

Updated MiniBooNE Results

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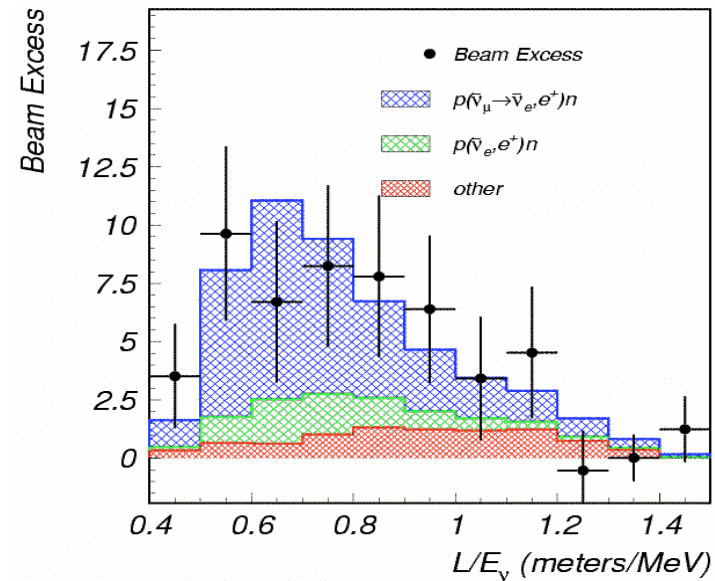
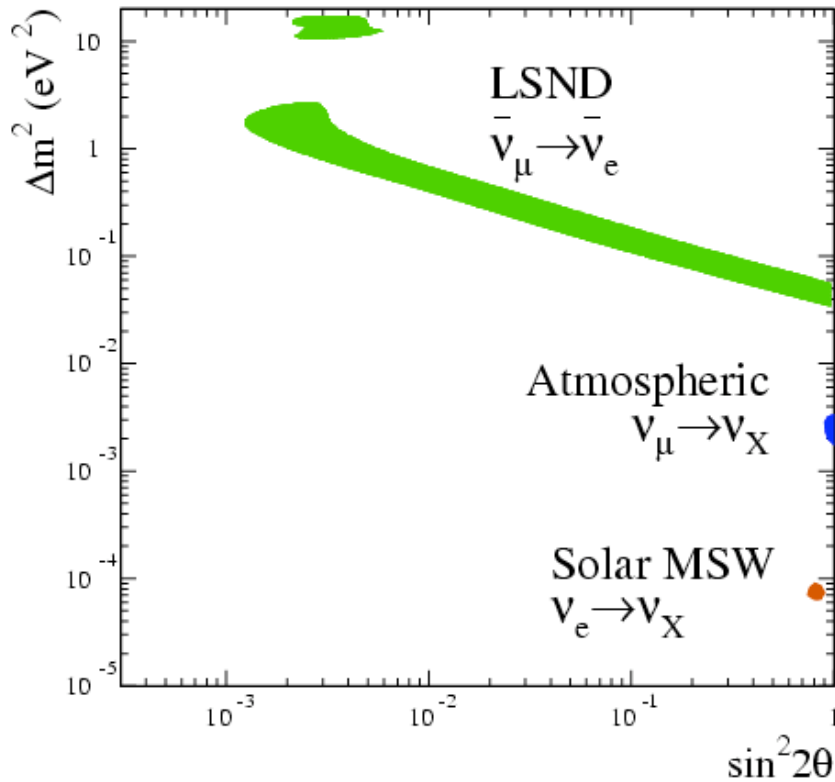


NuFact2010: 12th International Workshop on Neutrino Factories, Superbeams and Beta Beams
October 20-25, 2010. Mumbai, India

Outline

- MiniBooNE Experiment Description
- MiniBooNE's Neutrino Results
- MiniBooNE's Anti-neutrino Results
- Next Steps and Summary

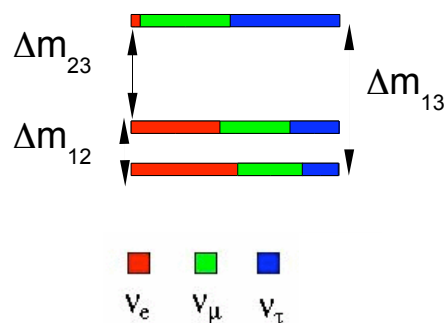
Oscillation Status After LSND



This signal looks very different from the others...

- Much higher $\Delta m^2 = 0.1 - 10 \text{ eV}^2$
- Much smaller mixing angle
- Only one experiment!

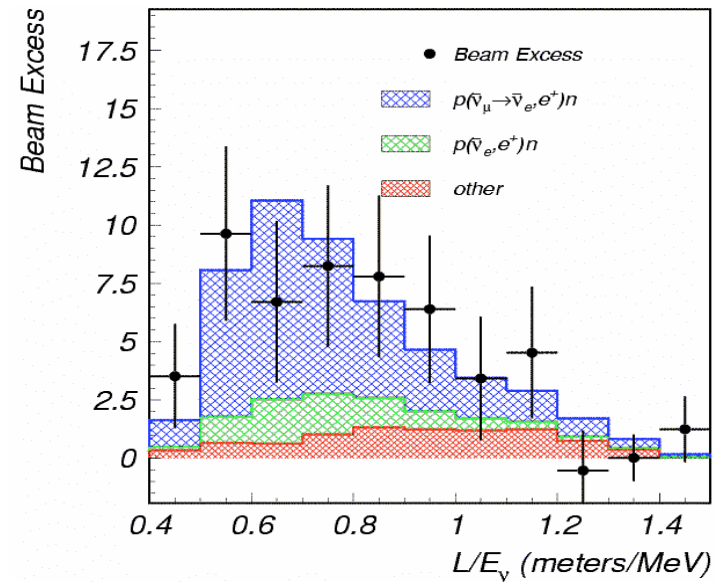
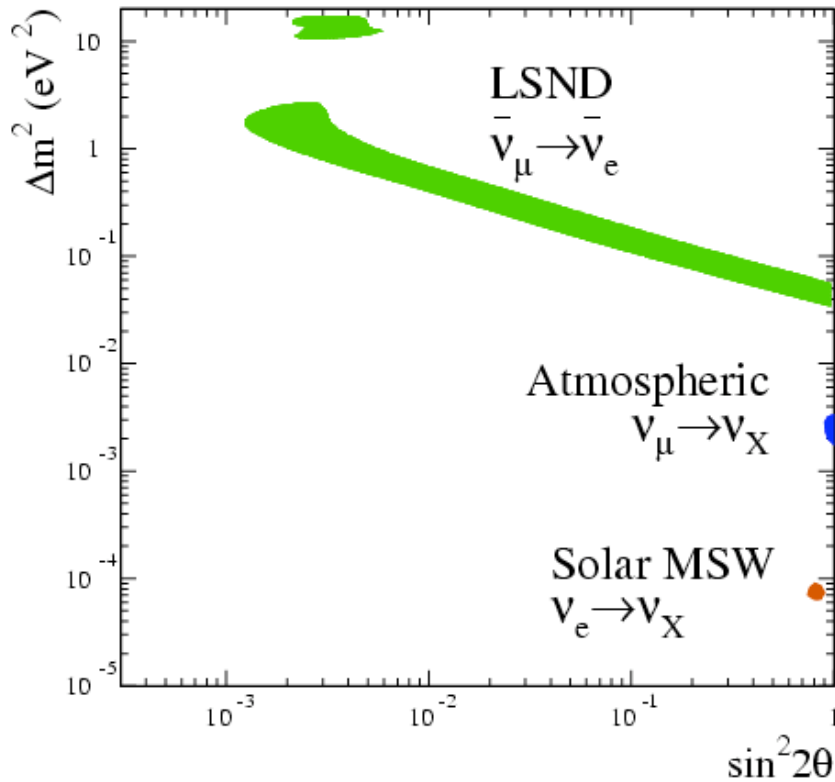
In SM there are only 3 neutrinos



- Three distinct neutrino oscillation signals, with $\Delta m_{solar}^2 + \Delta m_{atm}^2 \neq \Delta m_{LSND}^2$
- For three neutrinos, expect $\Delta m_{21}^2 + \Delta m_{32}^2 = \Delta m_{31}^2$

The three oscillation signals cannot be reconciled without introducing Beyond Standard Model Physics

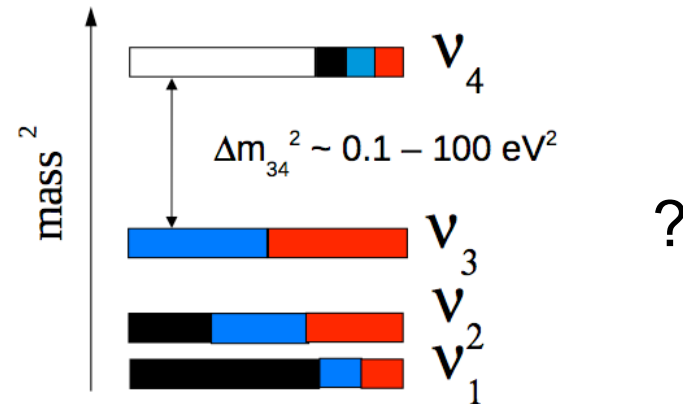
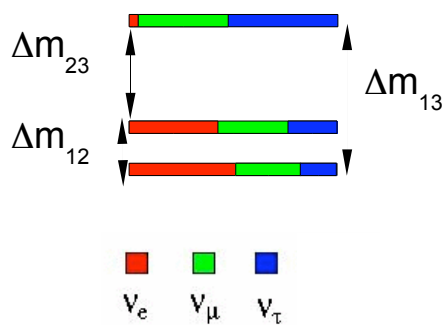
Oscillation Status After LSND



This signal looks very different from the others...

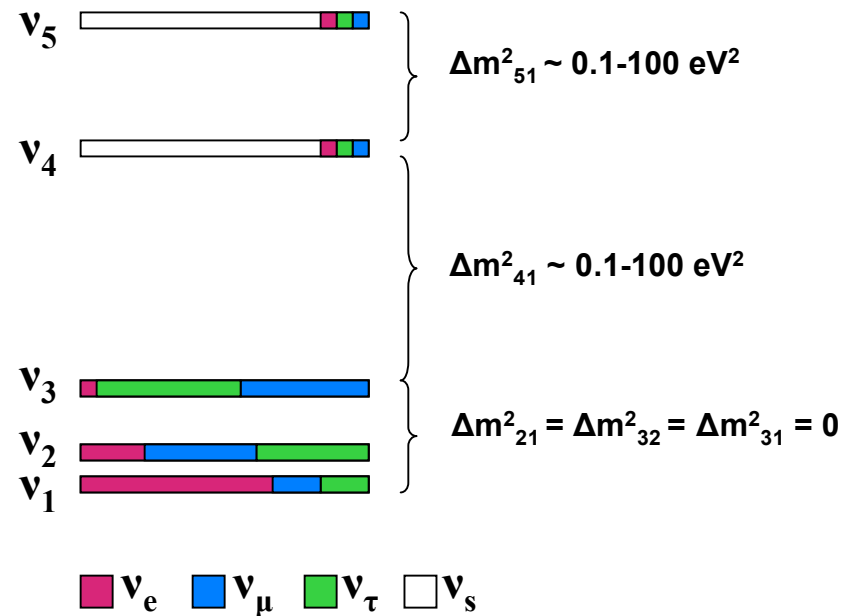
- Much higher $\Delta m^2 = 0.1 - 10 \text{ eV}^2$
- Much smaller mixing angle
- Only one experiment!

In SM there are only 3 neutrinos



Oscillation explanation of LSND in conjunction with the atmospheric and solar oscillation results needed more than 3 ν 's.

Models developed with 1 or more sterile ν 's (or other new physics models).



Simplified 3+2 Models for $\nu_\mu \rightarrow \nu_e$:

2 independent Δm^2

3 mixing parameters

1 Dirac CP phase

It was important to check LSND what was left to MiniBooNE

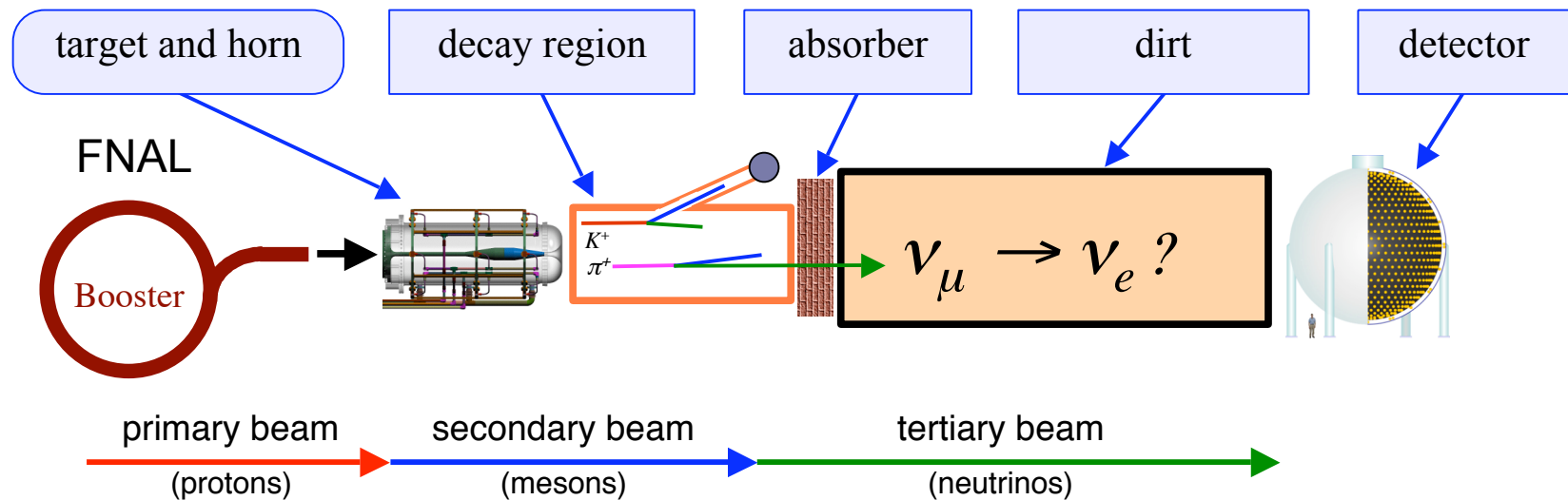
(**B**ooster **N**eutrino **E**xperiment)

MiniBooNE Setup

Keep L/E same as LSND while changing systematics, energy & event signature

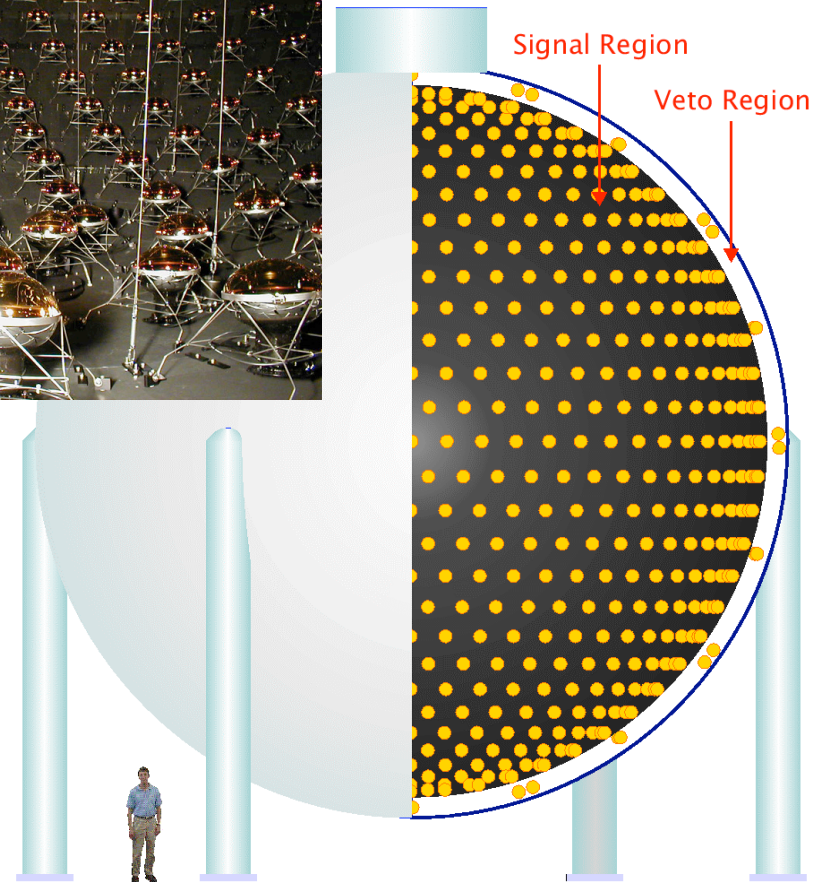
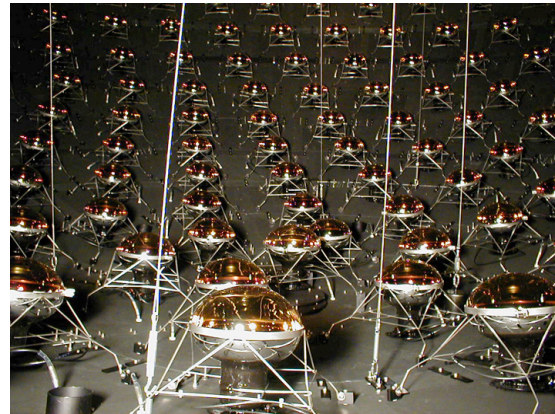
$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \sin^2(1.27 \Delta m^2 L/E) \rightarrow \text{Two neutrino fits}$$

LSND:	$E \sim 30 \text{ MeV}$	$L \sim 30 \text{ m}$	$L/E \sim 1$
MiniBooNE:	$E \sim 500 \text{ MeV}$	$L \sim 500 \text{ m}$	$L/E \sim 1$



Neutrino mode: search for $\nu_\mu \rightarrow \bar{\nu}_e$ appearance with $6.5E20$ POT \rightarrow assumes CP/CPT conservation
 Antineutrino mode: search for $\bar{\nu}_\mu \rightarrow \nu_e$ appearance with $5.66E20$ POT \rightarrow direct test of LSND

MiniBooNE Detector



MiniBooNE Detector:

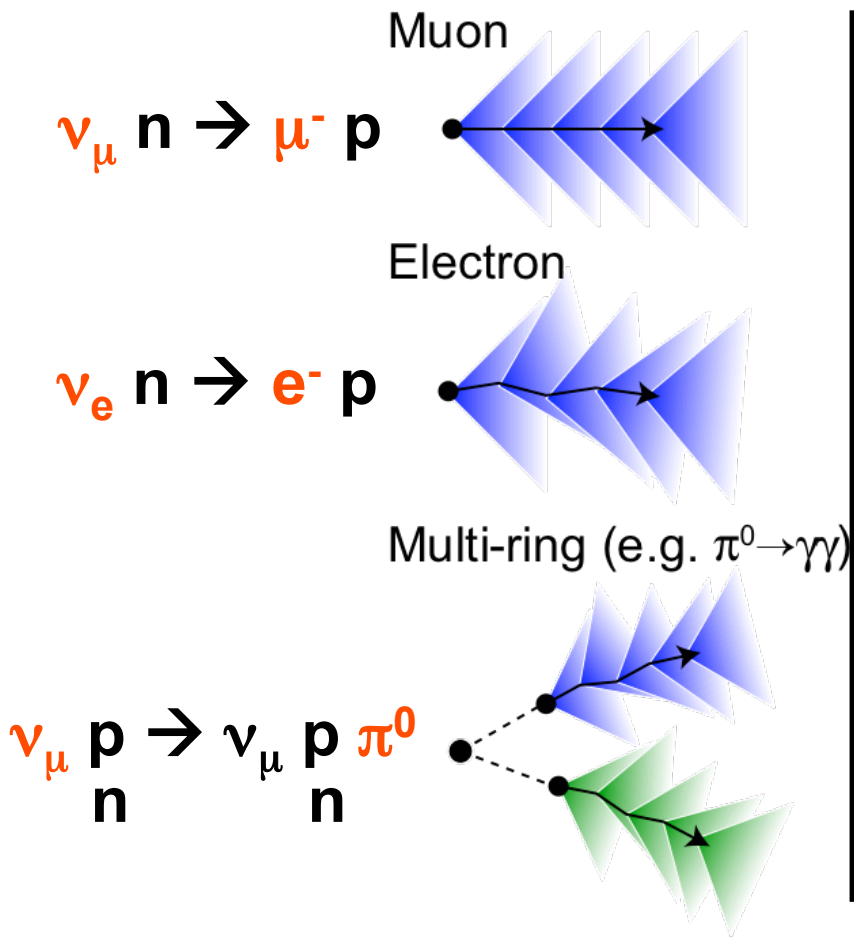
- 12m diameter sphere
- 950000 liters of oil(CH_2)
- 1280 inner PMTs
- 240 veto PMTs

Detector Requirements:

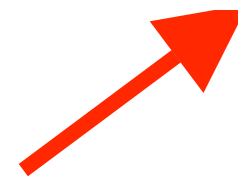
- Detect and Measure Events: Vertex, E_ν
- Separate ν_μ events from ν_e events.

Particle Identification

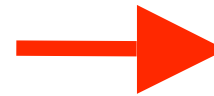
Čerenkov rings provide primary means of identifying products of ν interactions in the detector



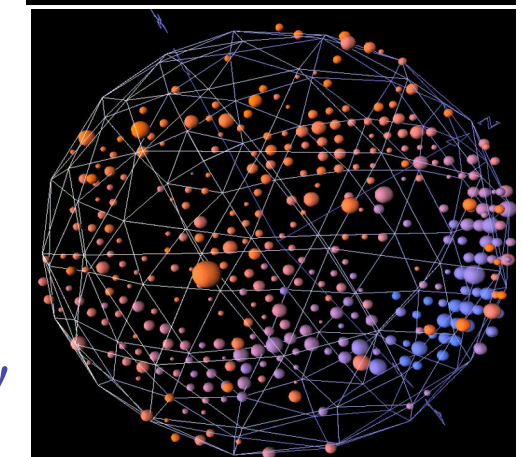
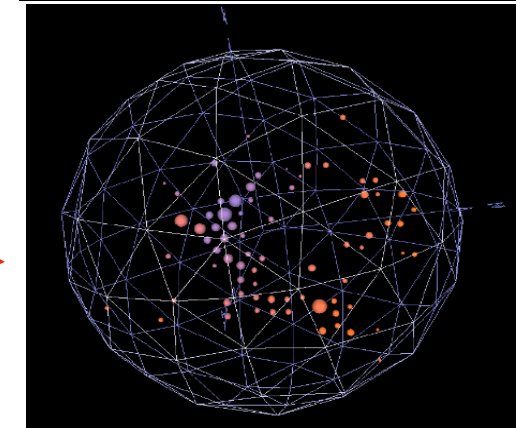
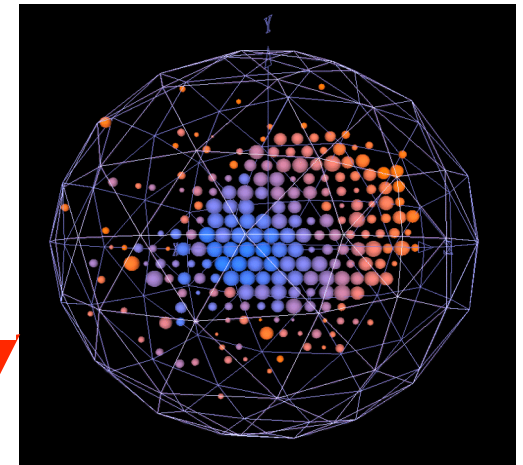
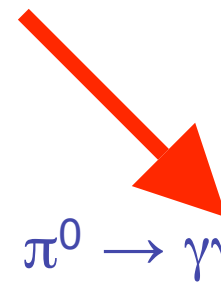
beam μ
candidate



μ -decay e-
candidate

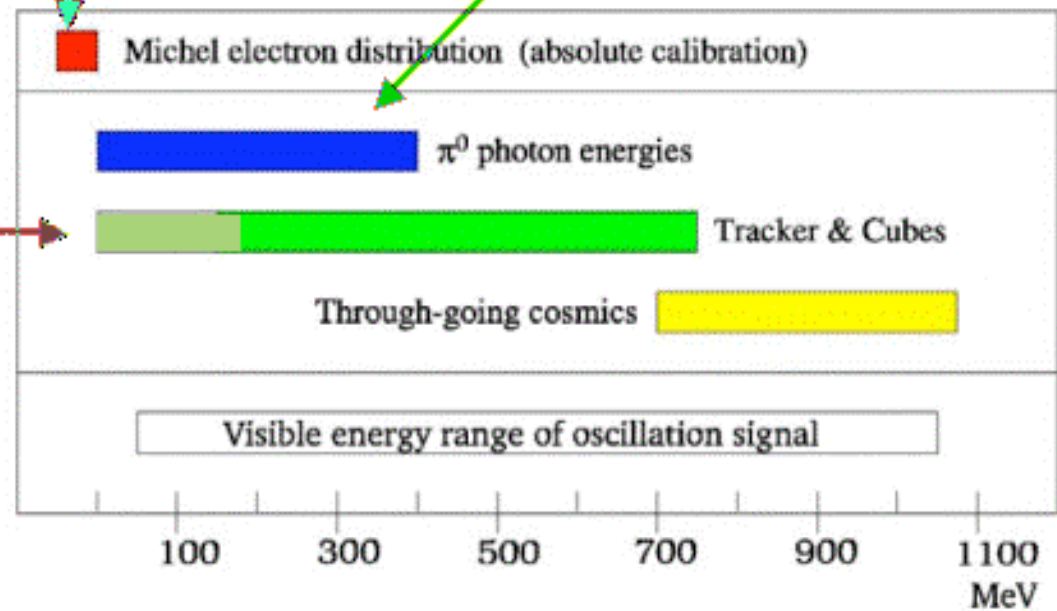
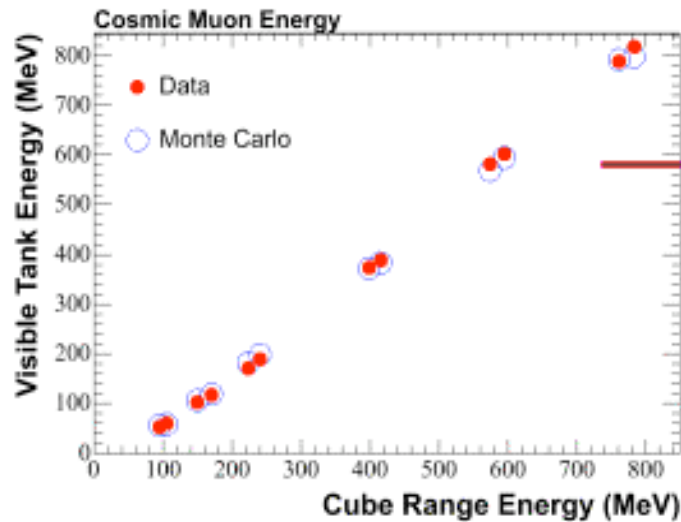
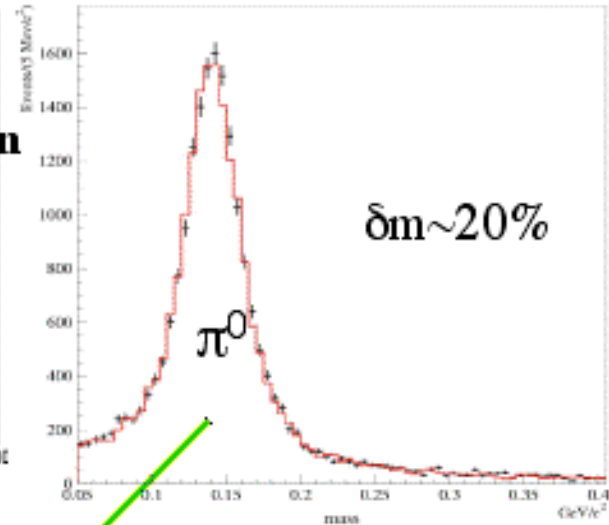
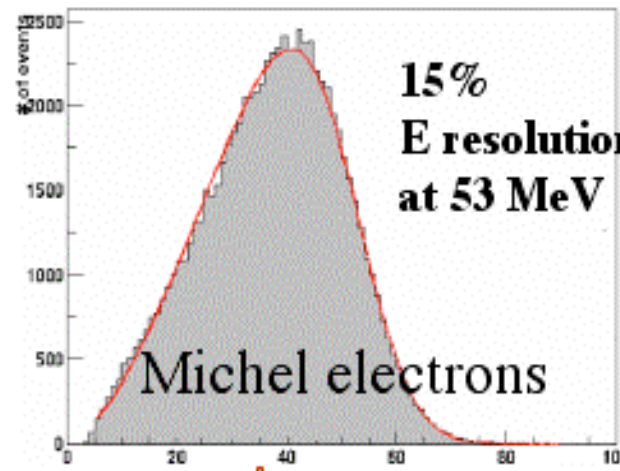
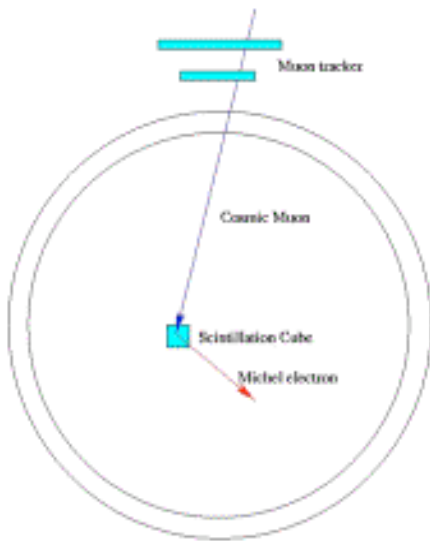


beam π^0
candidate



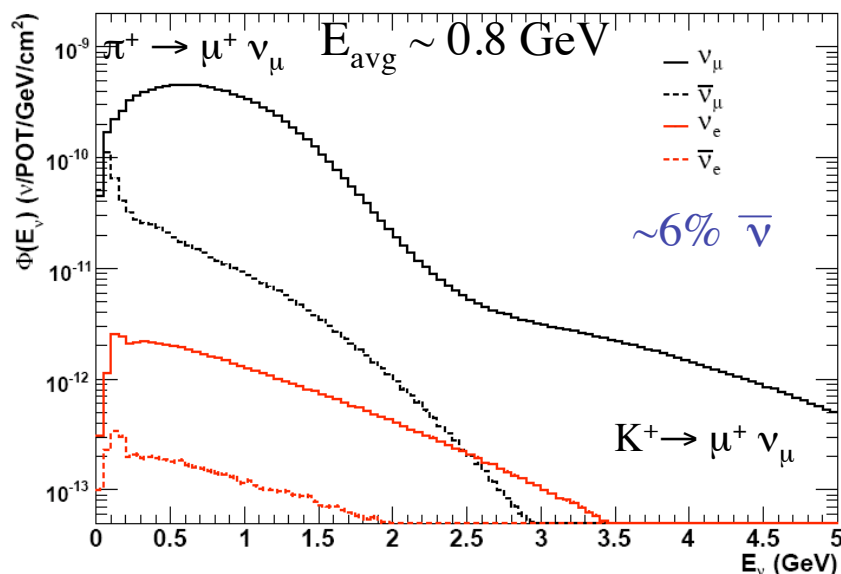
Energy Calibration

Tracker system

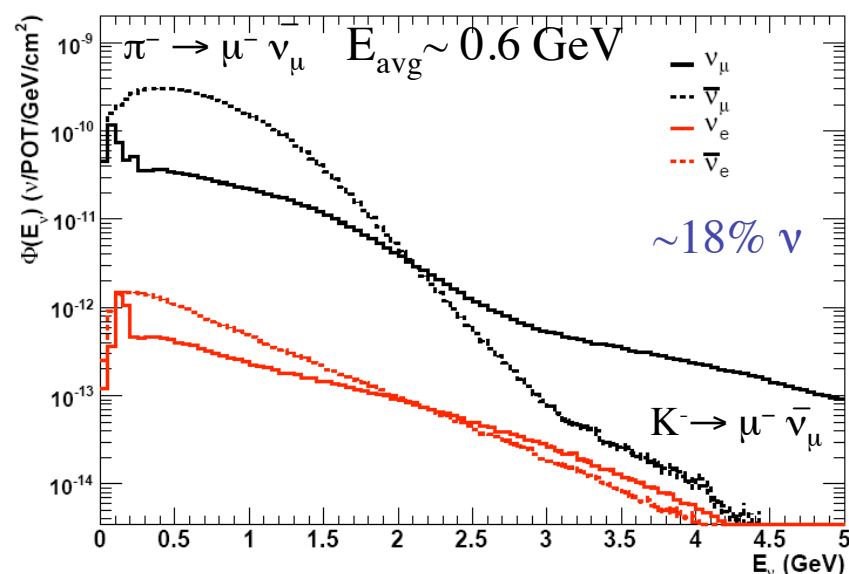


Booster Flux at MiniBooNE

Neutrino-Mode Flux



Antineutrino-Mode Flux



Subsequent decay of the μ^+ (μ^-) produces $\bar{\nu}_e$ (ν_e) intrinsics $\sim 0.5\%$

neutrino mode: $\nu_\mu \rightarrow \nu_e$ oscillation search

antineutrino mode: $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation search

Appearance experiment: it looks for an excess of electron neutrino events in a predominantly muon neutrino beam

$\nu_e, \bar{\nu}_e$ Event Rate Predictions

Events Rate = Flux x Cross-sections x Detector response

External measurements
(HARP, etc)

ν_μ rate constrained by
neutrino data

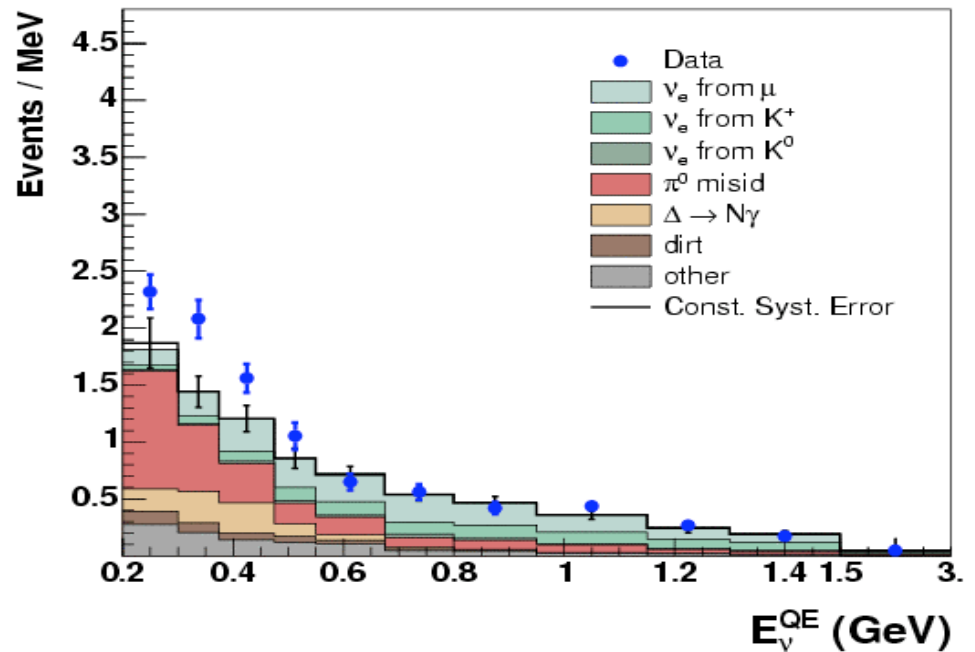
External and MiniBooNE
Measurements
 $\pi^0, \Delta \rightarrow N\gamma$, dirt, and intrinsic
 ν_e constrained from data.

Detailed detector
simulation and PID
Checked with neutrino
data and calibration
sources.

- A. A. Aguilar-Arevalo et al., “Neutrino flux prediction at MiniBooNE”, Phys. Rev. D79, 072002 (2009).
- A. A. Aguilar-Arevalo et al., “Measurement of Muon Neutrino Quasi-Elastic Scattering on Carbon”, Phys. Rev. Lett. 100, 032301 (2008).
- A. Aguilar-Arevalo et al., “First Observation of Coherent π^0 Production in Neutrino Nucleus Interactions with Neutrino Energy < 2 GeV”, Phys. Lett. 664B, 41 (2008).
- A. A. Aguilar-Arevalo et al., “Measurement of the Ratio of the ν_μ Charged-Current Single-Pion Production to Quasielastic Scattering with a 0.8 GeV Neutrino Beam on Mineral Oil”, Phys. Rev. Lett. 103, 081801 (2009).
- A. A. Aguilar-Arevalo et al., “Measurement of ν_μ and $\bar{\nu}_\mu$ induced neutral current single π^0 production cross sections on mineral oil at $E_\nu \sim 1$ GeV”, Phys. Rev. D81, 013005 (2010).
- A. A. Aguilar-Arevalo et al., “Measurement of the ν_μ charged current π^+ to quasi-elastic cross section ratio on mineral oil in a 0.8 GeV neutrino beam”. Phys.Rev. Lett. 103:081801 (2010).
- A. A. Aguilar-Arevalo et al., “First Measurement of the Muon Neutrino Charged Current Quasielastic Double Differential Cross Section”, Phys. Rev. D81, 092005 (2010), arXiv: 1002.2680 [hep-ex].
- A. A. Aguilar-Arevalo et al., “Measurement of the Neutrino Neutral-Current Elastic Differential Cross Section”, submitted Phys. Rev. D, arXiv:1007.4730 [hep-ex].
- A. A. Aguilar-Arevalo et al., “The MiniBooNE Detector”, Nucl. Instr. Meth. A599, 28 (2009).
- P. Adamson et al., “Measurement of ν_μ and ν_e Events in an Off-Axis Horn-Focused Neutrino Beam”, Phys. Rev. Lett. 102, 211801 (2009).
- R.B. Patterson et al., “The Extended-Track Event Reconstruction for MiniBooNE”, Nucl. Instrum. Meth. A608, 206 (2009).

Neutrino Mode MiniBooNE Results (2009)

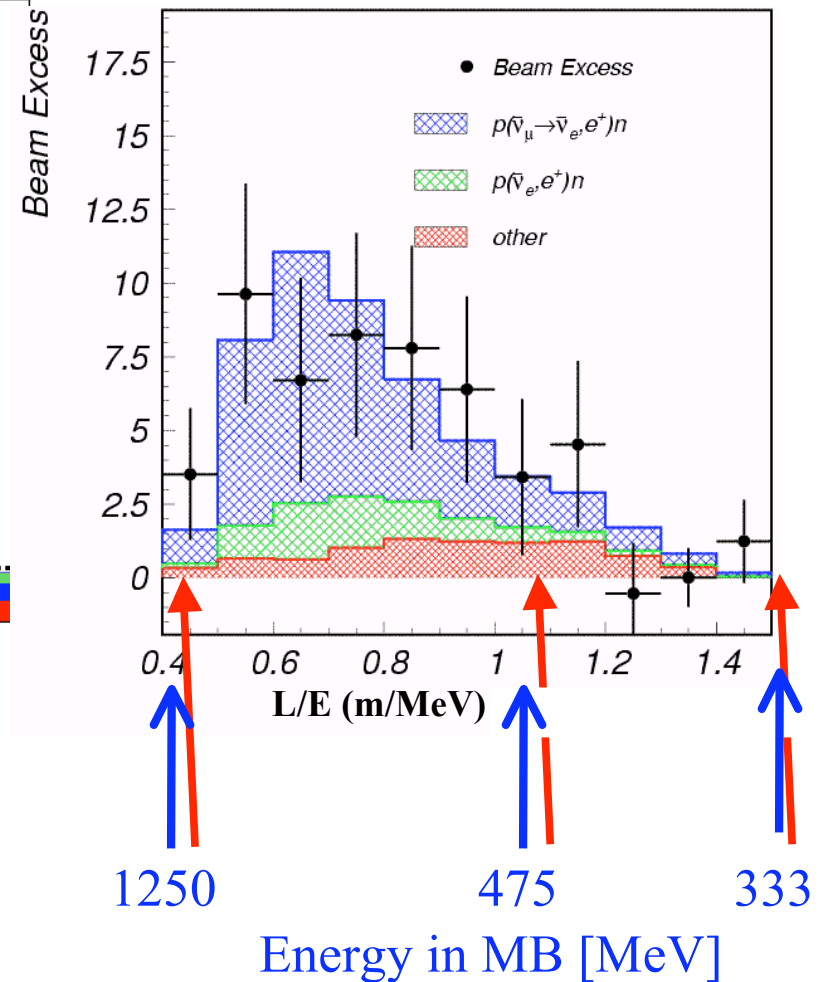
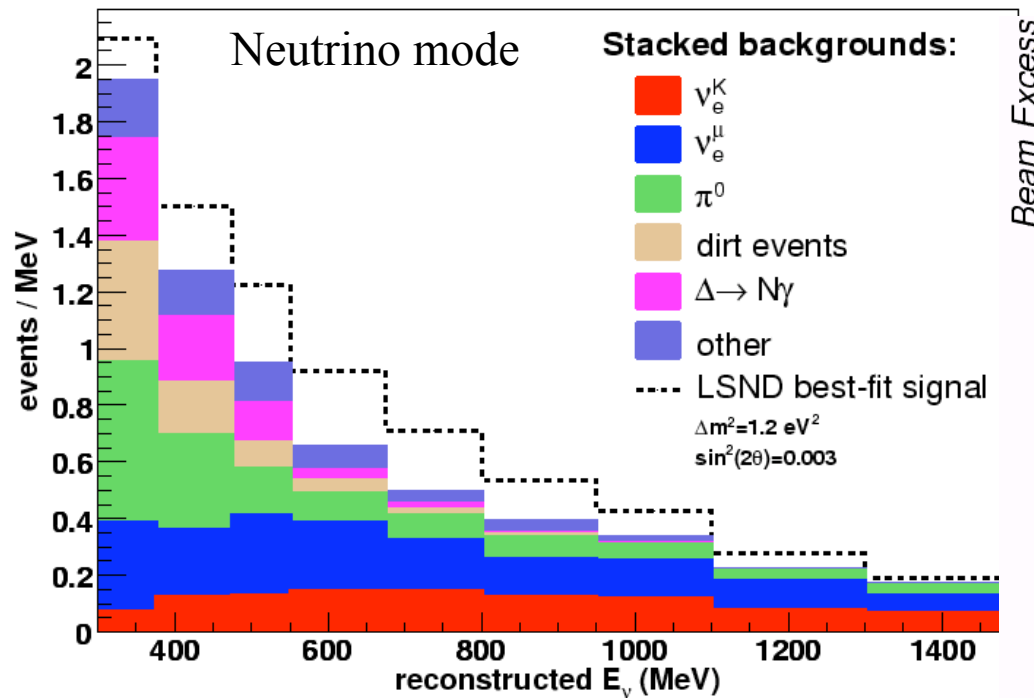
- **6.5E20 POT** collected in neutrino mode
- $E > 475$ MeV data in good agreement with background prediction
 - Energy region has reduced backgrounds and maintains high sensitivity to LSND oscillations.
 - A two neutrino fit rules out LSND at the 90% CL assuming CP conservation.
- $E < 475$ MeV, statistically large (6σ) excess
 - Reduced to 3σ after systematics, shape inconsistent with two neutrino oscillation interpretation of LSND. Excess of 129 ± 43 (stat+sys) events is consistent with magnitude of LSND oscillations.



E_ν [MeV]	200-300	300-475	475-1250
total background	186.8±26	228.3±24.5	385.9±35.7
ν_e intrinsic	18.8	61.7	248.9
ν_μ induced	168	166.6	137
NC π^0	103.5	77.8	71.2
NC $\Delta \rightarrow N\gamma$	19.5	47.5	19.4
Dirt	11.5	12.3	11.5
other	33.5	29	34.9
Data	232	312	408
Data-MC	45.2±26	83.7±24.5	22.1±35.7
Significance	1.7σ	3.4σ	0.6σ

Reminder of Some Pre-unblinding Choices

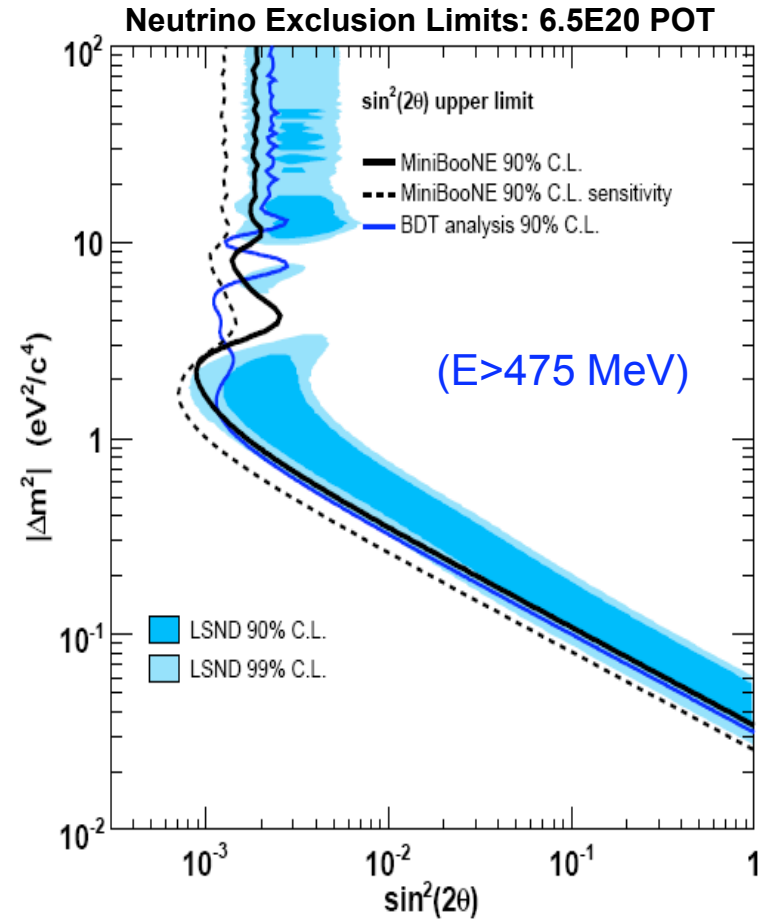
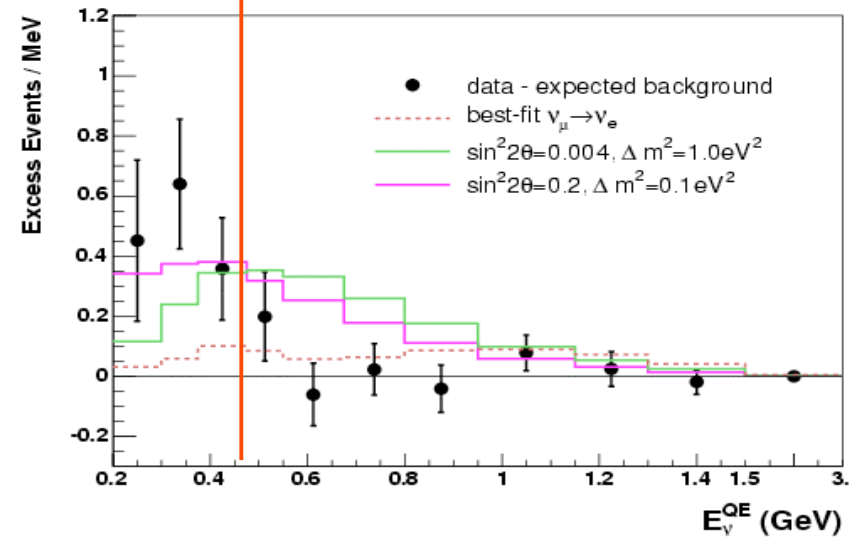
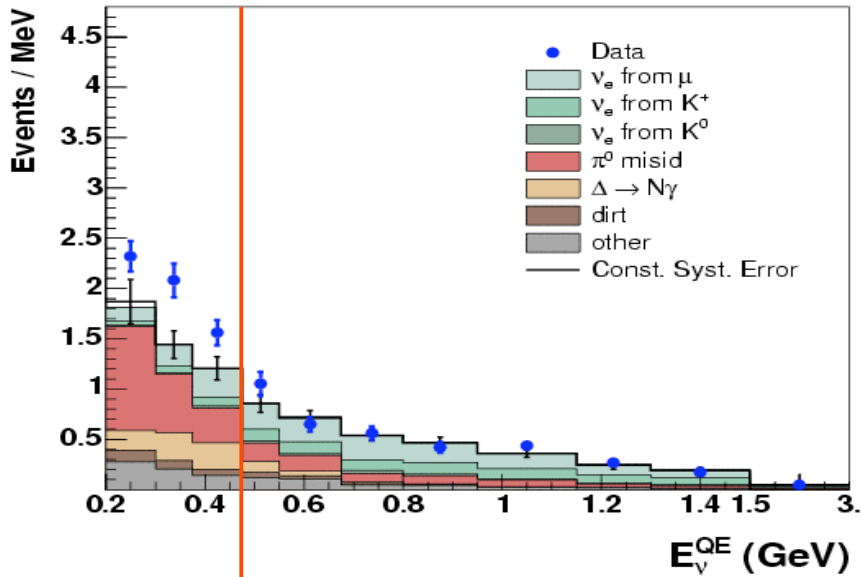
We are using energy range $E_\nu > 475$ MeV in oscillation analysis.



Why is the 200-475 MeV region unimportant for oscillation search?

- Large backgrounds from mis-ids reduce S/B.
- Many systematics grow at lower energies.
- Most importantly, not a region of L/E where LSND observed a significant signal

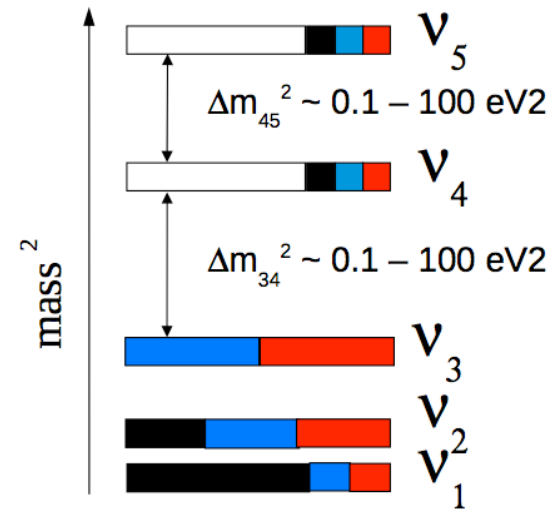
Neutrino Mode MiniBooNE Results (2009): Limit



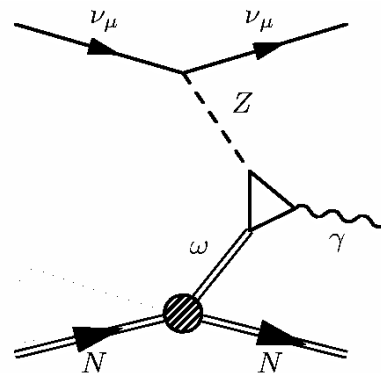
Range of possible explanations for observed excess

Several possible explanations have been put forth by the physics community, attempting to reconcile the MiniBooNE neutrino mode result with LSND and other appearance experiments...

- 3+2 with CP violation
[Maltoni and Schwetz, hep-ph0705.0107 ; G. K., NuFACT 07 conference]
- Anomaly mediated photon production
[Harvey, Hill, and Hill, hep-ph0708.1281]
- New light gauge boson
[Nelson, Walsh, Phys. Rev. D 77, 033001 (2008)]
- Neutrino decay
[hep-ph/0602083]
- Extra dimensions
[hep-ph/0504096]
- CPT/Lorentz violation
[PRD(2006)105009]
- ...



$$\nu_{\mu} \rightarrow \nu_e \neq \bar{\nu}_{\mu} \rightarrow \bar{\nu}_e \quad ?$$



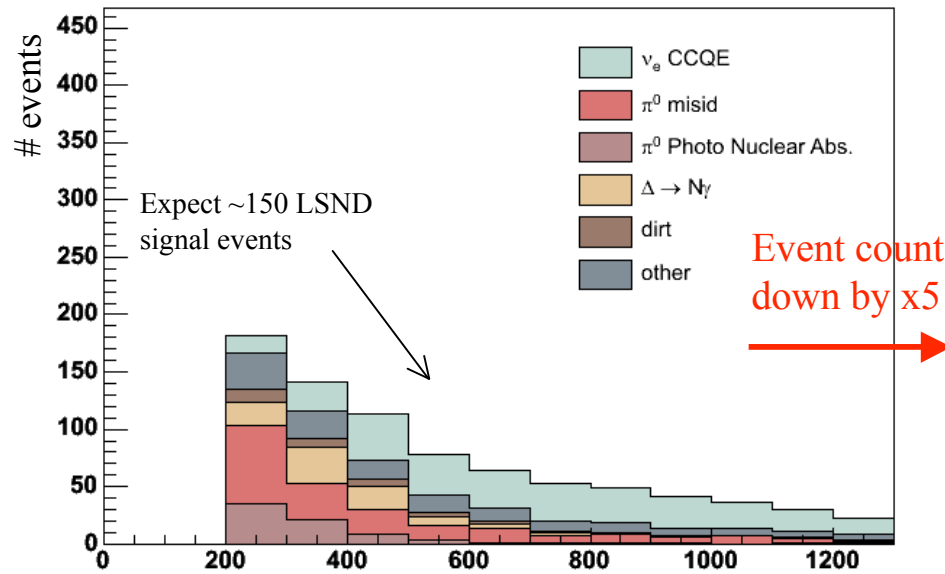
$\bar{\nu}_e$ Event Rate Predictions in Appearance Analysis

We have collected about $\sim 1/5$ the number of interactions as in neutrino mode

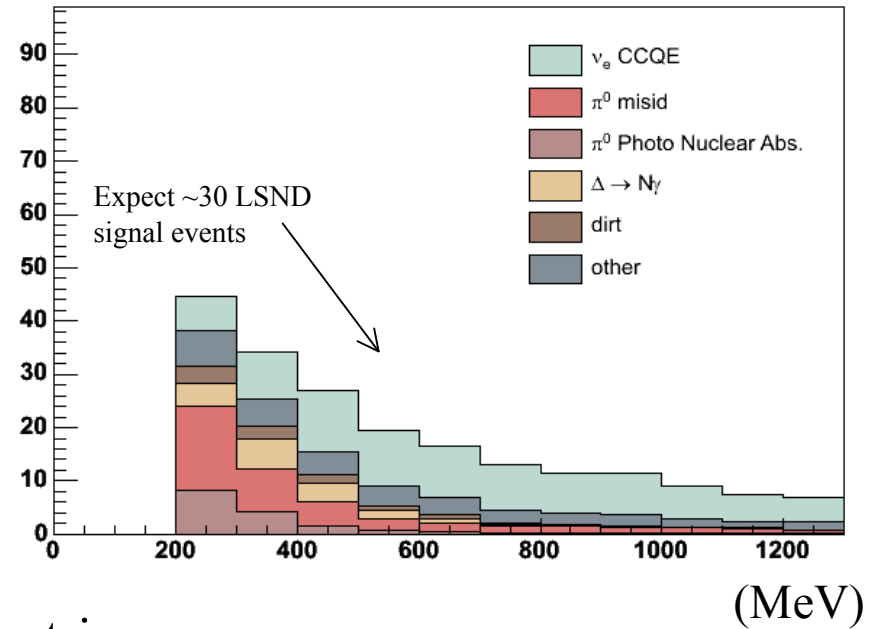
- Similar number of protons on target so far in two modes
- The flux per proton on target is lower ($\sim \times 1.5$) in $\bar{\nu}$ mode
- The cross section is lower ($\sim \times 3$) in $\bar{\nu}$ mode
- Background types and relative rates are similar for neutrino and antineutrino mode.
 - except inclusion of 15.9% wrong-sign neutrino flux component in antineutrino mode
- Fit analysis and errors are similar.

$\nu_e, \bar{\nu}_e$ Background Predictions After Reconstruction and Selection

Neutrino 6.5×10^{20} POT



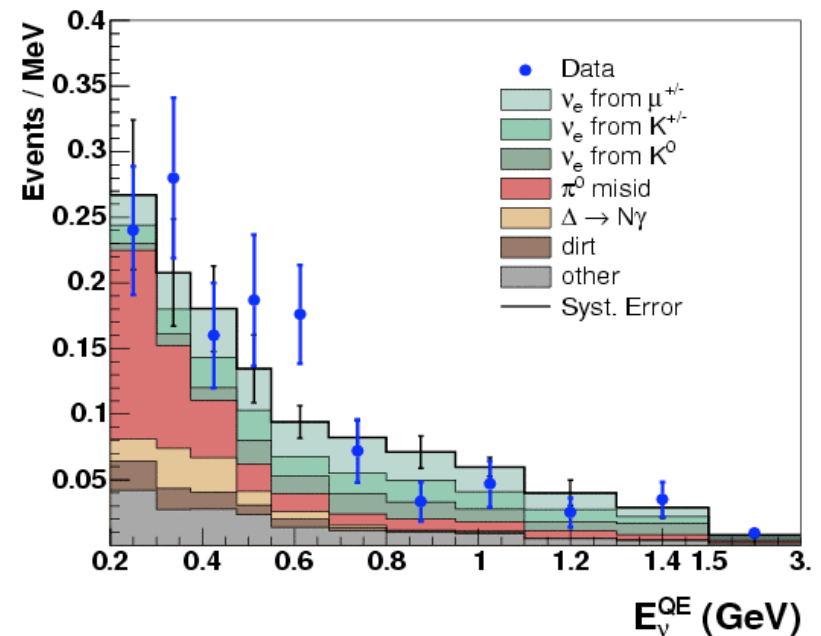
AntiNeutrino 5.66×10^{20} POT



E_{ν}^{QE} = Reconstructed neutrino energy

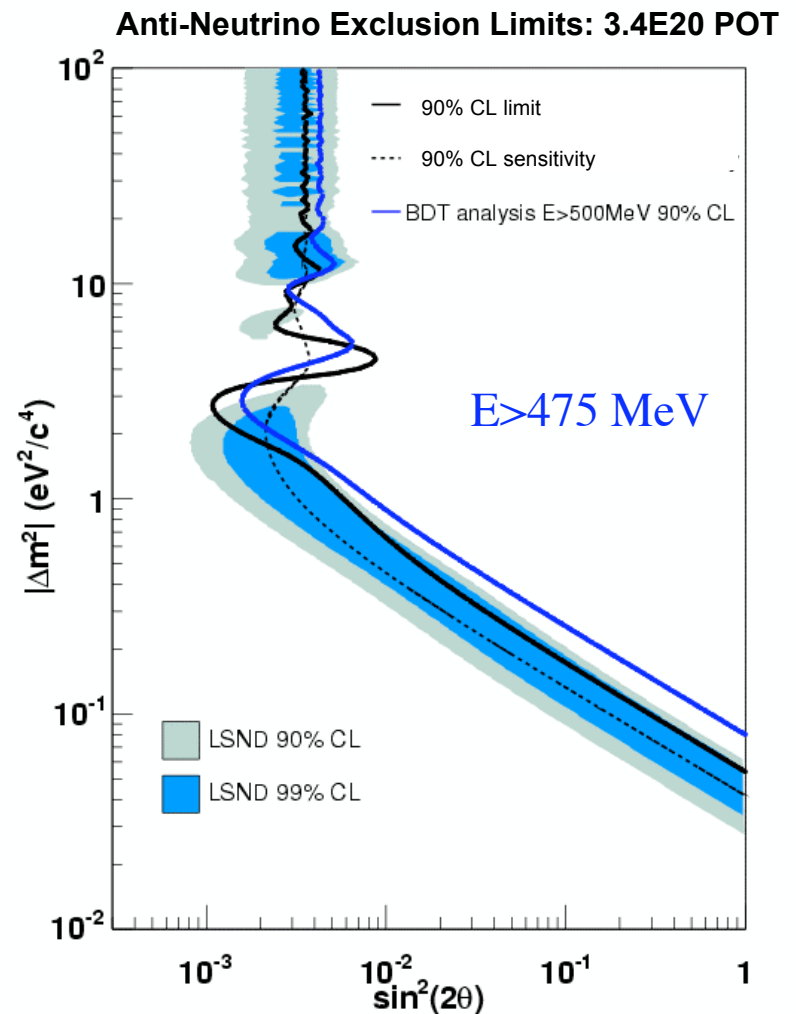
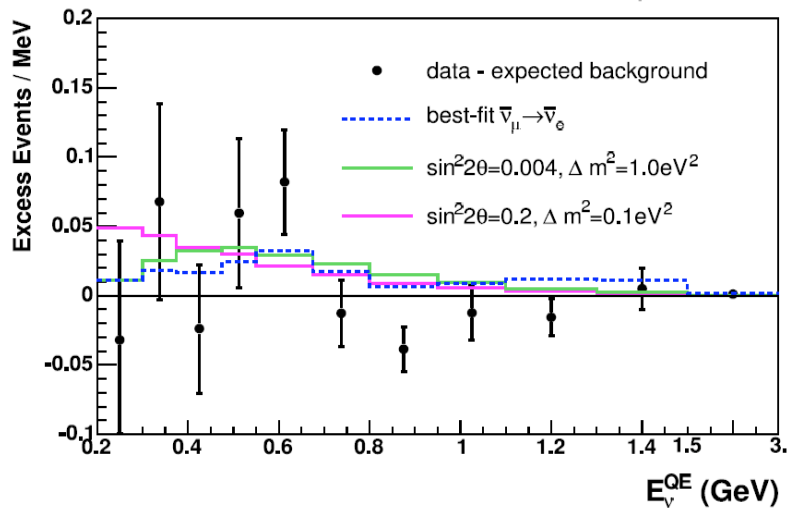
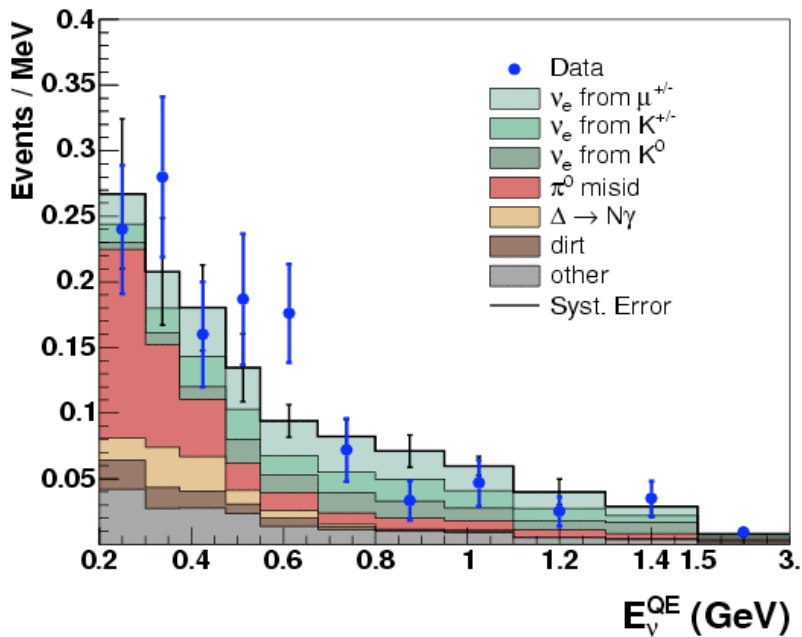
First Anti-neutrino Mode Results (2009): 3.4E20 POT

- **3.4E20 POT** collected in anti-neutrino mode
- From 200-3000 MeV excess is 4.8 ± 17.6 (stat+sys) events.
- Statistically small excess in 475-1250 MeV region
 - Only antineutrino's allowed to oscillate in fit
 - Limit from two neutrino fit excludes less area than sensitivity due to fit adding a LSND-like signal to account for wiggle
 - Stat error too large to distinguish LSND-like from null
- No significant excess $E < 475$ MeV.



E_ν [MeV]	200-475	475-1250
total background	60.29	57.78
ν_e intrinsic	17.74	43.23
ν_μ induced	42.54	14.55
NC π^0	24.60	7.17
NC $\Delta \rightarrow N\gamma$	6.58	2.02
Dirt	4.69	1.92
CCQE	2.86	1.24
other	3.82	2.20
LSND best fit	4.33	12.63

First Anti-neutrino Mode Results (2009): 3.4E20 POT



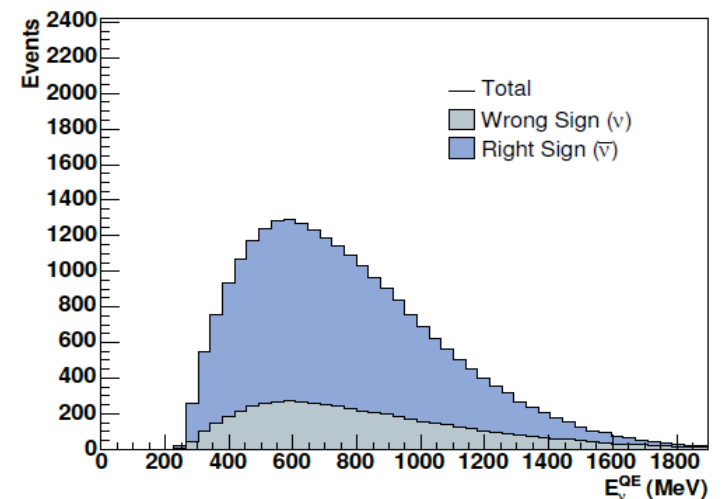
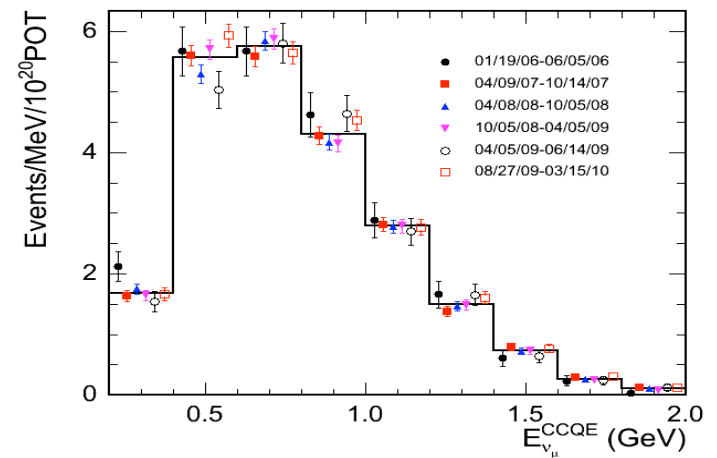
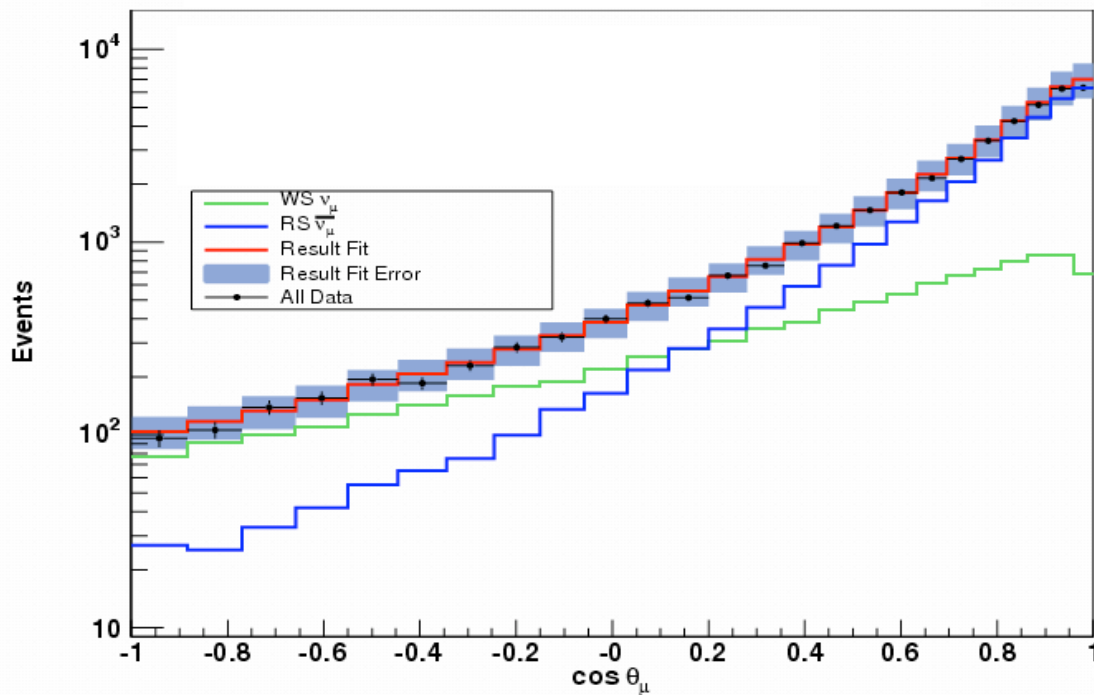
New Anti-neutrino mode results: $5.66E20$ POT
(70% more data)

Data Checks

- Beam and Detector low level stability checks; beam stable to 2%, and detector energy response to 1%.
- $\bar{\nu}_\mu$ rates and energy stable over entire antineutrino run.
- Independent measurement of π^0 rate for antineutrino mode.
- Measured dirt rates are similar in neutrino and antineutrino mode.
- Measured ν wrong sign component stable over time and energy.
- Checked off axis rates from NuMI beam.
- Above 475 MeV, about two thirds of the electron (anti)neutrino intrinsic rate is constrained by simultaneous fit to $\bar{\nu}_\mu$ data.
- New SciBooNE neutrino mode K^+ weight = $0.75 \pm 0.05(\text{stat}) \pm 0.30(\text{sys})$.
- One third of electron neutrino intrinsic rate come from K^0 , where we use external measurements and apply 30% error.
 - Would require $>3\sigma$ increase in K^0 normalization, but shape does not match well the excess.

Signal Prediction

- Assuming only right sign oscillates ($\bar{\nu}_\mu$)
- Need to know wrong sign vs right sign
- We measure it
- $\bar{\nu}_\mu$ CCQE gives more forward peaked muon

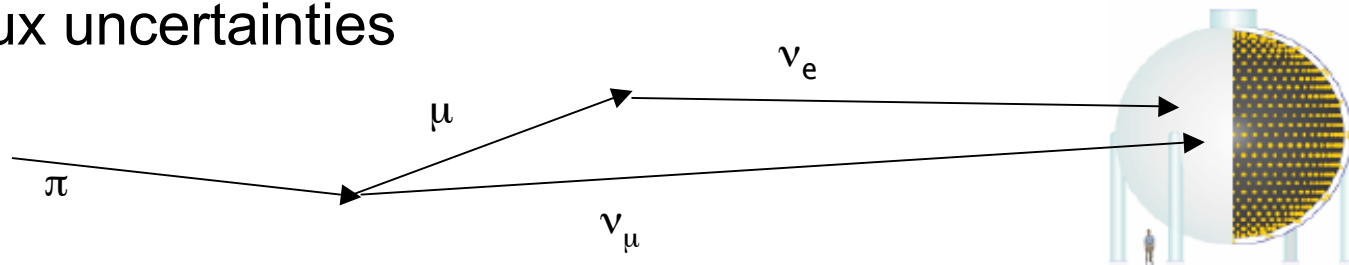


Oscillation Fit Method

- Maximum likelihood fit:

$$-2 \ln(L) = (x_1 - \mu_1, \dots, x_n - \mu_n) M^{-1} (x_1 - \mu_1, \dots, x_n - \mu_n)^T + \ln(|M|)$$

- Simultaneously fit
 - ν_e CCQE sample
 - High statistics ν_μ CCQE sample
- ν_μ CCQE sample constrains many of the uncertainties:
 - Flux uncertainties

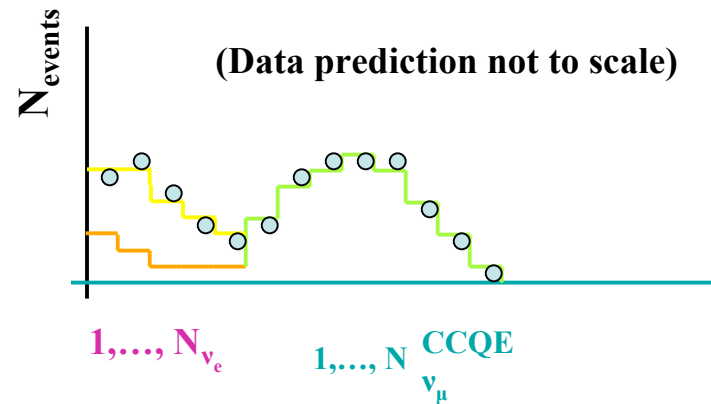
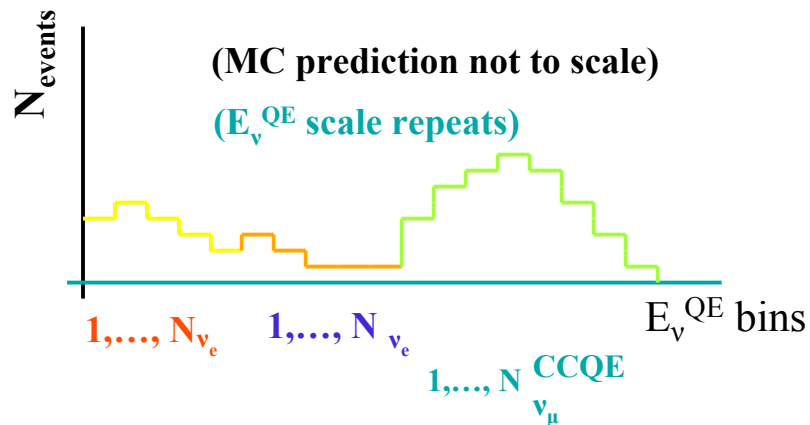


- Cross section uncertainties (CCQE process)

Constrained Fit

The following three distinct samples are used in the oscillation fits:

1. Background to ν_e oscillations
2. ν_e Signal prediction (dependent on Δm^2 , $\sin^2 2\theta$)
3. ν_μ CCQE sample, used to constrain ν_e prediction (signal+background)



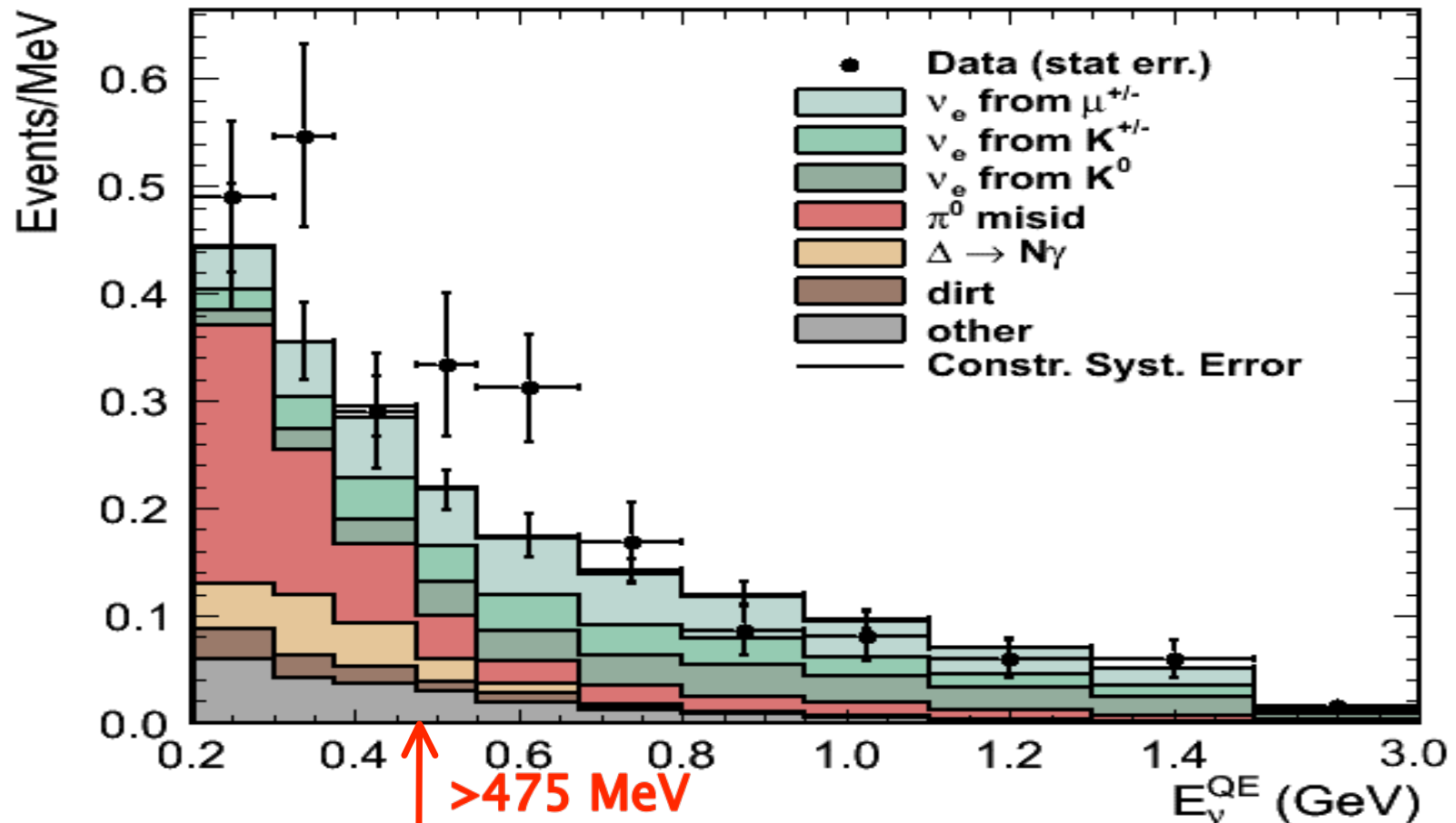
$$-2 \ln(L) = (x_1 - \mu_1, \dots, x_n - \mu_n) M^{-1} (x_1 - \mu_1, \dots, x_n - \mu_n)^T + \ln(|M|)$$

M_{ij} = full syst+stat covariance matrix at best fit prediction

logL calculated using both datasets (ν_e and ν_μ CCQE), and corresponding covariance matrix

New Anti-neutrino mode results: 5.66E20 POT

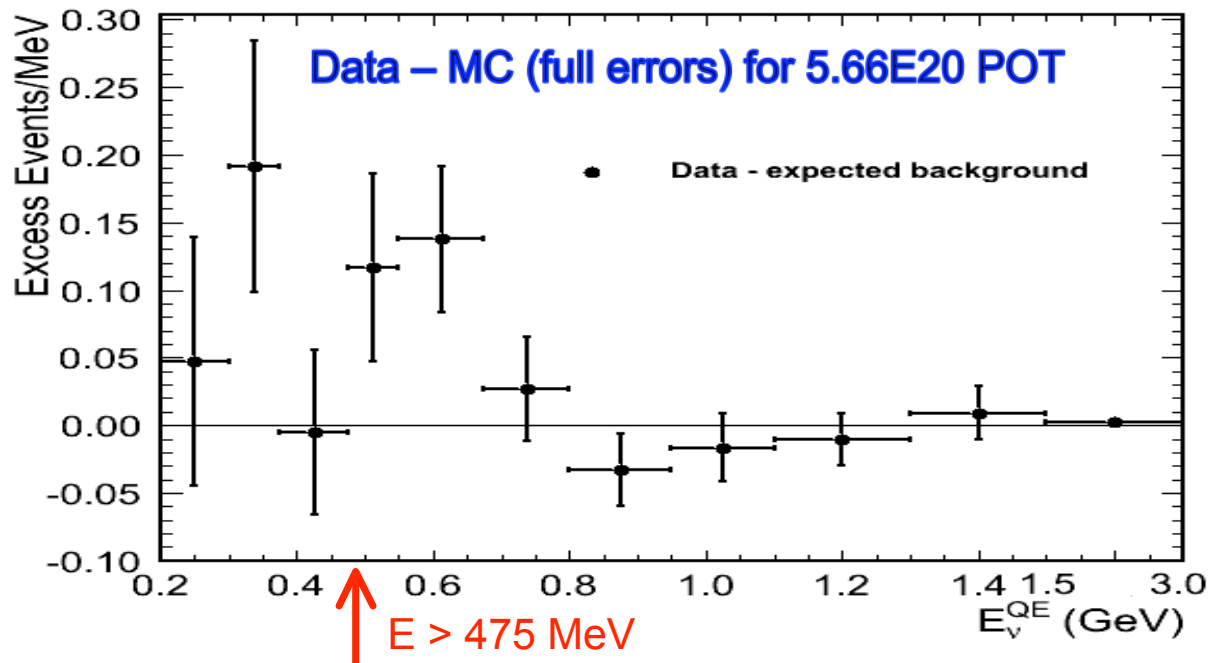
	200-475 MeV	475-1250 MeV	200-3000 MeV
Data	119	120	277
MC (stat+sys)	100.5 ± 14.3	99.1 ± 13.9	233.8 ± 22.5
Excess (stat)	$18.5 \pm 10.0 (1.9\sigma)$	$20.9 \pm 10.0 (2.1\sigma)$	$43.2 \pm 15.3 (2.8\sigma)$
Excess (stat+sys)	$18.5 \pm 14.3 (1.3\sigma)$	$20.9 \pm 13.9 (1.5\sigma)$	$43.2 \pm 22.5 (1.9\sigma)$



Testing the Null Hypothesis

- Model independent.
- At null look at the χ^2 distribution of fake experiments (thrown from null error matrix).

	chi2/NDF	probability
E>475MeV	26.8/14.9	3.0%
E>200MeV	33.2/18.0	1.6%



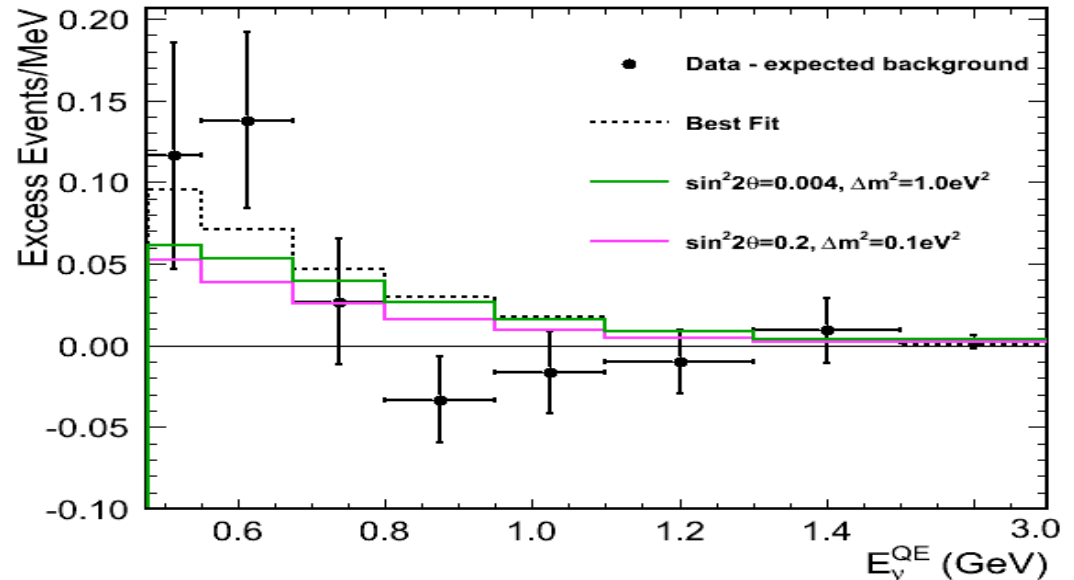
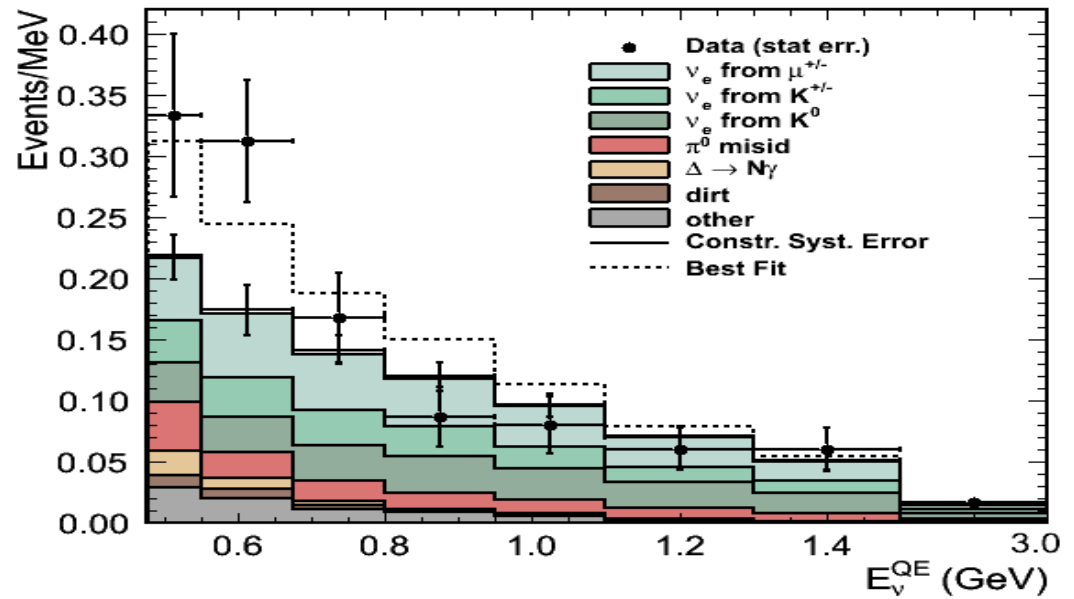
Comparison to LSND

	$E_\nu(\text{QE})$ [MeV]		
	200-475	475-1250	1250-3000
MC Background	100.5	99.1	34.2
Data	119	120	38
Excess	18.5 ± 14.3	20.9 ± 13.9	3.8 ± 5.8
LSND Best Fit	7.6	22.0	3.5
Expectation from ν low-E excess	11.6	0	0
LSND + Low-E	19.2	22.0	3.5

- Errors quoted here are stat+sys.
- Excess consistent with the expectation from LSND and adding the low energy excess scaled for neutrinos (wrong-sign).
- Expected 67 events at low energy (200-475 MeV) if neutrino low E excess is due to a Standard Model NC gamma-ray mechanism, e.g. Axial Anomaly.

Oscillation Fit

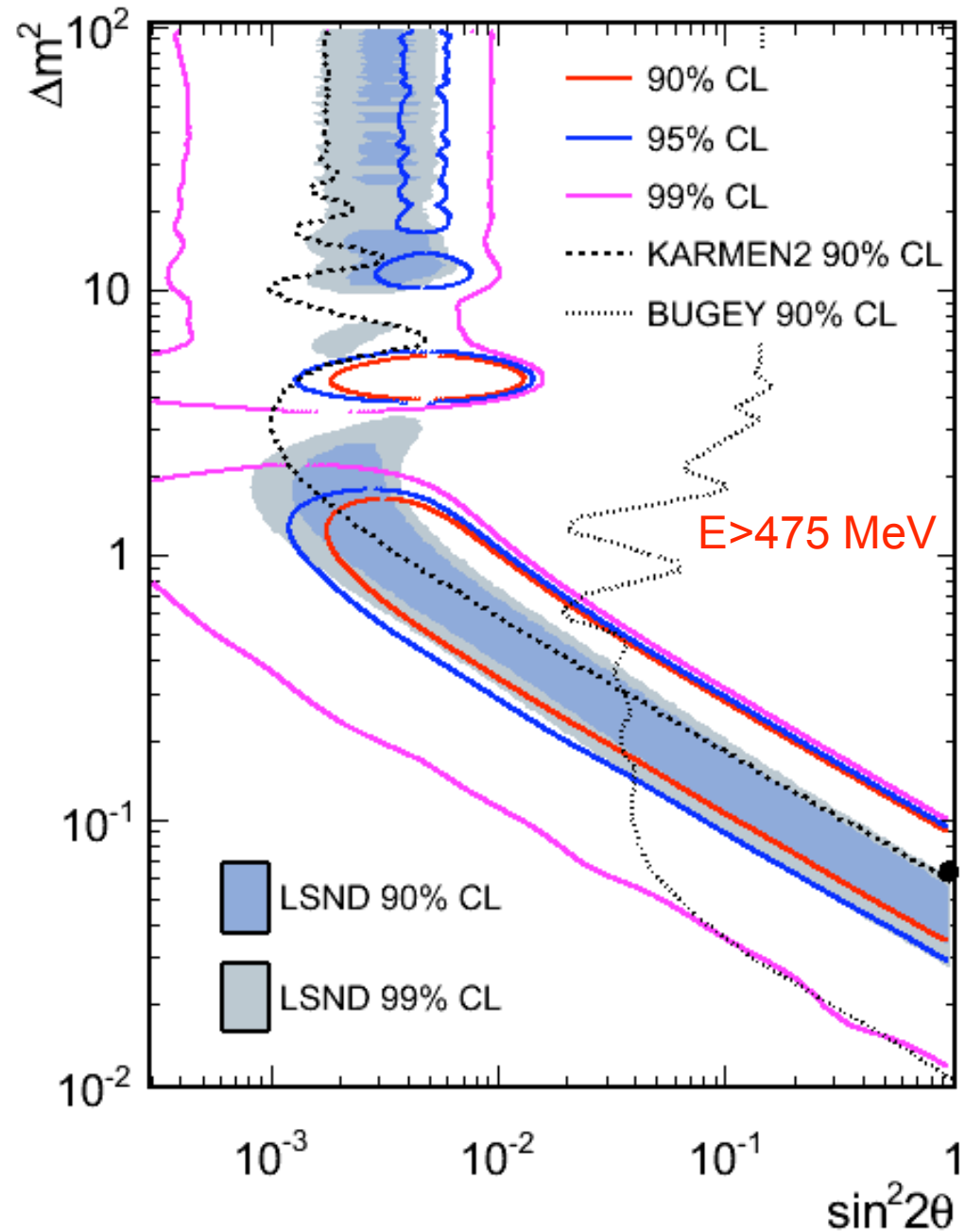
- Results for **5.66E20 POT**.
- Maximum likelihood fit.
- Only antineutrinos allowed to oscillate.
- $E > 475$ MeV region is free of effects of low energy neutrino excess. This is the same official oscillation region as in neutrino mode.
- Results published.



Accepted by PRL
(arXiv:1007.1150 [hep-ex])

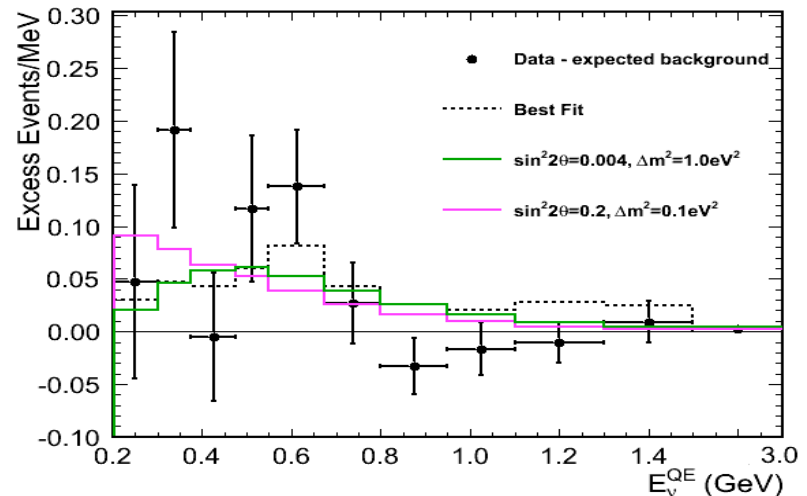
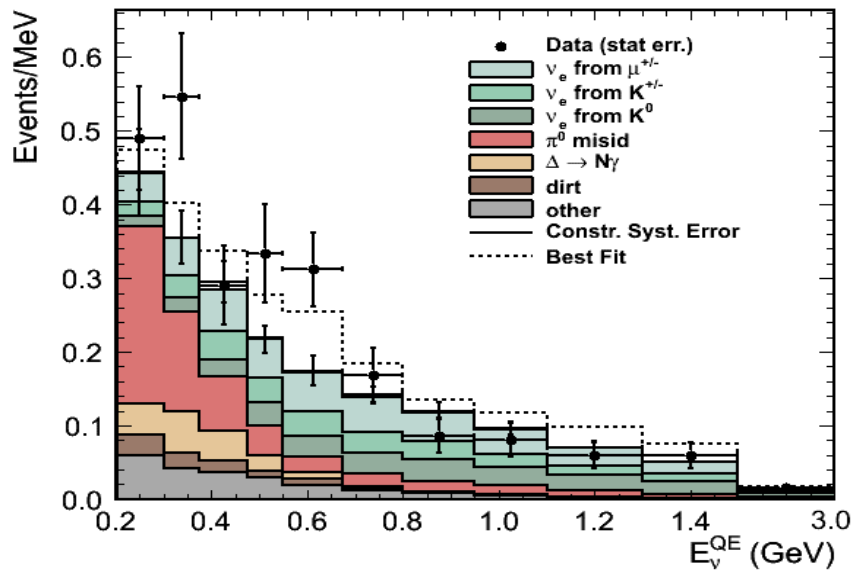
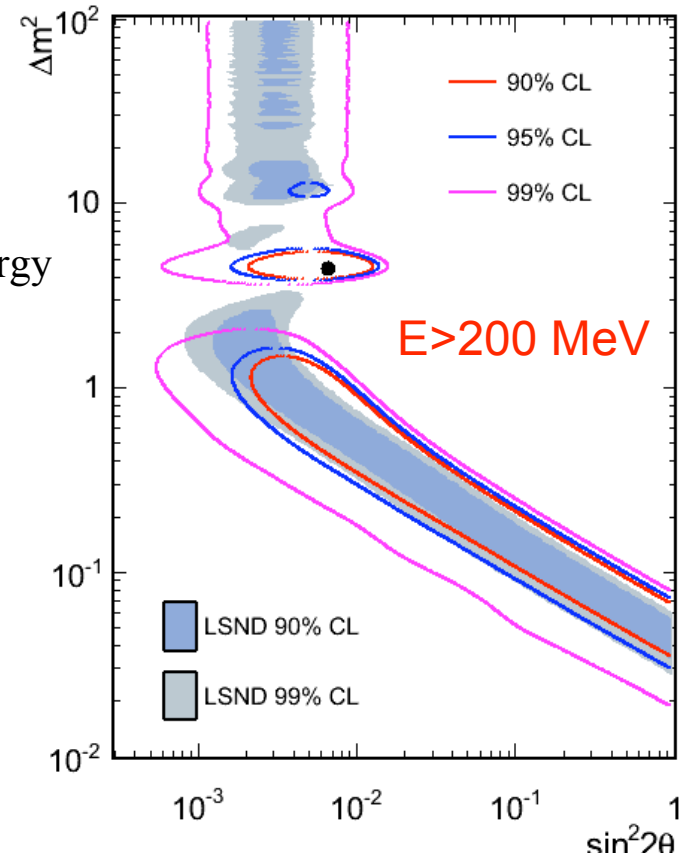
Oscillation Fit

- Results for **5.66E20 POT**
- Maximum likelihood fit.
- Null excluded at 99.4% with respect to the two neutrino oscillation fit.
- Best Fit Point
($\Delta m^2, \sin^2 2\theta$) =
(0.064 eV², 0.96)
 χ^2/NDF = 16.4/12.6
 $P(\chi^2) = 20.5\%$



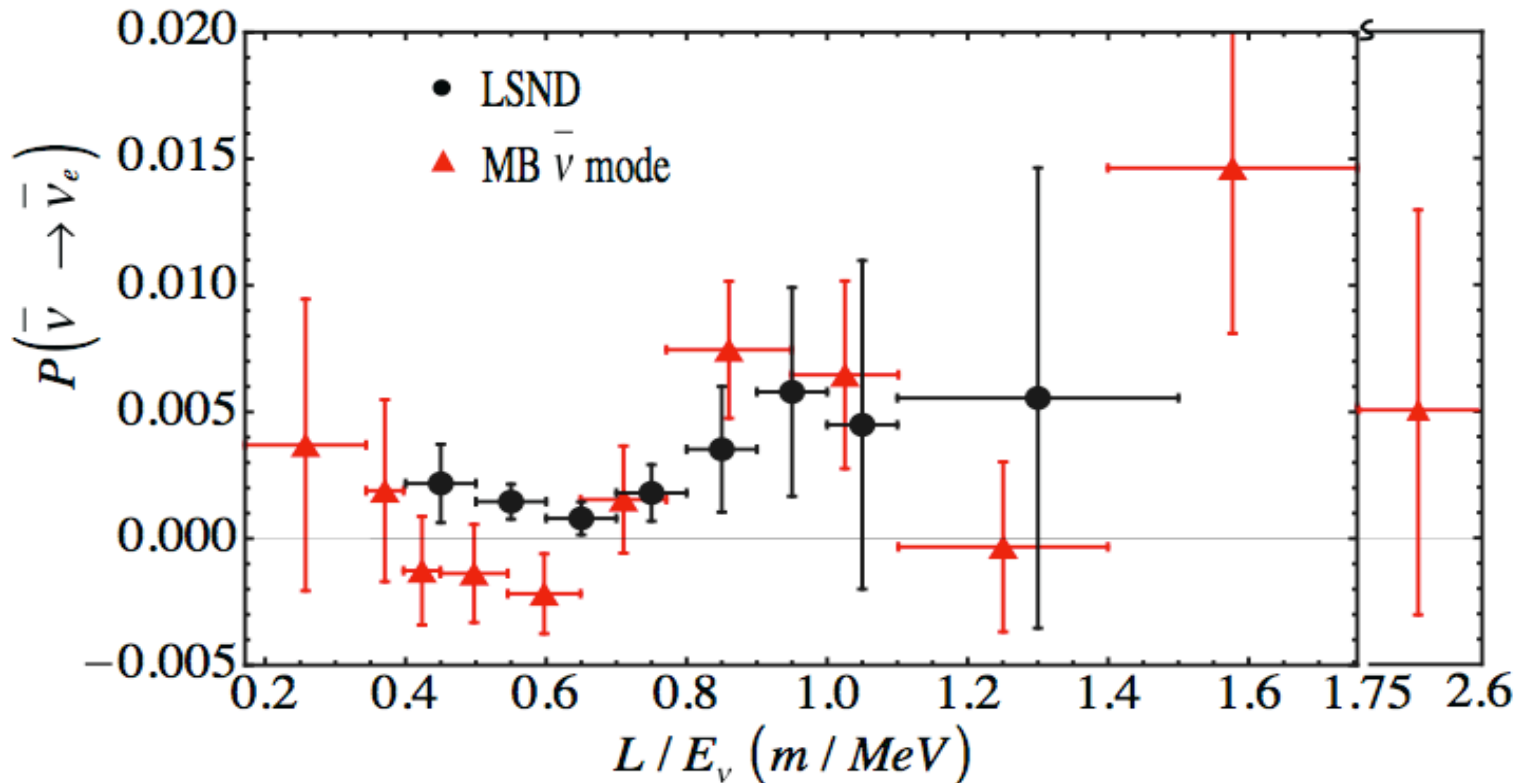
Oscillation Fit with $E_\nu > 200$ MeV

- Results for **5.66e20 POT**.
- Does not include effects (subtraction) of neutrino low energy excess.
- Maximum likelihood fit method.
- Null excluded at 99.6% with respect to the two neutrino oscillation fit (model dependent).
- Best Fit Point $(\Delta m^2, \sin^2 2\theta) = (4.42 \text{ eV}^2, 0.0066)$
 $\chi^2/\text{NDF} = 20.4/15.3$, $P(\chi^2) = 17.1\%$



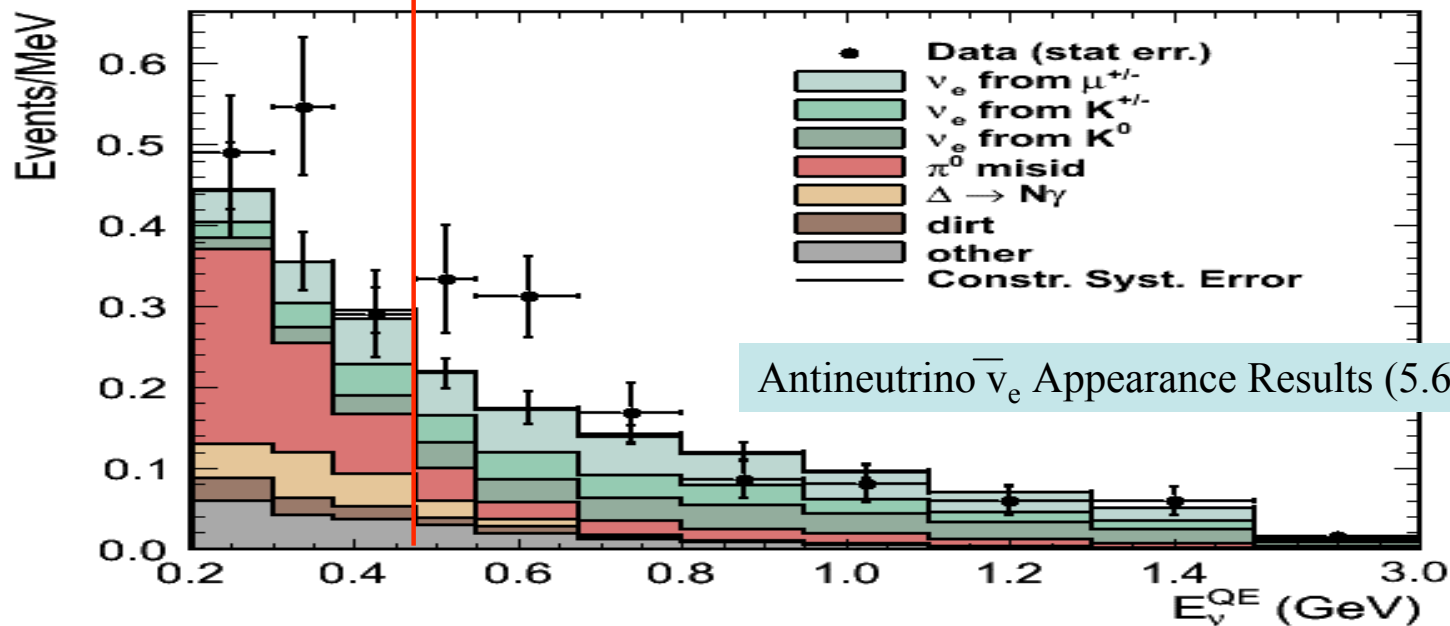
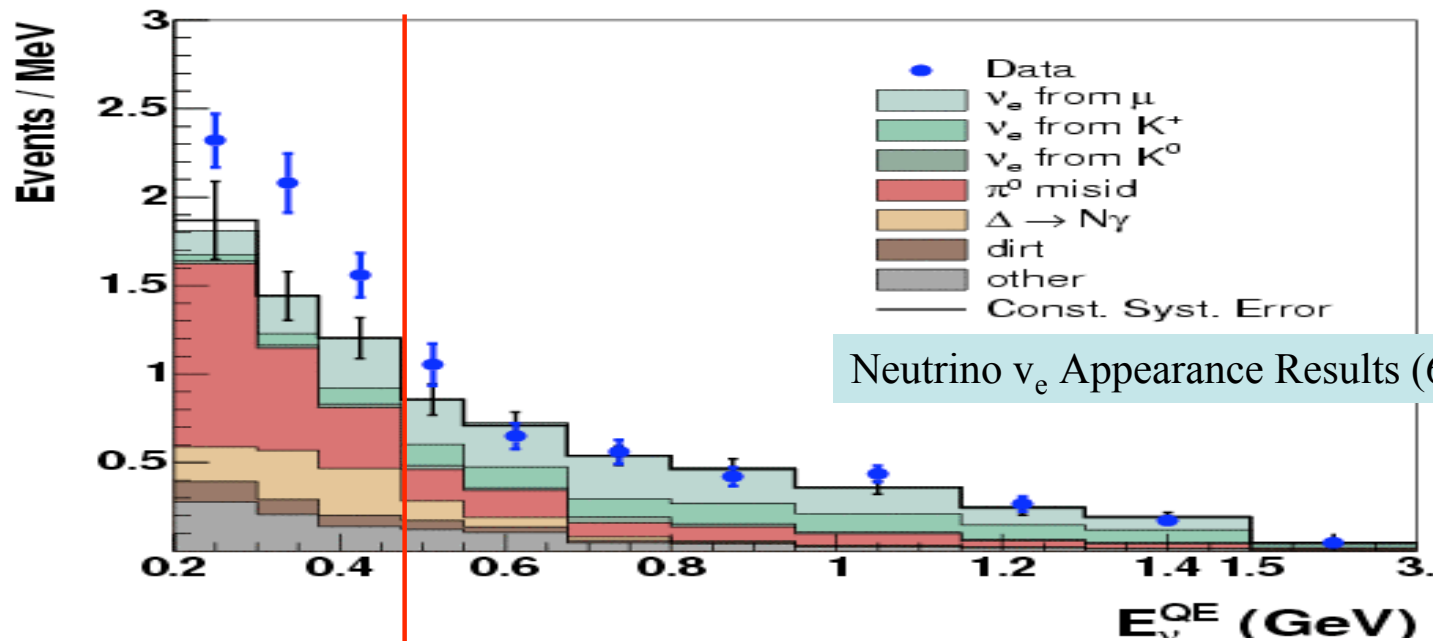
L/E Plot

- Data used for LSND and MiniBooNE correspond to $20 < E_\nu < 60$ MeV and $200 < E_\nu < 3000$ MeV, respectively.
- Oscillation probability is event excess divided by the number of events expected for 100% $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transformation.
- L is reconstructed distance travelled by the antineutrino from the mean neutrino production point to the interaction vertex; E_ν is the reconstructed antineutrino energy.



The data points include both statistical and systematic errors.

Comparison of ν_e and $\bar{\nu}_e$ Appearance Results



Summary of Results

- The MiniBooNE ν_e and $\bar{\nu}_e$ appearance picture starting to emerge is the following:

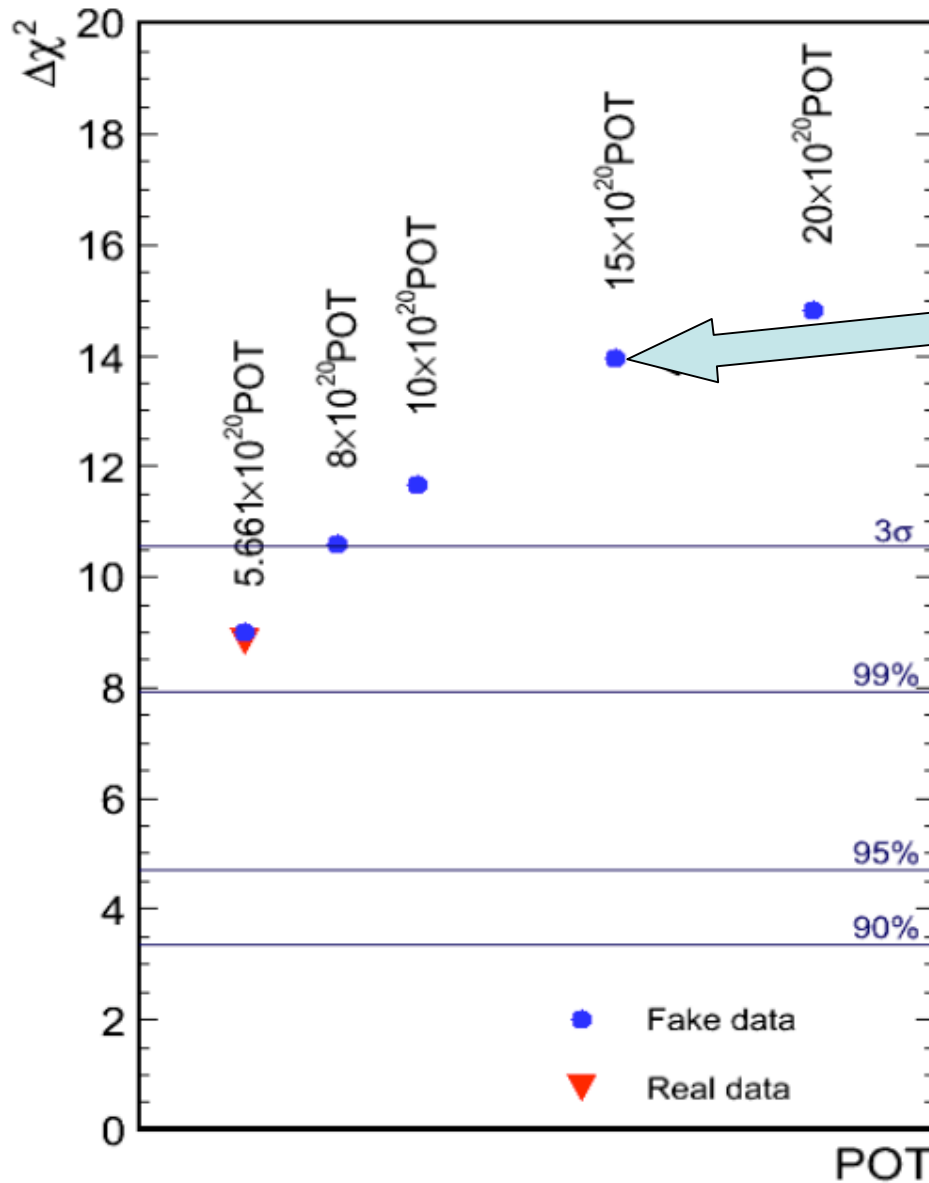
1) Neutrino Mode:

- a) $E < 475$ MeV: An unexplained 3σ electron-like excess.
- b) $E > 475$ MeV: A two neutrino fit rules out LSND at the 98% CL.

2) Anti-neutrino Mode:

- a) $E < 475$ MeV: A small 1.3σ electron-like excess.
- b) $E > 475$ MeV: An excess that is 3.0% consistent with null. Two neutrino oscillation fits consistent with LSND at 99.4% CL relative to null.

Future MiniBooNE Running



Signal significance could continue to significantly grow up to 15×10^{20} POT where it is 3.7σ .

MiniBooNE Collaboration requested 15×10^{20} POT to complete the run in current configuration.

Essentially no gain from statistics after.

Future Prospects

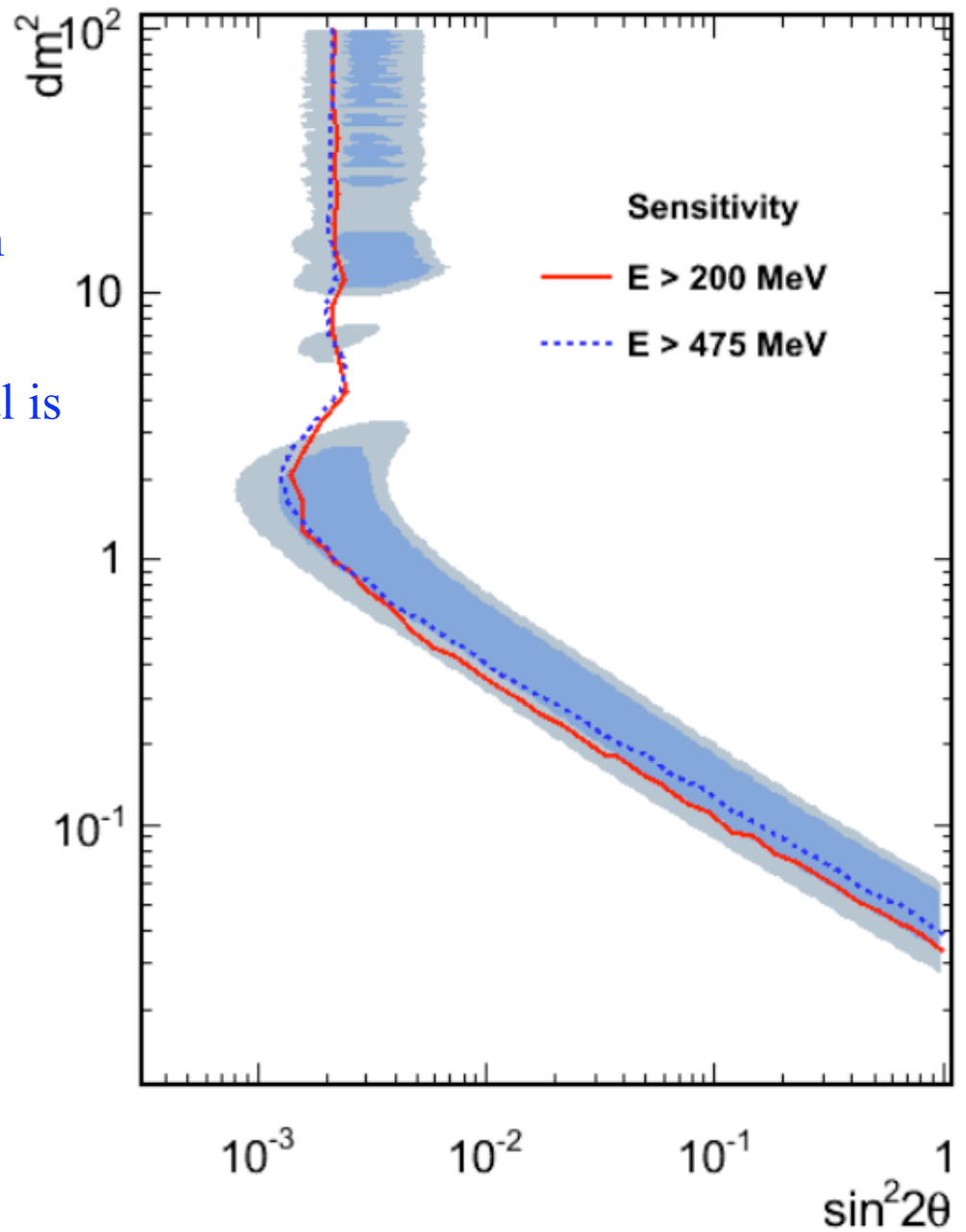
- **Need more statistics**
 - MiniBooNE is running to double antineutrino data set for a total of $\sim 10 \times 10^{20}$ POT by Spring 2011.
 - If signal continues at current rate, two neutrino best fit will be $\sim 3\sigma$ (with $> 8 \times 10^{20}$ POT).
 - Requested $\sim 15 \times 10^{20}$ POT to achieve $\sim 4\sigma$ evidence.
- **There are follow on experiments at FNAL and elsewhere**
 - μ Boone has CD-1 approval.
 - BooNE (LOI). A MB-like near detector at 200 m (when MiniBooNE finished in current configuration).
 - Proposal (Carlo Rubia) for two detector LAr detector at CERN PS ring.
 - Various other ideas (DAEdALUS or accelerator at few km distance from Gd-loaded SK, etc).
 - Ideas involving NO ν A near detector(s).

Thank you!

Backup Slides

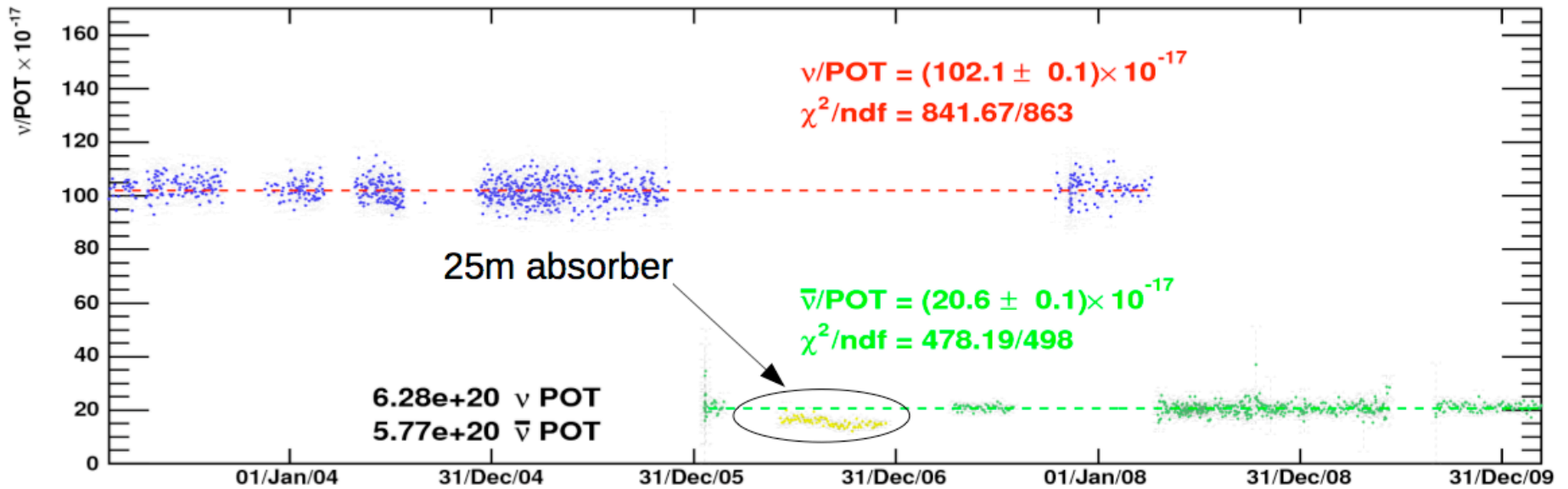
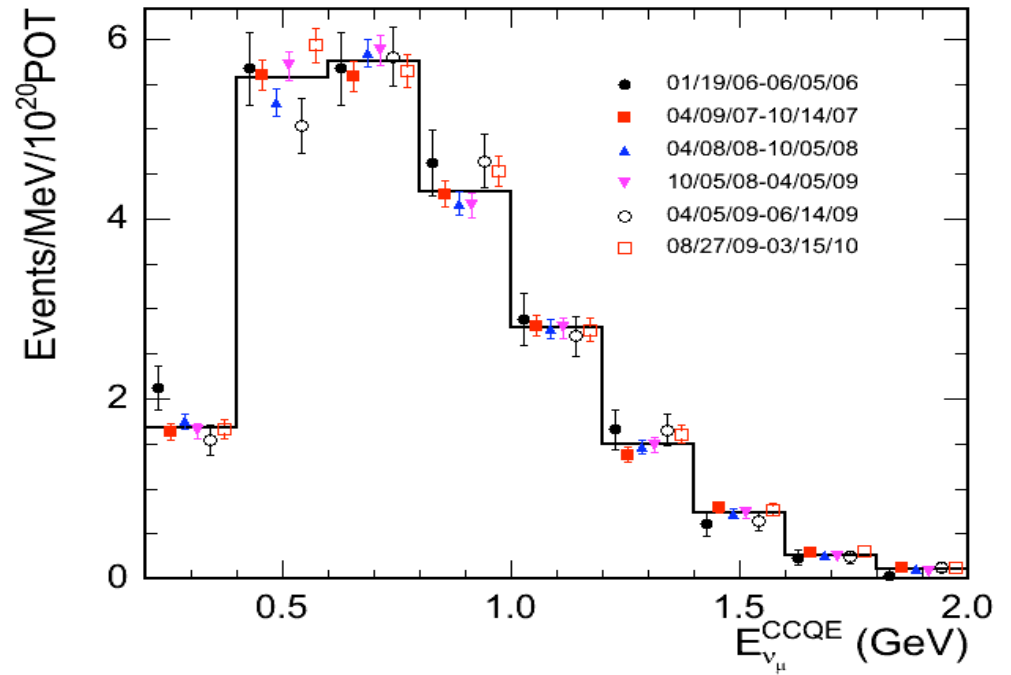
Sensitivity

- MiniBooNE uses $E_\nu > 475$ MeV region for oscillation fits.
- Energy region where LSND-type signal is expected.
- If $E_\nu < 475$ MeV:
 - Large backgrounds
 - Big systematics
 - Not sensitive to LSND oscillation signal



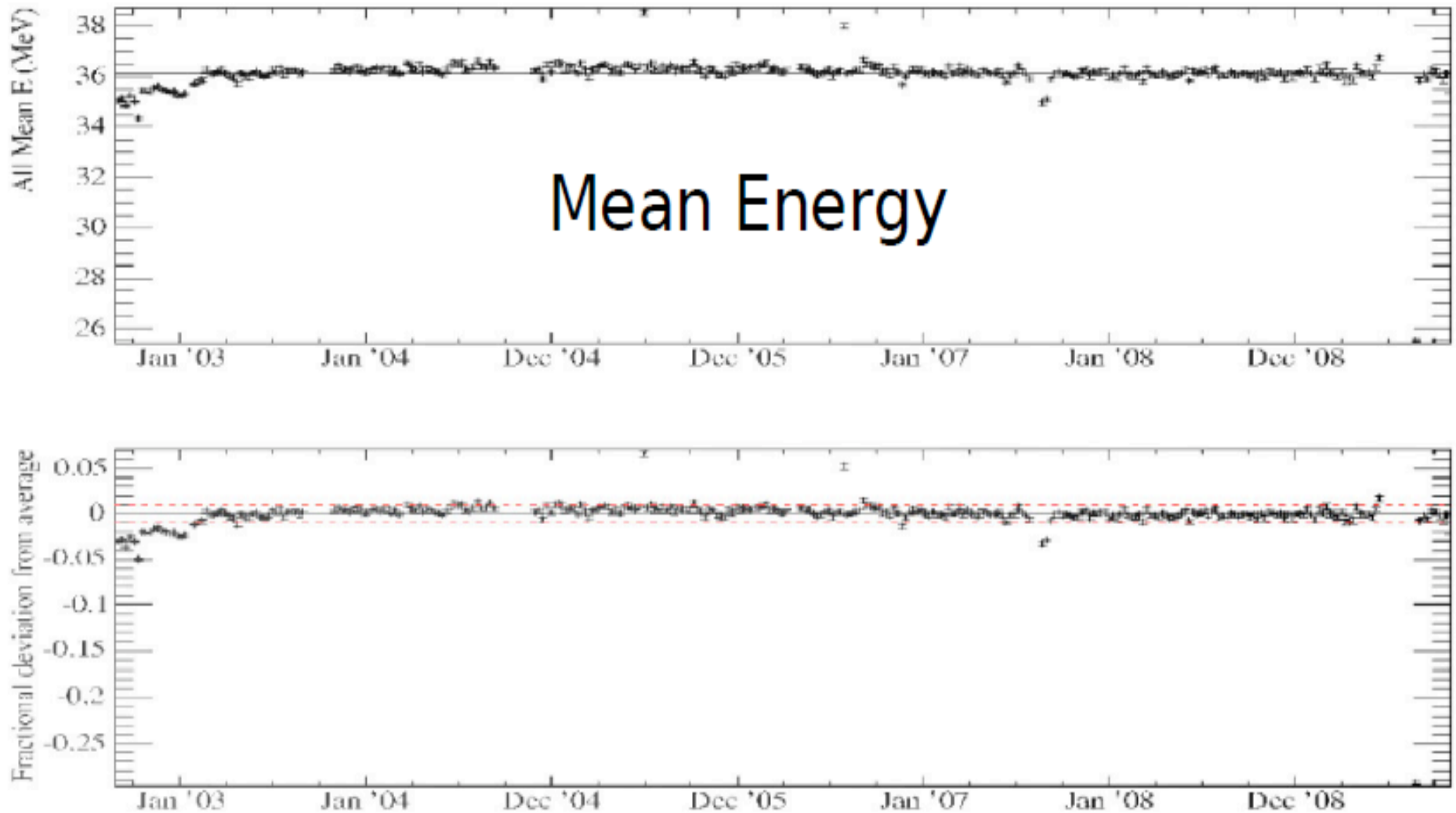
Beam and Data Stability

Very stable throughout the run.

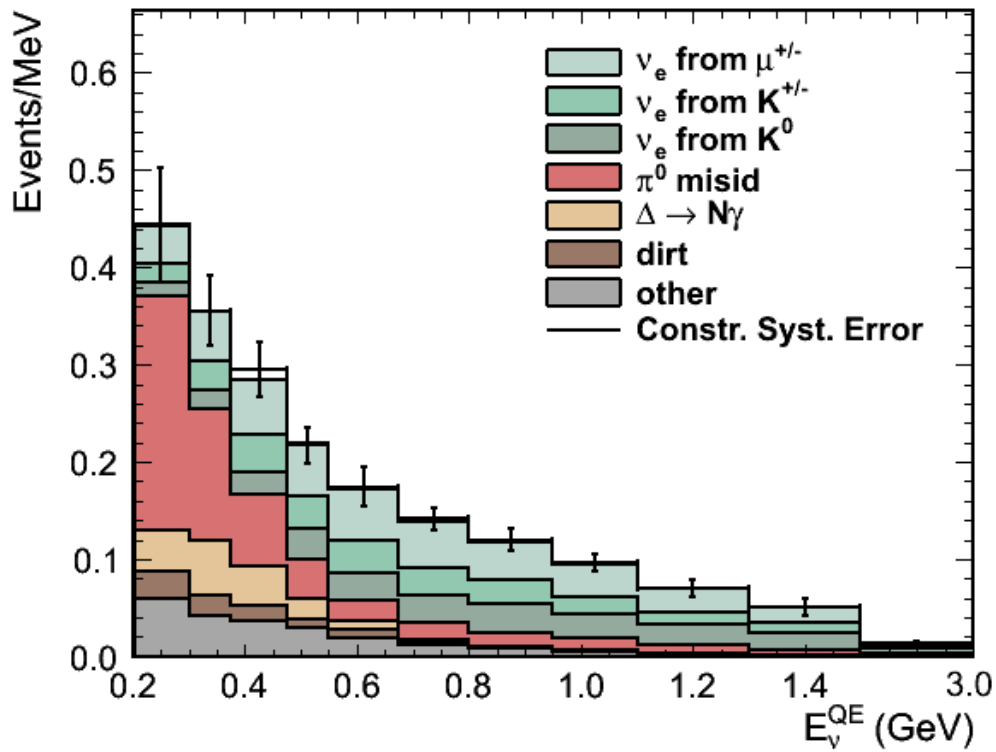


Energy Scale Stability

Michel electron mean energy is within 1% since the start of data taking.

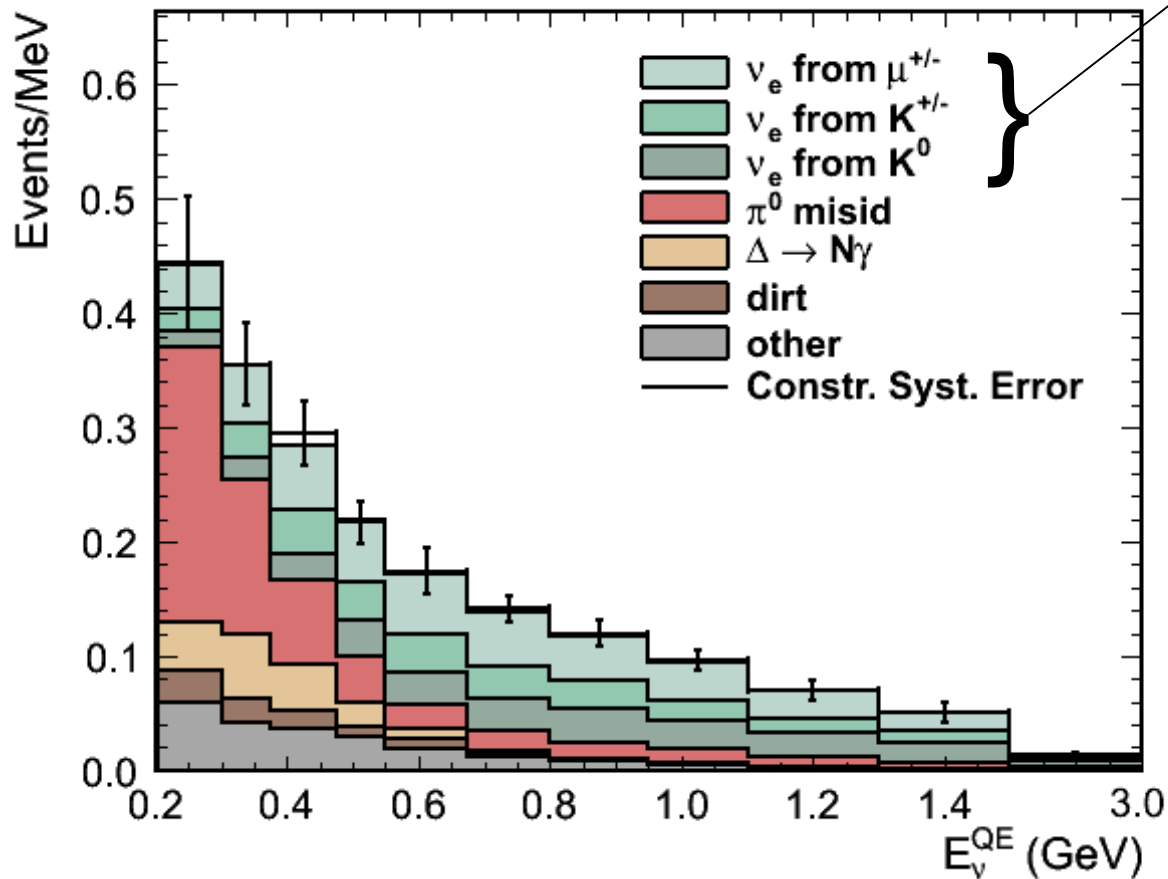


Background Prediction



	200-475	475-1250	
μ^\pm	13.45	31.39	Intrinsic ν_e
K^\pm	8.15	18.61	
K^0	5.13	21.2	
Other ν_e	1.26	2.05	
NC π^0	41.58	12.57	Mis-ID
$\Delta \rightarrow N\gamma$	12.39	3.37	
dirt	6.16	2.63	
ν_μ CCQE	4.3	2.04	
Other ν_μ	7.03	4.22	
Total	99.45	98.08	

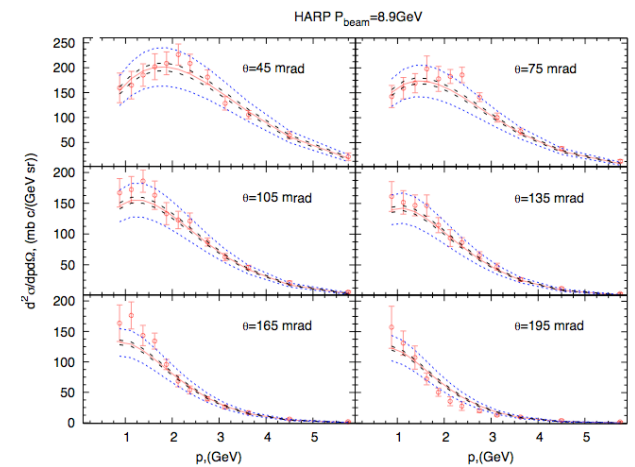
Background Prediction



- Intrinsic ν_e

External measurements

-HARP $p+\text{Be}$ for π^\pm



-Sanford-Wang fits to world K^+/K^0 data

Published PRD 79, 072002 (2009)

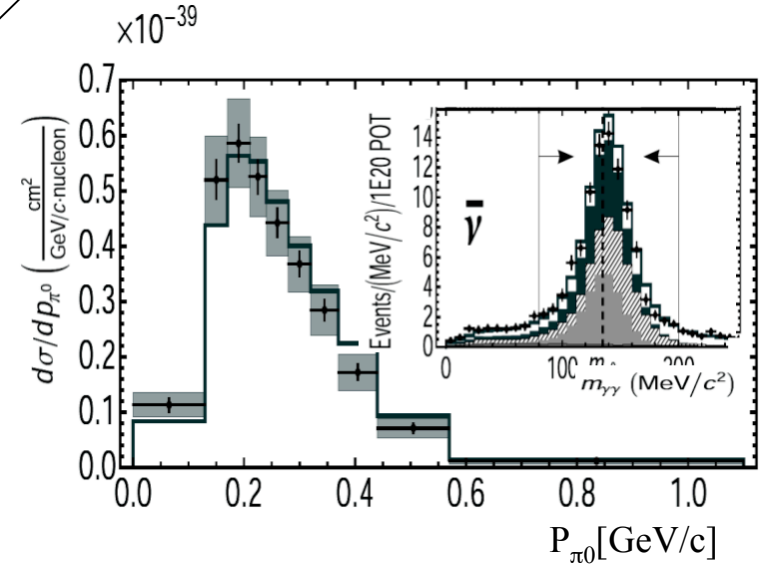
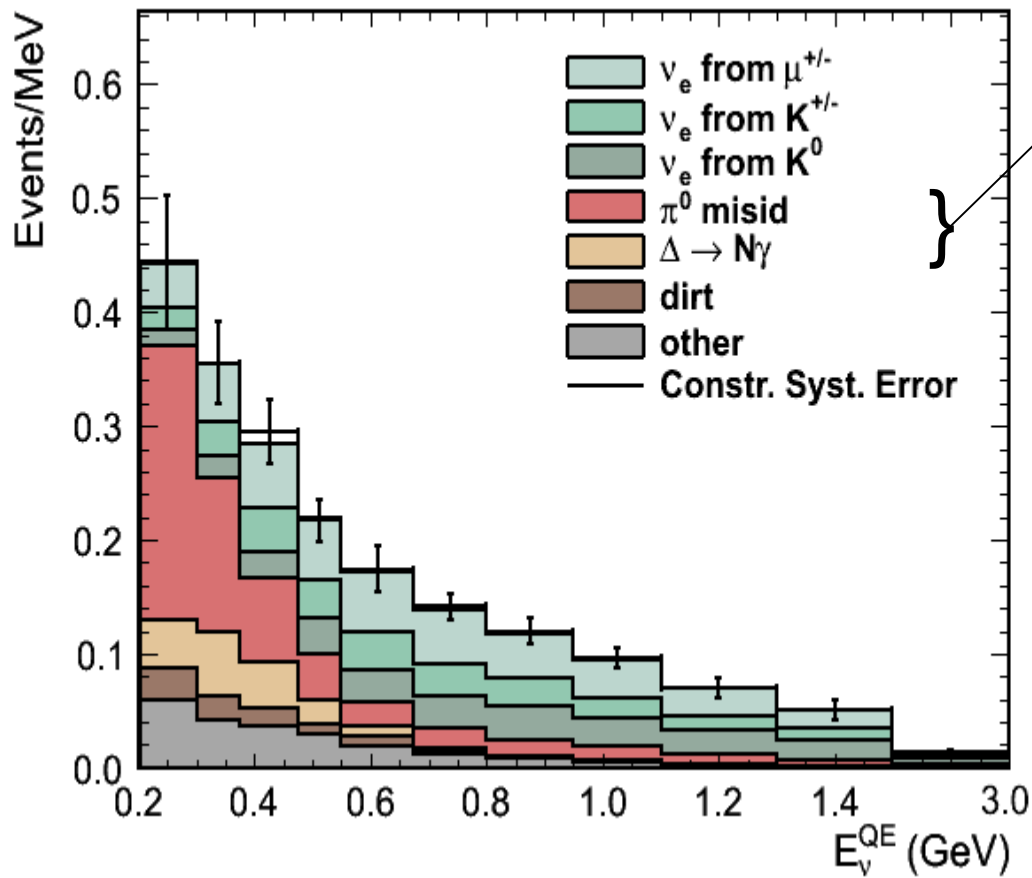
MiniBooNE data
constrained

Background Prediction

Neutral Current π^0

MiniBooNE NC π^0

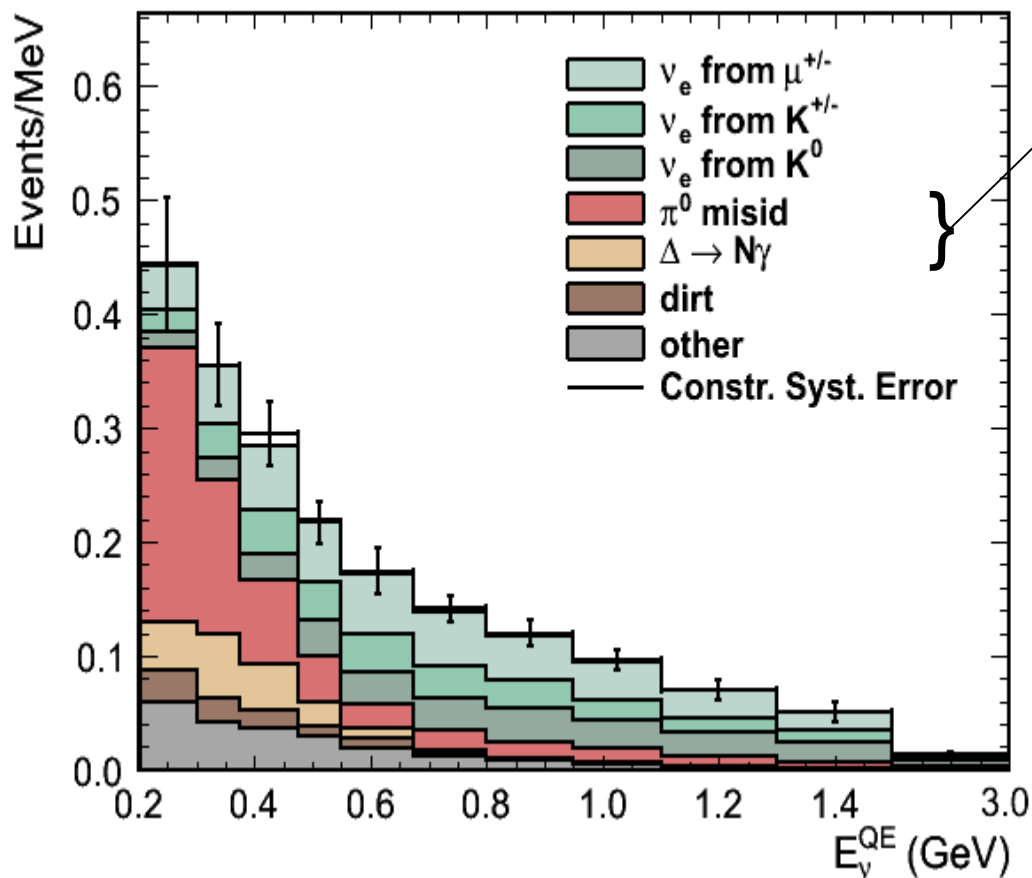
measurement



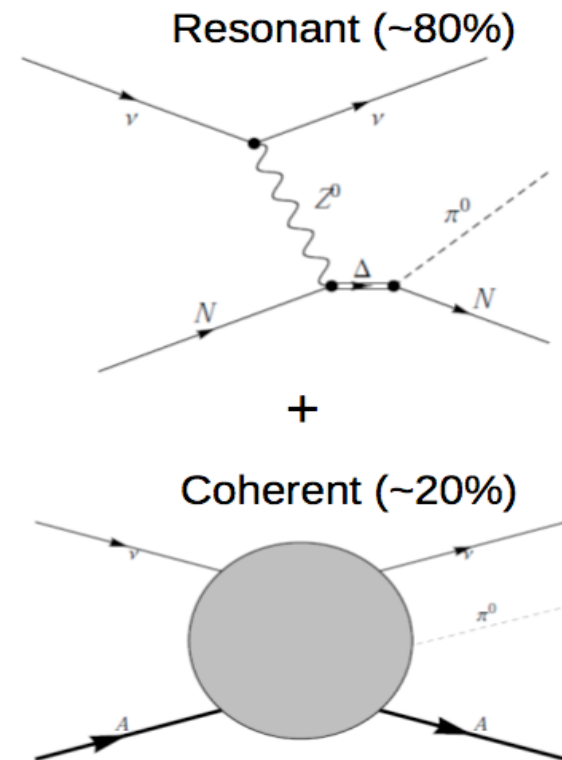
Published PRD 81, 013005 (2010)

Constrains radiative
 Δ decays as well

Background Prediction



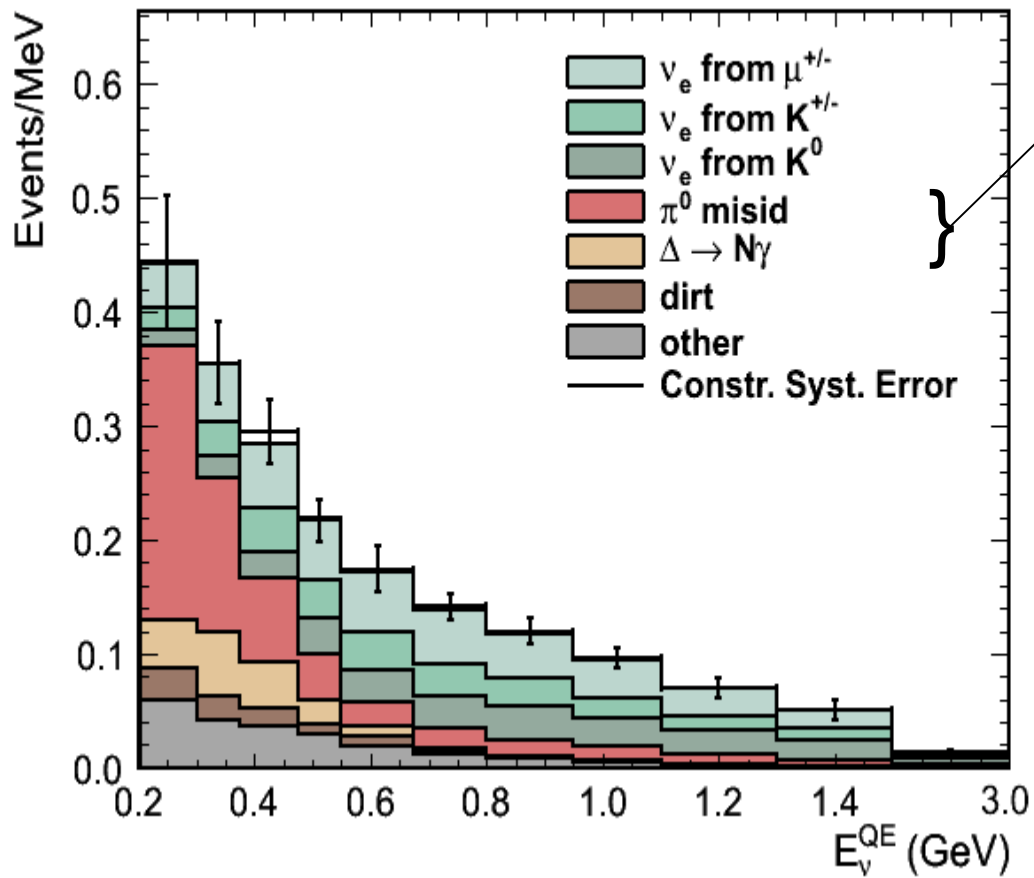
Neutral Current π^0



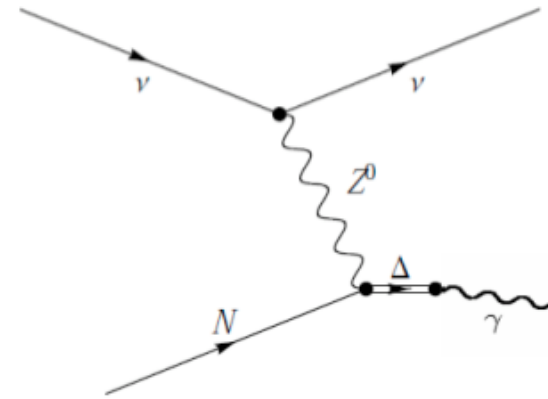
Measured Coherent/
Resonant Ratio

Published PLB 664, 41 (2008)

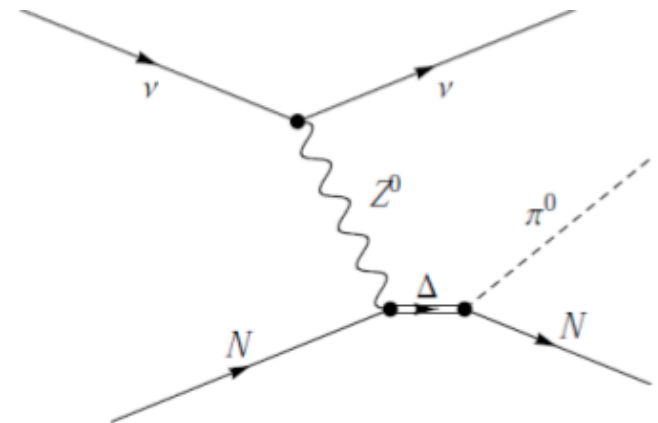
Background Prediction



$$\Delta \rightarrow N\gamma$$

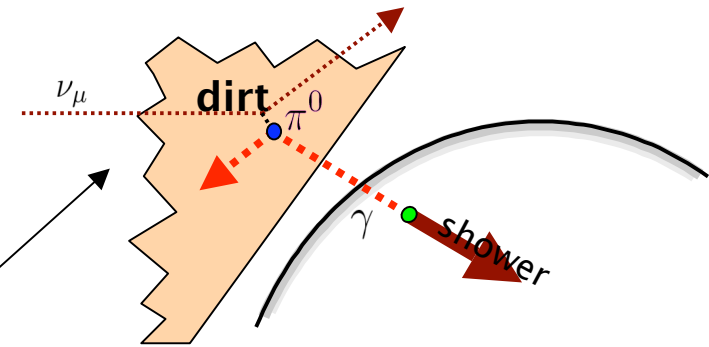
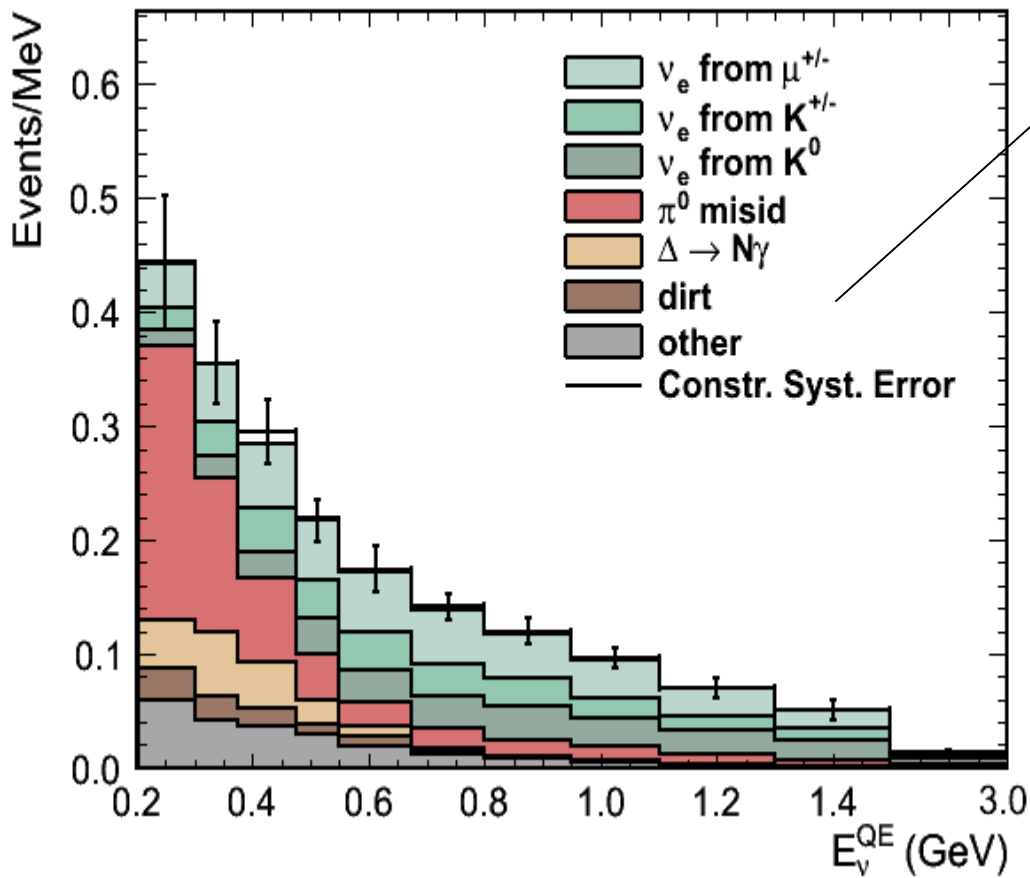


Use MiniBooNE NC π^0 measurement to constrain



Background Prediction

Dirt Events



- Events at high radius pointing toward center
- low energy depositions

MiniBooNE measurement

Background Uncertainties

Uncertainty (%)	200-475MeV	475-1100MeV
π^+	0.4	0.9
π^-	3	2.3
K^+	2.2	4.7
K^-	0.5	1.2
K^0	1.7	5.4
Target and beam models	1.7	3
Cross sections	6.5	13
NC pi0 yield	1.5	1.3
Hadronic interactions	0.4	0.2
Dirt	1.6	0.7
Electronics & DAQ model	7	2
OM	8	3.7
Total	13.43	16.02

-Unconstrained
 ν_e background
uncertainties

-Propagate input
uncertainties from
either MiniBooNE
measurement or
external data

Background Uncertainties

Uncertainty (%)	200-475MeV	475-1100MeV
π^+	0.4	0.9
π^-	3	2.3
K^+	2.2	4.7
K^-	0.5	1.2
K^0	1.7	5.4
Target and beam models	1.7	3
Cross sections	6.5	13
NC pi0 yield	1.5	1.3
Hadronic interactions	0.4	0.2
Dirt	1.6	0.7
Electronics & DAQ model	7	2
Optical Model	8	3.7
Total	13.43	16.02

-Uncertainty determined by varying underlying cross section model parameters (M_A , Pauli blocking, ...)

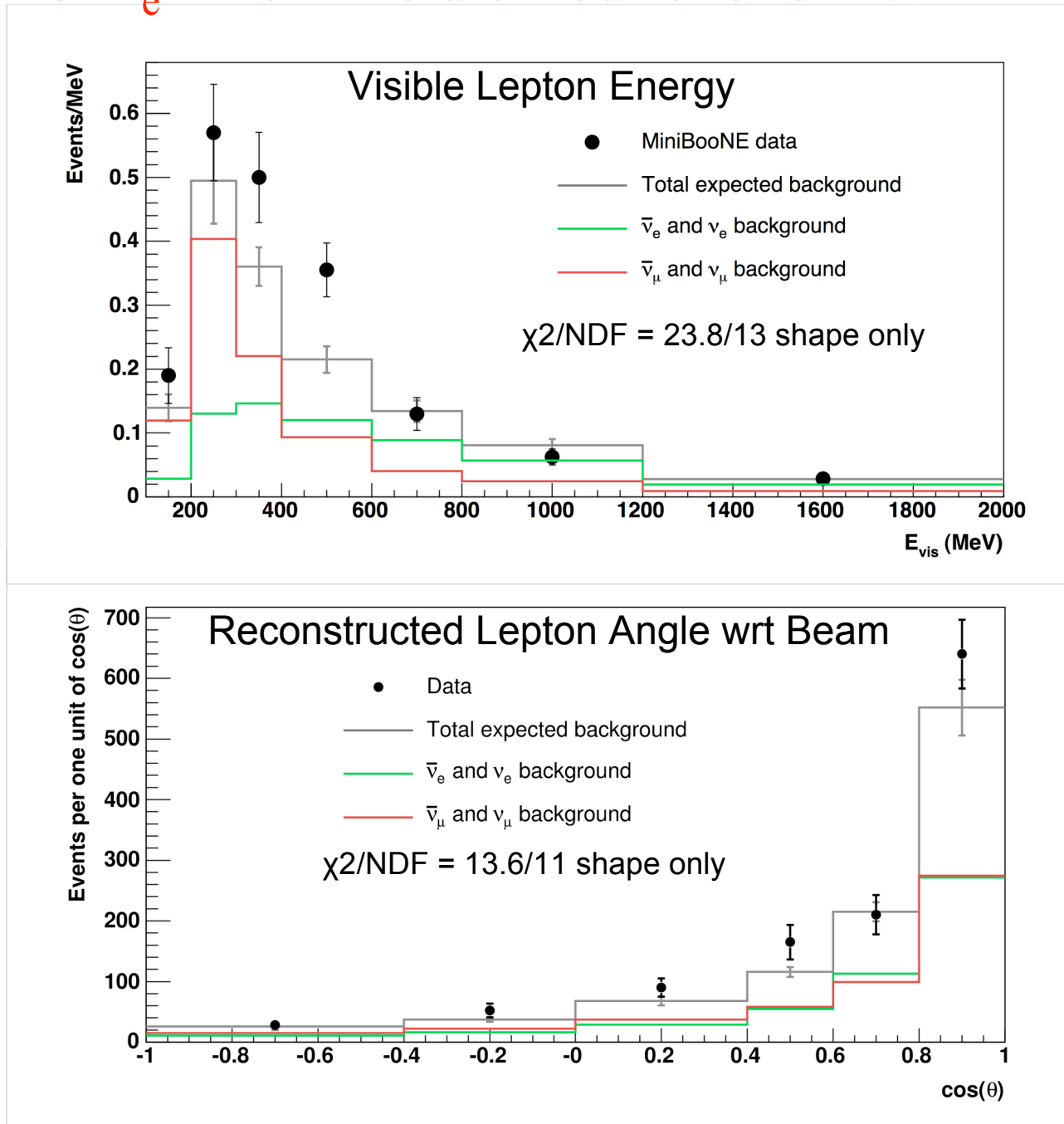
-Many of these parameters measured in MiniBooNE

Background Uncertainties

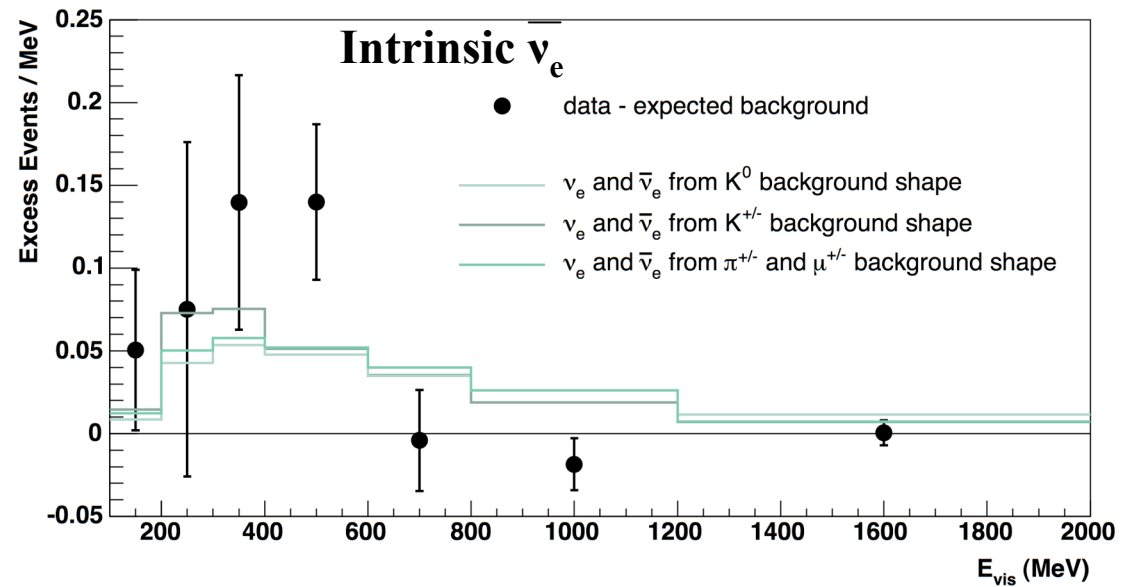
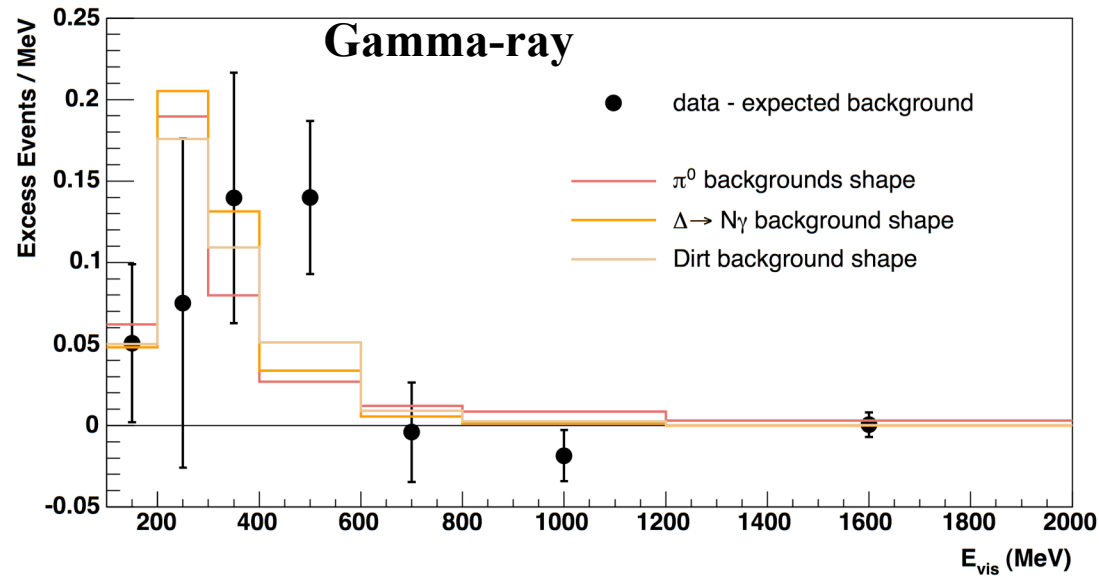
Uncertainty (%)	200-475MeV	475-1100MeV
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Cross sections	6.5	13
NC pi0 yield	1.5	1.3
Hadronic interactions	0.4	0.2
Dirt	1.6	0.7
Electronics & DAQ model	7	2
Optical Model	8	3.7
Total	13.43	16.02

-Uncertainty in light creation, propagation and detection in the detector

Other $\bar{\nu}_e$ kinematic distributions for 5.66E20 POT

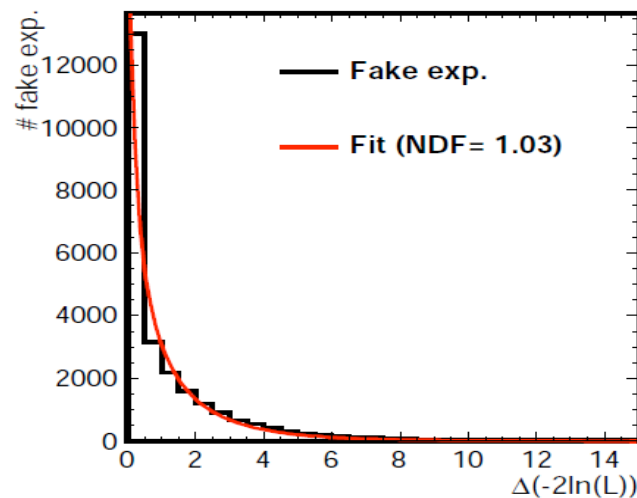


Background $\bar{\nu}_e$ Evis distributions for 5.66E20 POT



Drawing Contours

- Use frequentist approach.
- Generate fake data experiments on grid of $(\sin^2 2\theta, \Delta m^2)$ points.
- At each point find the cut on likelihood ratio for $X\%$ confidence level such that $X\%$ of experiments below cut
- Fitting two parameters, so naively expect χ^2 distribution with 2 degrees of freedom, in reality at null it looks more like 1 degree of freedom.



Oscillation Fit with $E_\nu > 200$ MeV (include low E_ν ν -mode effects)

- Results for $5.66e20$ POT.
- Assume simple scaling of neutrino low energy excess; subtract 11.6 events from low energy region (200-475 MeV).
- Maximum likelihood fit method.
- Null excluded at 99.6% with respect to the two neutrino oscillation fit (model dependent).
- Best Fit Point $(\Delta m^2, \sin^2 2\theta) = (4.42 \text{ eV}^2, 0.0066)$
 $\chi^2/\text{NDF} = 21.6/15.3$, $P(\chi^2) = 13.7\%$.

