Unified description of fermion masses with quasi-degenerate neutrinos in SO(10)

Anjan S. Joshipura and Ketan M. Patel

Physical Research Laboratory, Navarangpura, Ahmedabad - 380 009, India

Abstract. Obtaining a unified description of the quasi-degenerate neutrino mass spectrum together with the hierarchical charged fermions is a challenging task. In this talk, we discuss two distinct possible scenarios leading to such spectra in the supersymmetric $SO(10)$ grand unified framework. Consistency of both scenarios is demonstrated through detailed fits to fermion masses and mixing angles, all of which can be explained with reasonable accuracy in a model with the most general Yukawa sector of $SO(10)$. The origin of large neutrino mixing angles is linked to neutrino mass degeneracy in both the scenarios.

Keywords: Neutrino mass, Neutrino mixing angle, Grand unified theory, Flavor symmetry
PACS: 12.10.-g, 12.10.Kt, 12.15.Ff, 14.60.Pq

INTRODUCTION

Neutrino oscillation experiments over the years have unveiled the facts that neutrinos are massive, with very small measured mass squared differences, and contrary to the quark sector, large flavor mixing. Unfortunately, such experiments cannot tell us about the overall scale of neutrino masses and no other laboratory experiment has unambiguously detected such a scale so far. At present, the most stringent constraint on the neutrino mass scale comes from cosmological observations. Recent results from the WMAP and surveys of large scale structure have set a limit on the sum of neutrino masses $\sum m_i \leq 0.3-2$ eV [1]. All these observations thus allow a possibility in which all three neutrinos masses are nearly degenerate ($m_1 \simeq m_2 \simeq m_3 \equiv m_0$), having a quasi-degenerate mass m_0 in the range $0.1-0.7$ eV. There exists an interesting link between the quasi-degenerate neutrino mass spectrum and the largeness of neutrino mixing angles. The mixing angles remain undefined in the exact degenerate limit. A small perturbation that leads to splittings in neutrino masses can also stabilize all or some of the mixing angles to large values. So the theory, which predicts quasi-degeneracy, has a built-in mechanism to explain large mixing angles. Nevertheless the construction of such a theory or framework which obtains a quasi-degenerate neutrino mass spectrum within the conventional picture of neutrino mass generation is non-trivial. This becomes more challenging in unified approaches like $SO(10)$ models due to their quark-lepton unifying nature. In this talk, we discuss $SO(10)$-based scenarios leading to hierarchical charged fermions and quasi-degenerate neutrino masses.

The renormalizable supersymmetric theories based on the $SO(10)$ group are quite powerful in constraining the fermionic mass structures. The standard fermions are assigned to the 16 dimensional representation of the $SO(10)$ group and they can obtain masses through symmetric couplings with 10 and $\overline{126}$ and antisymmetric couplings with the 120 dimensional representation of the Higgs fields. Neutrino masses arise in these models either from the vacuum expectation value (vev) of the left-handed triplet (type-II seesaw) or from the right-handed triplet (type-I seesaw) Higgs components of $\overline{126}$ field. Starting with a supersymmetric $SO(10)$, an effective minimal supersymmetric standard model (MSSM) is obtained by assuming fine-tuning, which keeps only two Higgs doublets light. Further, the electroweak symmetry is broken after these light MSSM doublets acquire vev and they then generate the fermion masses. The resulting mass formulae for different fermion masses can be suitably written as [2, 3]

$$\begin{aligned} M_d = H + \ F + \ G \quad &, \quad M_u = r(H + \ sF + t_u G), \\ M_l = H - 3F + t_l G \quad &, \quad M_D = r(H - 3sF + t_D G), \\ M_L = r_L\, F \quad &, \quad M_R = r_R^{-1}\, F \end{aligned} \tag{1}$$

and the light neutrino mass matrix is given by

$$\mathscr{M}_\nu \equiv \mathscr{M}_\nu^{II} + \mathscr{M}_\nu^{I} = r_L F - r_R M_D F^{-1} M_D^T \tag{2}$$

where H, F and G arise from the fermionic Yukawa couplings to 10, $\overline{126}$ and 120 Higgs fields respectively. r, s, t_u, t_l, t_D, r_L and r_R are complex parameters.

Quasi-degenerate Neutrino Spectrum

Depending on which seesaw mass term dominates in eq.(2), a quasi-degenerate neutrino spectrum can be obtained with the following assumptions.

Type-II seesaw dominance

It was pointed out long ago [4, 5] that eq.(2) can provide an interesting framework for quasi-degenerate neutrinos if the type-II seesaw term dominates over the type-I contribution. In this approach, a degenerate neutrino spectrum can be obtained with an assumption

$$F = c_0\, I \tag{3}$$

where I is an identity matrix in generation space. The sub-dominant type-I contribution can then lead to the neutrino mass differences and large neutrino mixing angles. This simple and attractive scenario is realizable only if the type-II contribution dominates, which is not always the case. In fact many detailed studies [6] of the minimal model find that parameter space favored by the overall fit to fermion masses suppresses the type-II contribution compared to type-I. An alternative possibility is that both degeneracy and its breaking arise from a single source, namely type-I seesaw mechanism.

Type-I seesaw dominance

Obtaining degenerate neutrinos through type-I seesaw requires a peculiar structure for the right-handed neutrino mass matrix. It has been pointed out recently that such structure can arise from the application of the minimal flavor violation hypothesis to the lepton sector [7]. Following such a strategy, quasi-degenerate neutrinos in type-I dominated $SO(10)$ models can be obtained by imposing

$$F = a\, H^2 \tag{4}$$

Without loss of generality, we can express the mass matrices in (1) in an $SO(10)$ basis with diagonal H. This can be done by the replacements

$$H \to D_H \;\text{ and }\; H^2 \to D_H V^* D_H \tag{5}$$

where D_H is a diagonal matrix with real elements and V is a symmetric unitary matrix. G retains its antisymmetric form. Implementing ansatz (4), the light neutrino mass matrix (2) in a diagonal basis of H is given by

$$\mathscr{M}_\nu^I = \frac{r_R r^2}{a}(V - 6saD_H + t_D(GD_H^{-1}V - VD_H^{-1}G) + \mathscr{O}(s^2, t_D^2)) \tag{6}$$

Retaining only the H contribution to the Dirac neutrino mass matrix M_D, the above equation implies a degenerate neutrino mass spectrum. It is interesting to note that in this limit

- Correct $b-\tau$ unification is obtained which is favored by the observations extrapolated at the GUT scale.
- The CKM matrix is unity while the neutrino mixing angle is determined from V. As was pointed out in [8], the diagonalisation of V leads to two arbitrary angles (θ_{23}, θ_{12}) and vanishing θ_{13}.

Thus ansatz (4) can lead to a correct description of the quark and leptonic mixing angles to zeroth order. Further, the contributions from $\overline{126}$ and 120-plets induce nonzero quark mixing angles and reproduce the correct mass spectrum of light fermions. It is shown in reference [3] that the proposed ansatz (4) can be obtained in effective $SO(10)$ theory from an extended model based on the three generations of the vectorlike fermions and an $O(3) \times U(1)$ flavor symmetry.

NUMERICAL ANALYSIS

We now discuss the viability of fermion mass relations of eq.(1) with ansatz (3) and (4) through detailed numerical study. We carried out a χ^2 analysis separately in each of these two cases. The χ^2 function is constructed as

$$\chi^2(\alpha_j) = \sum_i \left(\frac{X_i(\alpha_j) - O_i}{\sigma_i} \right) \tag{7}$$

where X_i are the fermion masses and mixing angles as complex nonlinear functions of parameters α_j calculated at the GUT scale. $O_i(\sigma_i)$ are the input mean values (1σ errors) of respective observables extrapolated at M_{GUT}=2 $\times$ 10^{16} GeV for tanβ=10. The list of such input values in the quark sector is given in [2]. We included the RG evolution in the neutrino mass matrix and obtained its low energy form [3]. For input values of neutrino masses and lepton mixing angles, we used the updated low energy values given in [9]. Then the data are fitted by minimizing the χ^2 function with respect to parameters α_j using an algorithm based on the downhill simplex method.

The results of the minimization obtained assuming type-I seesaw dominance and ansatz (4) are displayed as solution 1 and 2 in table(1). Solution 3 corresponds to the solution obtained assuming type-II seesaw dominance followed by ansatz (3). All three solutions provide a good fit over the entire fermion spectrum and are acceptable from a statistical point of view. The best fit value of $\chi^2 = 2.038$ is obtained for type-I dominated quasi-degenerate neutrino spectrum which fits all observables within $\lesssim 0.9\sigma$. The obtained fit in the type-II case $\chi^2 = 6.0$ is also acceptable, however, not as good as in the case of a pure type-I seesaw. The predictions of various observables are shown in bold fonts. Clearly, all solutions predict large CP-violation in the lepton sector. The initial value of θ_{13} was zero as discussed in the previous section. This becomes nonzero but remains small in

TABLE 1. The best fit solutions for fermion masses and mixing obtained assuming the type-I seesaw dominance (solutions (1) and (2)) and type-II seesaw dominance (solution(3)). Various observables and their pulls obtained at the minimum are shown (See text for details). The bold faced quantities are predictions of the respective solutions.

		Solution 1		**Solution 2**		**Solution 3**	
No.	Observables	Fitted value	Pull	Fitted value	Pull	Fitted value	Pull
1	m_d [MeV]	0.653677	-0.917861	0.207819	**-2.00532**	0.868041	-0.395023
2	m_s [MeV]	17.5885	-0.386821	21.6923	0.402361	12.2829	**-1.40714**
3	m_b [GeV]	1.11131	0.418721	1.05832	-0.046348	1.25634	**1.69141**
4	m_u [MeV]	0.462718	0.0847896	0.450825	0.00549932	0.450489	0.0032611
5	m_c [GeV]	0.210603	0.0136849	0.211727	0.0695654	0.210393	0.00324503
6	m_t [GeV]	63.6891	-0.832404	67.6155	-0.658038	102.325	0.883371
7	m_e [MeV]	0.358503	0.00969691	0.358506	0.0206782	0.358502	0.00503107
8	m_μ [MeV]	75.6719	0.00734514	75.6711	-0.0083064	75.6709	-0.0111809
9	m_τ [GeV]	1.29219	-0.00814429	1.29223	0.0218404	1.29217	-0.0244576
10	$\Delta m^2_{sol}/\Delta m^2_{atm}$	0.0303514	0.050109	0.0303237	0.0377877	0.0302538	0.00659421
11	m_0 [eV]	**0.31**	-	**0.17**	-	**0.36**	-
12	$\sin\theta^q_{12}$	0.224205	-0.0592102	0.224306	0.00359473	0.224154	-0.0913125
13	$\sin\theta^q_{23}$	0.0351308	0.023704	0.0350426	-0.0441173	0.0351436	0.033571
14	$\sin\theta^q_{13}$	0.00319336	-0.0132867	0.00315871	-0.0825897	0.00326199	0.123983
15	$\sin^2\theta^l_{12}$	0.319801	-0.0619079	0.321124	0.0187774	0.321168	0.0214673
16	$\sin^2\theta^l_{23}$	0.481942	0.313909	0.436492	-0.178126	0.439779	-0.14255
17	$\sin^2\theta^l_{13}$	**0.0195266**	-	**0.00288176**	-	**0.0356836**	-
18	$\delta_{CKM}[^\circ]$	67.7227	0.247333	56.4935	-0.134071	49.7146	-0.429864
19	$\delta_{PMNS}[^\circ]$	**53.98**	-	**-66.99**	-	**-25.33**	-
20	$\alpha_1[^\circ]$	**146.55**	-	**-59.31**	-	**137.71**	-
21	$\alpha_2[^\circ]$	**-89.88**	-	**162.41**	-	**-33.44**	-
	χ^2		2.038		4.684		6.0

all three solutions displayed. However, almost the entire range of θ_{13} is found compatible as shown by all three solutions. The overall scale of degenerate neutrino mass m_0 is determined using the observed value of Δm^2_{atm}. The values of m_0 for all three solutions are seen to be $\gg \sqrt{\Delta m^2_{atm}}$ showing the consistency of ansatz. The predictions of m_0 are particularly interesting from an experimental point of view as they can be probed directly by KATRIN experiment (sensitivity $m_0 < 0.2$ eV) in the near future [10].

CONCLUSION

It is indeed possible to obtain the quasi-degenerate neutrino mass spectrum together with hierarchical charged fermions in an $SO(10)$ grand unified framework. Quasi-degenerate neutrinos arise in such a framework in two distinct possibilities based on purely type-I and the other on the mixture of type-II and type-I seesaw mechanism if they are supplemented with ansatz (4) and (3) respectively. Detailed numerical analysis shows that these ansatz are capable of explaining the entire fermionic spectrum and not just the quasi-degenerate neutrinos. Further the large neutrino mixing angles emerge as a consequence of neutrino mass degeneracy in both the cases.

REFERENCES

1. S. Hannestad, Prog. Part. Nucl. Phys. **65**, 185 (2010) [arXiv:1007.0658 [hep-ph]].
2. A. S. Joshipura, B. P. Kodrani and K. M. Patel, Phys. Rev. D **79**, 115017 (2009) [arXiv:0903.2161 [hep-ph]].
3. A. S. Joshipura and K. M. Patel, Phys. Rev. D **82**, 031701 (2010) [arXiv:1005.0045 [hep-ph]].
4. D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993);
   A. S. Joshipura, Z. Phys. C **64**, 31 (1994);
   A. S. Joshipura, Phys. Rev. D **51**, 1321 (1995);
   S. T. Petcov and A. Y. Smirnov, Phys. Lett. B **322**, 109 (1994).
5. A. Ioannisian and J. W. F. Valle, Phys. Lett. B **332**, 93 (1994) [arXiv:hep-ph/9402333];
   D. G. Lee and R. N. Mohapatra, Phys. Lett. B **329**, 463 (1994) [arXiv:hep-ph/9403201].
6. See for example B. Bajc, I. Dorsner and M. Nemevsek, JHEP **0811**, 007 (2008) [arXiv:0809.1069 [hep-ph]] and references therein.
7. A. S. Joshipura, K. M. Patel and S. K. Vempati, Phys. Lett. B **690**, 289 (2010) [arXiv:0911.5618 [hep-ph]].
8. G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Rev. Lett. **82**, 683 (1999) [arXiv:hep-ph/9810328].
9. M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP **1004**, 056 (2010) [arXiv:1001.4524 [hep-ph]].
10. J. Wolf (KATRIN Collaboration), Talk given at this workshop (NuFact 2010).