

Sterile neutrinos beyond LSND at NF

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Collaborators: Walter Winter, Davide Meloni, Many thanks to members in the IDS-NF collaboration.

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Outline



- Sterile neutrinos
- Recent progress at IDS-NF

2 Sensitivities at NF

- Sterile neutrinos with arbitrary masses
- Sterile neutrinos with particular masses

3 Summary

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What is the sterile neutrino?

- No SM interactions but massive
- Not absolute ghost
 - Gravitational interactions
 - New NSIs beyond SM
 - **3** Neutrino oscillations: ν_e , ν_μ , $\nu_\tau \rightarrow \nu_s$.
- Observables
 - Gravitational signals: Extremely hard!
 - 2 Disappearance of active neutrinos $u_e\,,
 u_\mu\,,
 u_ au$
 - Indirect evidence from a global analysis of neutrino oscillation data
 - Any others?

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The simple analysis of neutrino oscillation

• Standard oscillation pictures with $\mathcal{J}_{ij}^{\alpha\beta} \equiv U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}$: $\mathcal{P}_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \operatorname{Re}(\mathcal{J}_{ij}^{\alpha\beta}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) + 2 \sum_{i < j} \operatorname{Im}(\mathcal{J}_{ij}^{\alpha\beta}) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right)$

• For 3+1 &
$$\Delta m_{41}^2 \gg \Delta m_{31}^2$$
, $\mathcal{P}_{e\mu} \approx 4|U_{e4}|^2|U_{\mu4}|^2\sin^2\left(\frac{\Delta m_{41}^2L}{4E}\right)$

- Focus on $\sin^2\left(\frac{\Delta m_{41}^2 L}{4E}\right) \Longrightarrow L_{\text{osc}} = 2.47 \frac{E[\text{GeV}]}{\Delta m^2 [\text{eV}^2]} \text{km}.$ If $E \sim \text{GeV}$ and $L \sim \text{km} \Longrightarrow$ discovery reach of Δm^2 at eV^2 .
- LSND: $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ $E \sim 30 \text{ MeV} \text{ and } L \sim 30 \text{ m}$ MiniBooNE: $\nu_{\mu} \rightarrow \nu_{e} \& \bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ $E \sim 500 \text{ MeV} \text{ and } L \sim 500 \text{ m}$

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Why do we need sterile neutrinos? Are there any sterile neutrinos in Nature? LSND v.s MiniBooNE



ref: www-boone.fnal.gov/

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Anomaly?



ref: www-boone.fnal.gov/

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Motivations Sterile neutrinos

Point at sterile neutrinos!



ref: 1007.1150 [hep-ex] by MiniBooNE Collaboration Other explanations: CPT violations...



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IDS-NF

Initial IDS-NF set-up (E = 25 GeV, $L_1 \approx 4000$ km and $L_2 \approx 7500$ km). Wrong-sign muon events at NF:

$$\Gamma(\mu^+ \to e^+ \bar{\nu}_{\mu} \nu_e) \times P_{\nu_e \to \nu_{\mu}} \times \sigma(\nu_{\mu} N \to \mu^- X)$$



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ND at the NF

Parameter	ND1	ND2	ND3	ND4	ND5
Diameter D	17 m	4 m	4 m	0.32 m	6.8 m
Distance d	80 m	1000 m	80 m	80 m	1000 m
Mass	450 t	25 t	25 t	0.2 t	2000 t



ref: Phys.Rev.D80:053001 JT,W.Winter

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Detectors

Magetized Iron Detector(MIND)

A hybrid Near Detector(ND-Hybrid)



ref: Detector R&D (a) The 6th plenary meeting of IDS-NF by P. Soler.



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Parametrization and mass schemes

Our parametrization explicitly reads

 $U = R_{34}(\theta_{34}, 0) R_{24}(\theta_{24}, 0) R_{14}(\theta_{14}, 0) R_{23}(\theta_{23}, \delta_3) R_{13}(\theta_{13}, \delta_2) R_{12}(\theta_{12}, \delta_1)$





Systematic analysis of mass ordering A

$\Delta m^2_{41} [{ m eV}^2]$	$\sin^2 2\theta_{14}$	$\sin^2 2\theta_{24}$	$\sin^2 2\theta_{34}$	$\theta_{14}[^{\circ}]$	$\theta_{24}[^{\circ}]$	$\theta_{34}[^{\circ}]$
0.001	0.403	0.029	0.042	19.7	4.9	5.9
0.01	0.224	0.004	0.044	14.1	1.9	6.1
0.1	0.054	0.013	0.047	6.7	3.3	6.3
1	0.001	0.0009	0.047	0.9	0.8	6.3
10	0.0002	0.0002	0.047	0.5	0.5	6.3
100	0.0043	0.0006	0.047	1.9	0.7	6.3
1000	0.0168	0.015	0.047	3.7	3.5	6.3





Generalized exclusion limits





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Outline

Motivations

- Sterile neutrinos
- Recent progress at IDS-NF

2 Sensitivities at NF

- Sterile neutrinos with arbitrary masses
- Sterile neutrinos with particular masses

3 Summary

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Sterile neutrinos with particular masses

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LSND-motivated Δm^2_{41}



Bounds from ref: 0812.3703



Sensitivities at NF

Sterile neutrinos with particular masses

FD4000 km@NF v.s MINOS





Sensitivities at NF

Sterile neutrinos with particular masses

NDFD@NF v.s MINOS





Summary

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- We invstigate sensitivities to sterile neutrinos with arbitrary masses at NF. Two mass regions are available:
 - one is LSND-type by ND;
 - the other one is close to atmospheric Δm^2_{31} by FD.
- Four mass schemes are allowed but have different $\sin^2 2\theta_{i4} \Delta m_{41}^2$ dependence.
- For the special $\Delta m^2_{41} \sim \mathcal{O}(1) \text{ eV}^2$, we discuss whether τ detections are necessary or not. Meanwhile, we compare our results with those in MINOS. ND takes an important role in the game.

Vielen Dank!



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Osc. Probabilities at ND

$$\begin{aligned} \mathcal{P}_{e\mu} &= \mathcal{P}_{\mu e} = 4c_{14}^2 s_{14}^2 s_{24}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{e\tau} &= 4c_{14}^2 c_{24}^2 s_{14}^2 s_{34}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{es} &= 4c_{14}^2 c_{24}^2 c_{34}^2 s_{14}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{ee} &= 1 - \sin^2(2\theta_{14}) \sin^2(\Delta_{41}) \\ \mathcal{P}_{\mu e} &= 4c_{14}^2 s_{14}^2 s_{24}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{\mu \tau} &= 4c_{14}^4 c_{24}^2 s_{24}^2 s_{34}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{\mu s} &= 4c_{14}^4 c_{24}^2 c_{34}^2 s_{24}^2 \sin^2(\Delta_{41}) \\ \mathcal{P}_{\mu \mu} &= 1 - c_{14}^2 s_{24}^2 \left[3 + 2c_{14}^2 \cos(2\theta_{24}) - \cos(2\theta_{14}) \right] \sin^2(\Delta_{41}) \end{aligned}$$