2540 km: Bimagic baseline for neutrino oscillation parameters

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Abstract.
We consider a low energy neutrino factory setup with a detector placed at a distance of 2540 km. At this baseline, for both normal and inverted hierarchical neutrino mass spectrum, the wrong-sign muon event spectrum is almost independent of CP phase δ and the mixing angle θ_{13} in a specific energy window. This leads to an unambiguous determination of the hierarchy. In addition, a part of the muon spectrum remain sensitive to δ and θ_{13}, so that the same setup can be used to probe these parameters as well using a broadband beam.

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INTRODUCTION

The past and present neutrino oscillation experiments have established that neutrinos have mass and there is mixing between the three flavours. The oscillation experiments have measured the two independent mass squared differences as $\Delta m^2_{21} \sim 7.5 \times 10^{-5}$ eV^2 and $\Delta m^2_{31} \sim 2.3 \times 10^{-3}$ eV^2; the two mixing angles as $\sin^2\theta_{12} \sim 1/3$, $\sin^2\theta_{23} = \pi/4$. The third mixing angle is bounded as $\sin^2\theta_{13} < 0.056$ at 3σ. The imminent goals are to determine the value of the third mixing angle θ_{13}, the sign of $|\Delta m^2_{31}|$ commonly known as neutrino mass hierarchy and the CP violation in the lepton sector.

The oscillation probability $P_{\mu e}$ or $P_{e\mu}$ which is responsible for the wrong sign muon signal in a neutrino factory is sensitive to all the three parameters listed above and hence it was hailed as the 'Golden Channel'. However this probability also contains the CP-phase which is completely undetermined. This gives rise to degenerate solutions posing problems in the unambiguous determination of the other oscillation parameters. A solution to this problem is to have the detector at $\sim$ 7500 km the so called"the magic baseline" [1]) from the source, where the effect of CP violation vanishes for both the hierarchies. However, a detector at this baseline then has no handle on the CP phase. Moreover an experiment at the magic baseline requires a powerful beam and a strong collimation to have sufficient flux at the detector. Also to determine the CP phase one needs to design another experiment at a different baseline. The ideal situation would be to have a shorter baseline where the effect of CP phase in determination of hierarchy and θ_{13} can somehow be evaded in a particular window of the energy spectrum. At the same time one can possibly use another energy window at the same baseline to probe CP violation and θ_{13}. In the context of a ν_μ superbeam, it was recently proposed in [2] that the baseline of 2540 km fulfills the above criterion for IH at a neutrino energy of 3.3 GeV and a narrow band neutrino beam was therefore deemed desirable. In a subsequent paper [3] it was shown that this baseline *also* obeys the required condition for NH, at the energy 1.9 GeV. The two energies at which the no-CP condition is satisfied were termed as magic energies, and the baseline was referred to as "bimagic". It is really remarkable that the bi-magic baseline 2540 km also happens to be close to the distance between Brookhaven and Homestake [4], as well as that between CERN and Pyhasalmi mine [5], which is one of the proposed sites for the LENA detector. Probing the three hitherto unknown parameters at this baseline demands a a broadband beam covering the range 1–4 GeV. Such beams can be obtained from a low energy neutrino factory (LENF) with a muon energy of 5 GeV [6]. Because of the bi-magic property a single polarity beam woulld suffice in determination of all three parameters.

THE BI-MAGIC BASELINE

The source beam from a neutrino factory that accelerates μ^+ consists of $\bar{\nu}_\mu$ and ν_e. Charged current interactions at the detector can give muons in two ways: the original $\bar{\nu}_\mu$ that survive as $\bar{\nu}_\mu$ give μ^+ (right-sign muons) while the original ν_e that oscillate to ν_μ give μ^- (wrong-sign muons). Assuming constant matter density, the oscillation probability $P_{\nu_e \to \nu_\mu}$, relevant for the wrong-sign muon signal, can be written as [7]

$$
\begin{aligned}
P_{e\mu} \quad = \quad & 4s_{13}^2 s_{23}^2 \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} + \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \frac{\sin^2 \hat{A}\Delta}{\hat{A}^2} \\
& + 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{\rm CP}) \times \\
& \frac{\sin \hat{A}\Delta}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} , \qquad (1)
\end{aligned}
$$

keeping terms up to second order in $\alpha \equiv \Delta m_{21}^2/\Delta m_{31}^2$ and s_{13}. Here $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. Also,

$$
\hat{A} \equiv 2\sqrt{2} G_F n_e E_\nu / \Delta m_{31}^2 , \quad \Delta \equiv \Delta m_{31}^2 L/(4E_\nu) , \qquad (2)
$$

where G_F is the Fermi constant and n_e is the electron number density. For neutrinos, the signs of $\hat{A}$ and Δ are positive for normal hierarchy and negative for inverted hierarchy. $\hat{A}$ picks up an extra negative sign for anti-neutrinos. The last term in Eq. (1) is responsible for the hierarchy degeneracy due to δ_{CP} [8]. There are two ways to evade this problem: (i) to have $\sin(\hat{A}\Delta) = 0$ or (ii) $\sin[(1-\hat{A})\Delta] = 0$. The first condition is satisfied at the magic baseline ($L \sim 7500$ km) for all E_ν and for both the hierarchies. The second condition, on the other hand, depends on hierarchy. This dependence can be utilized to our advantage if we demand $\sin[(1-\hat{A})\Delta] = 0$ for one of the hierarchies and $\sin[(1-\hat{A})\Delta] = \pm 1$ for the other.

In such a situation, only the $\mathscr{O}(\alpha^2)$ term in Eq. (1) survives for the hierarchy for which $\sin[(1-\hat{A})\Delta] = 0$, making $P_{e\mu}$ independent of both $\delta_{\rm CP}$ and θ_{13}. At the same time, for the other hierarchy the first term in Eq. (1) enhances the number of events as well as θ_{13} sensitivity, and the third term enhances the sensitivity to $\delta_{\rm CP}$. If we demand "IH-noCP" (no sensitivity to CP phase in IH), these conditions imply

$$
\begin{aligned}
(1+|\hat{A}|)\cdot|\Delta| &= n\pi \qquad \text{for IH} , \qquad (3) \\
(1-|\hat{A}|)\cdot|\Delta| &= (m-1/2)\pi \quad \text{for NH} , \qquad (4)
\end{aligned}
$$

where n, m are integers, $n > 0$. These two conditions are exactly satisfied at a particular baseline and energy, given by

$$
\begin{aligned}
\rho L(\text{km g/cc}) &\approx (n-m+1/2) \times 16300 , \qquad (5) \\
E_\nu(\text{GeV}) &= \frac{4}{5} \frac{\Delta m_{31}^2(\text{eV}^2)\, L(\text{km})}{(n+m-1/2)} . \qquad (6)
\end{aligned}
$$

Note that the relevant L is independent of any oscillation parameters. A viable solution for these set of equations (with $n = 1$ and $m = 1$) is $L \approx 2540$ km, $\rho = 3.2$ g/cc and $E_\nu \equiv E_{\rm IH} \approx 3.3$ GeV, as was first pointed out in [2]. On the other hand, one may demand "NH-noCP" (no sensitivity to CP phase in NH), which leads to the conditions

$$
\begin{aligned}
(1-|\hat{A}|)\cdot|\Delta| &= n\pi \qquad \text{for NH} , \qquad (7) \\
(1+|\hat{A}|)\cdot|\Delta| &= (m-1/2)\pi \quad \text{for IH} , \qquad (8)
\end{aligned}
$$

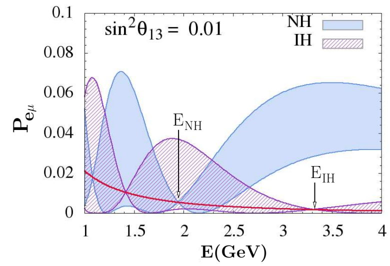

FIGURE 1. The conversion probability $P_{e\mu}$ for $L = 2540$ km. The bands are for $\delta_{\rm CP} \in (0, 2\pi)$. Other parameters are taken as $\Delta m_{21}^2 = 7.65 \times 10^{-5}$ eV^2, $|\Delta m_{31}^2| = 0.0024$ eV^2, $\sin^2\theta_{12} = 0.3$ and $\sin^2\theta_{12} = 0.5$. The red (solid) line corresponds to $\theta_{13} = 0$.

th n, m integers, $n \neq 0$ and $m > 0$. These lead to the same condition on L as in Eq. (5) except for an overall negative sign, while E_ν continues to be given by Eq. (6). These conditions are also satisfied at $L = 2540$ km (for $n = 1$ and $m = 2$) at $E_\nu \equiv E_{\rm NH} \approx 1.9$ GeV. The magic energies $E_{\rm IH}$ and $E_{\rm NH}$ can be obtained from a neutrino beam originating from a parent muon of energy 5 GeV at a neutrino factory.

Fig. 1 shows the probability $P_{e\mu}$ for $\sin^2\theta_{13} = 0, 0.01$. In this and all other plots, we have solved the exact neutrino propagation equation numerically using the Preliminary Reference Earth Model [9]. Clearly the IH-noCP and NH-noCP conditions are satisfied at the energies $E_{\rm IH}$ and $E_{\rm NH}$, respectively. At $E_{\rm IH}$, the probabilities $P_{e\mu}$ for NH and IH are distinct, hence a measurement of the neutrino spectrum around this energy can help in a clean distinction between the hierarchies. The oscillatory nature of $P_{e\mu}$ for non-zero θ_{13} vis-a-vis the monotonic behavior for $\theta_{13} = 0$ helps in the discovery of a nonzero θ_{13}. Finally, the significant widths of the bands (near $E_{\rm IH}$ for NH, and near $E_{\rm NH}$ for IH) imply sensitivity to $\delta_{\rm CP}$.

The simplified forms of probabilities at the magic energies offer insights into the CP sensitivity at this baseline. At $E_{\rm IH}$, we have

$$
\begin{aligned}
P_{e\mu}(\text{IH}) &\approx 18\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 , \\
P_{e\mu}(\text{NH}) &\approx 18\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 + 9 s_{13}^2 s_{23}^2 \\
& \quad -18\sqrt{2}\alpha s_{12} c_{12} s_{23} c_{23} s_{13} \cos(\delta_{\rm CP} + \pi/4) , \qquad (9)
\end{aligned}
$$

while at $E_{\rm NH}$, we have

$$
\begin{aligned}
P_{e\mu}(\text{NH}) &\approx 50\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 , \\
P_{e\mu}(\text{IH}) &\approx 50\alpha^2 s_{12}^2 c_{12}^2 c_{23}^2 + (25/9) s_{13}^2 s_{23}^2 \\
& \quad -(50\sqrt{2}/3)\alpha s_{12} c_{12} s_{23} c_{23} s_{13} \cos(\delta_{\rm CP} + \pi/4) (10)
\end{aligned}
$$

Near the magic energies, where the CP sensitivity is the highest, the $\delta_{\rm CP}$ values giving the highest and the lowest probabilities would be $3\pi/4$ and $7\pi/4$, respectively.

Results

We use a 25 kt magnetized totally active scintillator detector (TASD) with an energy threshold of 1 GeV. The parent muon energy is taken as 5 GeV with the number of useful muon decays as 5×10^{21} per year. We consider the running with only one polarity μ^+ of the parent muon, so that we have a neutrino flux consisting of $\bar{\nu}_\mu$ and ν_e. We assume a muon detection efficiency of 94% for energies above 1 GeV, 10% energy resolution for the whole energy range up to 5 GeV and a background level of 10^{-3} for the $\nu_e \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ channels. A 2.5% normalization error and 0.01% calibration error, both for throughout this study. The detector characteristics have been simulated by GLoBES [10].

Mass hierarchy determination.— Fig. 2 shows the hierarchy sensitivity of the bimagic neutrino factory setup. For each pair of $\sin^2\theta_{13}(\text{true})$–$\delta_{\text{CP}}(\text{true})$, we obtain χ^2_{min} by marginalizing over other parameters. We have taken 4% error on each of Δm^2_{21} and θ_{12}, and 5% error on each of θ_{23} and Δm^2_{31}, for calculating the priors. δ_{CP} has been varied over $(0, 2\pi)$. A 2% error has also been considered on the earth matter profile and marginalized over.

The contours in Fig. 2 suggest that if the true hierarchy is NH, then for favorable values of δ_{CP}, an exposure of $\approx 3 \times 10^{23}$ muons$\times$kt may determine the hierarchy at 3σ even for $\sin^2\theta_{13} \sim 3 \times 10^{-5}$. If the true hierarchy is IH then that can be established at 3σ for $\sin^2\theta_{13} \gtrsim 3 \times 10^{-4}$.

θ_{13} and δ_{CP} measurement.— The top panel of Fig. 3 shows that the exposure of $\approx 3 \times 10^{23}$ muons$\times$kt will be able to discover a nonzero θ_{13} to 3σ as long as $\sin^2\theta_{13} \gtrsim 10^{-3}$ for either hierarchy and for any δ_{CP} value. For NH and $\delta_{\text{CP}} \approx 3\pi/4$, the discovery of θ_{13} is possible even for $\sin^2\theta_{13}$ as low as 3×10^{-5}.

The bottom panel of Fig. 3 shows the δ_{CP} discovery reach with this setup. It shows that the exposure allows the discovery of nonzero δ_{CP} for NH for $\sin^2\theta_{13}$ as low as 10^{-4}, as long as $\delta_{\text{CP}} \approx 3\pi/4$. This is the δ_{CP} value at which we expect the highest deviation in the events spectrum from $\delta_{\text{CP}} = 0$, as indicated by Eqs. (9) and (10). For IH, the results are about one order of magnitude worse than those for NH.

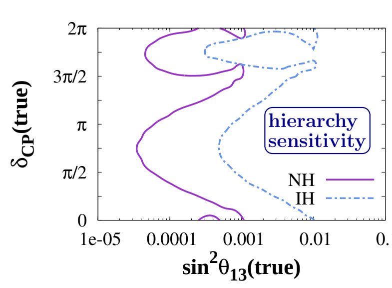

FIGURE 2. The 3σ hierarchy sensitivity contours. For parameters to the right of the contours, hierarchy can be determined.

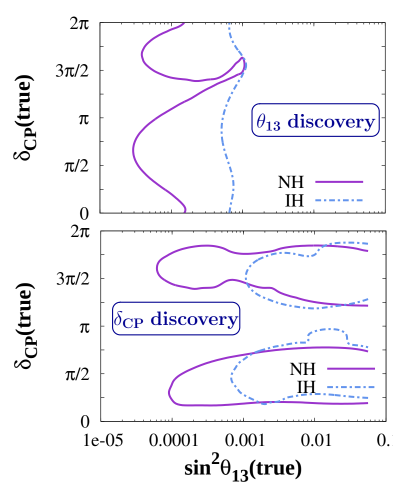

FIGURE 3. The 3σ discovery contours for θ_{13} (upper panel) and CP violating phase δ_{CP} (lower panel). The true hierarchies are as indicated.

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