken into account through the imaginary part of the Lindhard function for the particle hole excitations in the nuclear medium. The renormalization of the weak transition strengths are calculated in the random phase approximation (RPA) through the interaction of the p-h excitations as they propagate in the nuclear medium using a nucleon-nucleon potential described by pion and rho exchanges. The effect of the Coulomb distortion on the outgoing lepton has been taken into account in a modified effective momentum approximation(MEMA).

The total cross section $\sigma(E_\nu)$ for the charged current neutrino induced reaction on a nucleon inside the nucleus in a local Fermi gas model is written as [5]:

$$
\begin{aligned}
\sigma(E_\nu) \; = \; & -2G_F{}^2\cos^2\theta_c \int_{r_{min}}^{r_{max}} r^2 dr \int_{k'_{min}}^{k'_{max}} k'dk' \int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \\
& \times \; \frac{1}{E^2_{\nu_l}E_l} L_{\mu\nu}J^{\mu\nu} Im U_N(E_{\nu_l}-E_l-Q_r-V_c(r),\mathbf{q})
\end{aligned}
$$

(1)

where the leptonic tensor $L_{\mu\nu} = \sum L_\mu L_\nu{}^\dagger$ and the hadronic tensor $J^{\mu\nu} = \bar\sum\sum J^\mu J^{\nu\dagger}$. The leptonic current L_μ and the hadronic current J^μ are given by

$$
L_\mu = \bar{u}(k')\gamma_\mu(1-\gamma_5)u(k) \tag{2}
$$

$$
\begin{aligned}
J^\mu \; = \; & \bar{u}(p')[F_1(Q^2)\gamma^\mu + F_2(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M} + F_A(Q^2)\gamma^\mu\gamma_5 \\
& + \; F_P(Q^2)q^\mu\gamma_5]u(p).
\end{aligned} \tag{3}
$$

where q is the four momentum transfer and $Q^2 = -q^2$, M is the nucleon mass, G_F is the Fermi coupling constant. U_N is the Lindhard function for the particle hole

excitation [6]. The form factors F_1, F_2 are isovector electroweak form factors taken from the parametrization of Bradford et al. [9] with vector dipole mass M_V=0.84 GeV. The isovector axial form factor is taken as $F_A(q^2) = 1/(1+\frac{Q^2}{M_A^2})^2$ and the pseudoscalar form factor $F_p^V(Q^2)$ is given in terms of $F_A^V(Q^2)$ using Goldberger-Treiman relation. Inside the nucleus, the Q-value of the reaction and Coulomb distortion of outgoing lepton are taken into account by modifying the imaginary part of the Lindhard function $ImU_N(q_0,\mathbf{q})$ by $ImU_N(q_0-V_c(r)-Q_r,\mathbf{q})$. Furthermore, the renormalization of the weak transition strength in the nuclear medium in a random phase approximation (RPA) is taken into account by considering the propagation of particle hole(ph) as well as delta-hole(Δh) excitations. These considerations lead to a modified hadronic tensor $J_{RPA}^{\mu\nu}$, the expression for which is given in Ref. [6].

Inelastic Reaction

The cross sections for pion production are calculated using the delta dominance model [4],[5]. In this model, the weak hadronic currents interacting with the nucleons in the nuclear medium excite a Δ resonance which decays into pions and nucleons. The nuclear medium effects on the Δ properties lead to modification in its mass and width which have been discussed earlier by Oset et al. [10].

In the local density approximation the expression for the total cross section for the charged current one pion production is given by [7]

$$
\begin{aligned}
\sigma &= \frac{1}{(4\pi)^5} \int_{r_{min}}^{r_{max}} (\rho_p(r) + \frac{1}{9}\rho_n(r)) d\vec{r} \int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \\
&\times \int_0^\infty dk' \int_{-1}^{+1} d(cos\theta_\pi) \int_0^{2\pi} d\phi_\pi \frac{\pi|\vec{k}'||\vec{k}_\pi|}{ME_\nu^2 E_l} \\
&\times \frac{1}{E_p' + E_\pi\left(1 - \frac{|\vec{q}|}{|\vec{k}_\pi|}cos(\theta_\pi)\right)} \bar{\Sigma}\Sigma|\mathcal{M}_{fi}|^2
\end{aligned}
\tag{4}
$$

where the proton and neutron densities are given in terms of nuclear density $\rho(r)$[11].

The transition matrix element $\mathcal{M}_{fi}$ for incoherent process is given by

$$
\mathcal{M}_{fi} = \sqrt{3}\frac{G_F cos\theta_c}{\sqrt{2}} \frac{f_{\pi N\Delta}}{m_\pi} \bar{\Psi}(\mathbf{P}) k_\pi^\sigma \mathcal{P}_{\sigma\lambda} \mathcal{O}^{\lambda\alpha} L_\alpha u(\mathbf{p})
\tag{5}
$$

and for coherent process it is written as

$$
\begin{aligned}
\mathcal{M}_{fi} &= \frac{G_F}{\sqrt{2}} cos\theta_c l^\mu \sqrt{3}\frac{f_{\pi N\Delta}}{m_\pi} \sum_{r,s} \bar{u}_s(p) k_{\pi\sigma} \mathcal{P}^{\sigma\lambda} \mathcal{O}_{\lambda\mu} \\
&\times u_r(p)\mathscr{F}(\vec{q}-\vec{k}_\pi)
\end{aligned}
\tag{6}
$$

where L_α is the leptonic current and $\mathcal{O}^{\beta\alpha} = \mathcal{O}_V^{\beta\alpha} + \mathcal{O}_A^{\beta\alpha}$. $\mathcal{O}_V^{\beta\alpha}$ and $\mathcal{O}_A^{\beta\alpha}$ are the vector and axial vector N-Δ transition operators[7]. θ_W is the weak mixing angle.

$\mathscr{P}^{\sigma\lambda}$ is the Δ propagator in momentum space [7] and the delta decay width Γ is taken to be an energy dependent P-wave decay width [10]. $\mathscr{F}(\vec{q}-\vec{k}_\pi)$ is the nuclear form factor, which is calculated in Eikonal approximation to be [8]:

$$
\begin{aligned}
\tilde{\mathscr{F}}(\vec{q}-\vec{k}_\pi) &= 2\pi \int_0^\infty b\, db \int_{-\infty}^\infty dz\, \rho(\vec{b},z)\, J_0(k_\pi^t b) \\
&\times e^{i(|\vec{q}|-k_\pi^l)z} e^{-if(\vec{b},z)}
\end{aligned}
\tag{7}
$$

where $f(\vec{b},z) = \int_z^\infty \frac{1}{2|\vec{k}_\pi|}\Pi(\rho(\vec{b},z'))dz'$, Π is the self energy of the pion.

Inside the nuclear medium the mass and width of the delta are modified which in the present calculation are taken into account by using a modified mass $M_\Delta \rightarrow M_\Delta + Re\Sigma_\Delta$ and modified width $\Gamma_\Delta \rightarrow \tilde{\Gamma}_\Delta - 2Im\Sigma_\Delta$ from the model developed by Oset et al. [10], where $\tilde{\Gamma}_\Delta$ is the reduced width of the Δ due to Pauli blocking of nucleons in the $\Delta \rightarrow N\pi$ decay and Σ_Δ is the self energy of the Δ calculated in nuclear many body theory using the local density approximation [10]. The final state interaction(FSI) effect of the outgoing pions with the residual nucleus in the case of an incoherent process is taken into account by using a Monte Carlo simulation described in Ref. [12], while for the coherent pion production process this is treated by taking a distorted pion wave in an optical potential instead of the plane wave in the expression of the nuclear form factor given in Eq.7.

Results and Discussions

Our results for the CCQE cross sections without nucleon nucleon correlation effects agree within 1-2% with the different versions of Fermi gas model discussed in the literature [5] The results obtained with the nucleon nucleon correlation effects taken into account are consistent with the recent calculations performed by the various theoretical groups which were summarised at the last NUINT09 [13]. We find that the reduction in the cross section with the inclusion of nucleon nucleon correlation effects is around 25% at $E_\nu = 0.5$ GeV and around 15% at $E_\nu = 1$ GeV. In Fig.1(a), we have presented the results for the cross section in the case of quasielastic process obtained by using Eq.(1) with and without nucleon nucleon correlation effects. The several curves show the results obtained by using axial dipole mass $M_A = 1.35$ GeV and 1.6 GeV. If we take the value of axial dipole mass to be 1.35 GeV and calculate the cross section without nucleon nucleon correlation effects then our results are in good agreement with the experimental results reported by the MiniBooNE collaboration[2]. However, when we take RPA effects into account the theoretical cross sections are smaller than the experimentally measured ones. While when we take $M_A = 1.6$ GeV, then our results with nucleon nucleon correlation effects are in better agreement with the experimental results. In Fig.1(b), we have
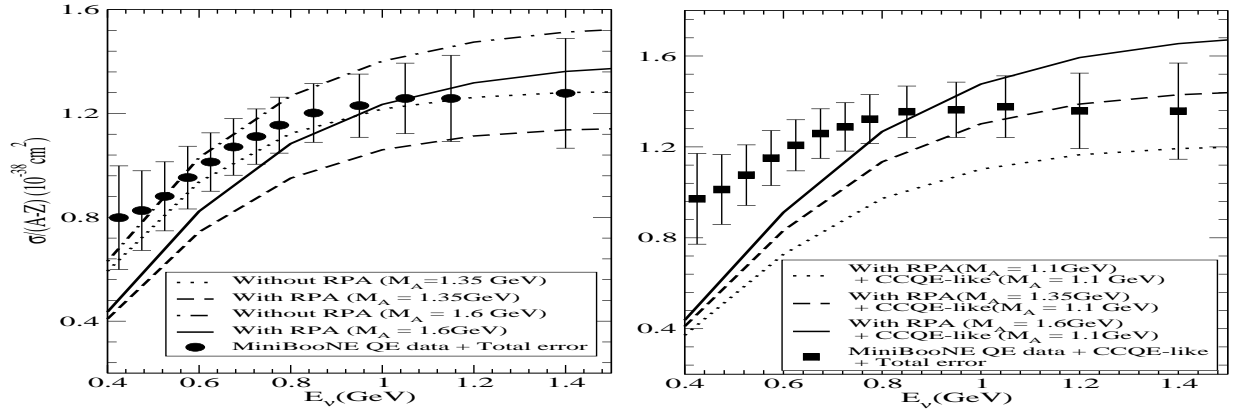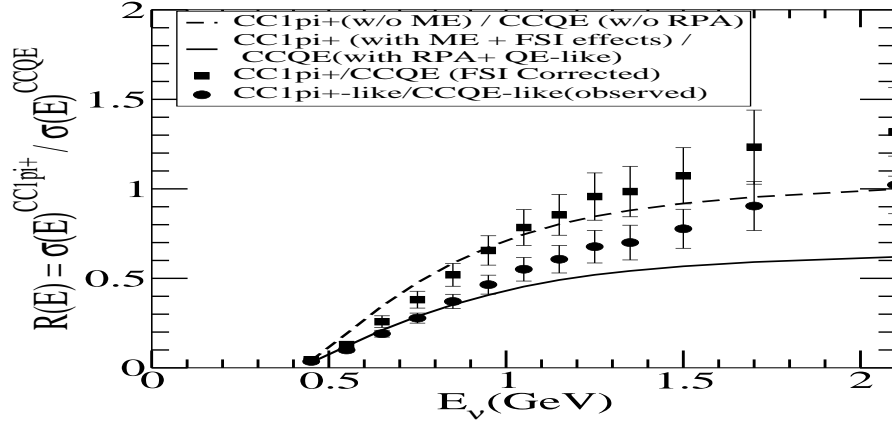
FIGURE 1. See text for the details.

FIGURE 2. Ratio of the cross sections for ν_μ induced charged current one pion production process to the charged current quasielastic process in mineral oil (CH_2)

presented the results of the CCQE cross section obtained in the local Fermi gas model(using M_A=1.1,1.35 and 1.6 GeV) with RPA effect along with the contribution of the CCQE-like events(using M_A=1.1 GeV) coming from the inelastic channel where a Δ disappears in the nuclear medium through the processes $\Delta N \rightarrow NN$ and $\Delta NN \rightarrow NNN$ without giving rise to pions, i.e. the pions produced in the inelastic processes get absorbed while coming out of the nucleus [10]. Here, we have presented the experimental results of the MiniBooNE collaboration[2] with CCQE + CCQE-like events. We find that CCQE-like events increases with the increase in neutrino energy, like the contribution is 12% at E_ν=0.6 GeV, 20% at E_ν=1 GeV which becomes 22% at E_ν=1.5 GeV, while the CCQE-like events shown by the MiniBooNE collaboration [2] decreases with the increase in the neutrino energy, for example it is around 20% at E_ν=0.6 GeV, 10% at E_ν=1 GeV which becomes around 5% at E_ν=1.5 GeV.

In Fig.(2), we compare our numerical results with the experimentally observed results and the FSI corrected results reported by the MiniBooNE collaboration [1].

The numerical results are obtained with M_A=1.1 GeV for CCQE process and CCQE-like process. We find that our theoretical results for the ratio $R(E) = \sigma^{CC1\pi^+}/\sigma^{CCQE}$ obtained without nuclear medium effect in the numerator, and the denominator is calculated in the local Fermi gas model without RPA effect are in agreement with the FSI corrected results of the MiniBooNE collaboration [1]. When, in the ratio R(E), we consider the nuclear medium and final state interaction effects in the numerator, and the denominator is calculated in the local Fermi gas model with RPA effect along with the contribution of CCQE-like events from the inelastic channel the numerical results are in agreement with the experimentally observed results reported by the MiniBooNE collaboration [1].

Table-1 summarizes our theoretical results for the lepton events obtained in the case of the atmospheric neutrino experiment performed at SuperK using 22.5 kT water fiducial mass on an exposure of 1489 days [3]. The event rates are calculated for the sub-GeV energy region by applying cuts on lepton momenta. We have studied the

TABLE 1. Ratio $R = \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu}$

Process	$\nu_e + \bar{\nu}_e$	$\nu_\mu + \bar{\nu}_\mu$	$R = \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu}$
Free case(QE+Inelastic)	3995	5984	0.667
FGM without RPA+Inelastic with nuclear medium and final state interaction effects	2911	4398	0.66
FGM with RPA +Inelastic with nuclear medium and final state interaction effects	2343	3661	0.64
SuperK experiment [3]	3353	3227	1.04

influence of nuclear medium on the number of events by taking into account nuclear medium modification effects on the cross sections in the case of $\nu(\bar{\nu})$ induced processes of quasielastic scattering, incoherent and coherent pion production in nuclei. It is found that the event rates are reduced by 25% when nuclear effects are included in a local Fermi gas model without taking into account the strong nucleon-nucleon correlation effects in nuclei. When these correlation effects are also included using a Random Phase Approximation (RPA), there is a further reduction of about 20% in the event rates. Our final results have been shown in row-III of Table-1 for electron and muon events. In the row-IV of Table-1 there are experimental numbers reported by the SuperK collaboration [3].

We find that the nuclear medium effects play an important role in the present accelerator experiments in the few GeV energy region as well as in the study of atmospheric neutrino experiments. In order to understand the lepton event rates in the accelerator as well as atmospheric neutrino experiments, a good theoretical understanding of the inelastic events leading to lepton production is needed in addition to the purely quasielastic events produced in neutrino nuclear reactions.

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