Quasi-degenerate Neutrinos in SO(10)

Ketan Patel

Physical Research Laboratory,
Ahmedabad, INDIA

October 22, 2010

NuFact10, TIFR, Mumbai.

- **With A. S. Joshipura ,Phys. Rev. D 82, 031701 (R) (2010) [arXiv:1005.0045].**

Quasi-degenerate Neutrinos: An allowed possibility

- Experiments over the years have revealed that Neutrino mass hierarchy is milder compared to quarks, and the extreme case of all neutrinos being quasidegenerate is still an allowed possibility.

Present information from Non-oscillation experiments

Experiments	Parameter probed	Present Bounds
β-decay	$m_\beta = \sqrt{\Sigma_i \lvert U_{ei}\rvert^2 m_i^2}$	$<$ 2 eV
$0\nu\beta\beta$	$m_{ee} = \Sigma_i U_{ei}^2 m_i$	$<$ (0.19 - 0.68) eV
Cosmology	$m_{cosmo} = \Sigma_i m_i$	$<$ (0.3 - 2) eV

- All the neutrinos may have a quasi-degenerate mass ($m_1 \simeq m_2 \simeq m_3 \equiv m_0$) in the range of $m_0 \sim 0.1 - 0.7$ eV.

Large Neutrino Mixing: A consequence of Quasi-degeneracy

- Neutrino Oscillation experiments have also revealed that two of the neutrino mixing angles are large as opposed to the small quark mixing angles.
- Large mixing angles become quite natural if neutrinos are almost degenerate.
- They remain undefined in the exact degenerate limit.
- A small perturbation that leads to differences in neutrino masses can also stabilize all/some of the mixing angles to large values.

INTRODUCTION

Large Neutrino Mixing: A consequence of Quasi-degeneracy

- Neutrino Oscillation experiments have also revealed that two of the neutrino mixing angles are large as opposed to the small quark mixing angles.
- Large mixing angles become quite natural if neutrinos are almost degenerate.
- They remain undefined in the exact degenerate limit.
- A small perturbation that leads to differences in neutrino masses can also stabilize all/some of the mixing angles to large values.

So the theory which predicts quasi-degeneracy, has a built-in mechanism to explain large mixing angles.

INTRODUCTION

SO(10) Models of Neutrino Masses:

- $SO(10)$ models provide a natural framework for understanding neutrino masses because of the seesaw mechanisms inherent in them.
- Neutrino masses arise in these models from two separate sources either from the vev of left-handed triplet (type-II) or from the right-handed triplet (type-I) Higgs.

INTRODUCTION

SO(10) Models of Neutrino Masses:

- $SO(10)$ models provide a natural framework for understanding neutrino masses because of the seesaw mechanisms inherent in them.
- Neutrino masses arise in these models from two separate sources either from the vev of left-handed triplet (type-II) or from the right-handed triplet (type-I) Higgs.
- Apart from gauge coupling unification, $SO(10)$ also unifies quarks and leptons at high scale and hence provides common and attractive platform to study the dissimilarities between quarks and leptons.
- Furthermore, the renormalizable models based on $SO(10)$ gauge group are quite powerful in constraining the fermion mass structure.

Fermion Masses in $SO(10)$

Yukawa interactions in renormalizable SUSY $SO(10)$ are

$$\mathcal{L}_Y = \mathbf{16}_i \, [H_{ij}\mathbf{10} \; + \; F_{ij}\overline{\mathbf{126}} \; + \; G_{ij}\mathbf{120}] \; \mathbf{16}_j + h.c.$$

Features of $SO(10)$

Fermion Masses in $SO(10)$

Yukawa interactions in renormalizable SUSY $SO(10)$ are

$$\mathcal{L}_Y = \mathbf{16}_i \, [H_{ij}\mathbf{10} \; + \; F_{ij}\overline{\mathbf{126}} \; + \; G_{ij}\mathbf{120}] \, \mathbf{16}_j + h.c.$$

- Decomposition under Pati-Salam ($SU(4)_{PS} \times SU(2)_L \times SU(2)_R$)

$$\begin{aligned}
16 &= (4,2,1) + (\overline{4},1,2) \\
10 &= \color{red}{(1,2,2)} + (6,1,1) \\
120 &= \color{red}{(1,2,2)} + (6,3,1) + (6,1,3) + \color{red}{(15,2,2)} + (10,1,1) + (\overline{10},1,1) \\
\overline{126} &= \color{blue}{(10,1,3)} + \color{blue}{(\overline{10},3,1)} + \color{red}{(15,2,2)} + (6,1,1)
\end{aligned}$$

- Starting from $SO(10)$, an effective MSSM is obtained by assuming that only two appropriate linear combinations of these Higgs doublets (red colored) remain light and are responsible for Fermion masses.
- VEV of $SU(2)_R(SU(2)_L)$ triplet (blue colored) generates light neutrino masses by type-I(type-II) seesaw mechanism.

Fermion Masses in $SO(10)$

After EWSB, the mass Lagrangian of the model is

$$-\mathcal{L}_{mass} = \overline{f}_L M_f f_R + \overline{\nu}_L M_D \nu_R + \frac{1}{2}\overline{\nu}_L M_L \nu_L^c + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + h.c.$$

Fermion Masses in $SO(10)$

After EWSB, the mass Lagrangian of the model is

$$-\mathcal{L}_{mass} = \overline{f}_L M_f f_R + \overline{\nu}_L M_D \nu_R + \frac{1}{2}\overline{\nu}_L M_L \nu_L^c + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + h.c.$$

The final fermion mass relations can suitably written as,

$$\begin{aligned} M_d &= H + \ F + \ G \quad , \quad M_u = r(H + s\ F + t_u\ G), \\ M_l &= H - 3F + t_l G \quad , \quad M_D = r(H - 3s\ F + t_D\ G), \\ M_L &= r_L\ F \quad , \quad M_R = r_R^{-1}\ F, \end{aligned}$$

The light neutrino mass matrix is given by,

$$\mathcal{M}_\nu = r_L F - r_R M_D F^{-1} M_D^T \equiv \mathcal{M}_\nu^{II} + \mathcal{M}_\nu^{I}$$

Quasidegenerate neutrinos in $SO(10)$

Ways to obtain quasi-degenerate neutrinos:

1. Some flavor symmetry leads to degenerate type-II contribution, and its breaking in the Dirac neutrino masses then leads to departure from degeneracy through type-I contribution. [Caldwell, Mohapatra(1993); Joshipura(1994); Petcov, Smirnov(1994); Ioannisian, Valle(1994); Lee, Mohapatra(1994)]

   $\Longrightarrow$ **realizable only if type-II dominates, which is not always the case.**

Quasidegenerate neutrinos in $SO(10)$

Ways to obtain quasi-degenerate neutrinos:

1. Some flavor symmetry leads to degenerate type-II contribution, and its breaking in the Dirac neutrino masses then leads to departure from degeneracy through type-I contribution. [Caldwell, Mohapatra(1993); Joshipura(1994); Petcov, Smirnov(1994); Ioannisian, Valle(1994); Lee, Mohapatra(1994)]

   $\Longrightarrow$ **realizable only if type-II dominates, which is not always the case.**

2. Both degeneracy and its breaking arise from a single source, namely type-I seesaw.

   $\Longrightarrow$ **requires a peculiar structure for M_R which can arise from**

   - "Dirac Screening" [Lindner, Schmidt and Smirnov (2005)] **OR**
   - Application of the Minimal Flavor Violation (MFV) hypothesis to the lepton sector. [Joshipura, Patel and Vempati (2010)]

Quasidegenerate neutrinos from MFV

Quasi-degenerate neutrinos through type-I seesaw:

- Generalization of the MFV principle to the leptonic sector.
- Assuming the flavor symmetry $\mathcal{G}_F$ at high scale

$$\mathcal{G}_F \equiv O(3)_l \times O(3)_e \times O(3)_\nu \times U(1)_R$$

- Symmetry decides the structure of M_R.

$$\begin{aligned} m_D &= \upsilon y_D \ , \\ M_R &= \Lambda y_D^T \left(c_0 + c_1 y_l y_l^T + d_1 y_l^* y_l^\dagger + d_2 (y_D y_D^\dagger + y_D^* y_D^T) + \ldots \right) y_D \ . \end{aligned}$$

Quasi-degenerate neutrinos through type-I seesaw:

- Generalization of the MFV principle to the leptonic sector.
- Assuming the flavor symmetry $\mathcal{G}_F$ at high scale

$$\mathcal{G}_F \equiv O(3)_l \times O(3)_e \times O(3)_\nu \times U(1)_R$$

- Symmetry decides the structure of M_R.

$$\begin{aligned} m_D &= \upsilon y_D \,, \\ M_R &= \Lambda y_D^T \left(c_0 + c_1 y_l y_l^T + d_1 y_l^* y_l^\dagger + d_2 (y_D y_D^\dagger + y_D^* y_D^T) + \ldots \right) y_D \,. \end{aligned}$$

- The light neutrino mass term is then given by:

$$\begin{aligned} \mathcal{M}_\nu &\equiv m_D M_R^{-1} m_D^T \,, \\ &\approx m_0 \left(1 - \frac{c_1}{c_0} y_l y_l^T - \frac{d_1}{c_0} y_l^* y_l^\dagger - \frac{d_2}{c_0} (y_D y_D^\dagger + y_D^* y_D^T) + \ldots . \right) \,. \end{aligned}$$

Quasidegenerate neutrinos from MFV

Quasi-degenerate neutrinos through type-I seesaw:

- Generalization of the MFV principle to the leptonic sector.
- Assuming the flavor symmetry $\mathcal{G}_F$ at high scale

$$\mathcal{G}_F \equiv O(3)_l \times O(3)_e \times O(3)_\nu \times U(1)_R$$

- Symmetry decides the structure of M_R.

$$\begin{aligned} m_D &= \upsilon y_D \, , \\ M_R &= \Lambda y_D^T \left(c_0 + c_1 y_l y_l^T + d_1 y_l^* y_l^\dagger + d_2 (y_D y_D^\dagger + y_D^* y_D^T) + \ldots \right) y_D \, . \end{aligned}$$

- The light neutrino mass term is then given by:

$$\begin{aligned} \mathcal{M}_\nu &\equiv m_D M_R^{-1} m_D^T \, , \\ &\approx m_0 \left(1 - \frac{c_1}{c_0} y_l y_l^T - \frac{d_1}{c_0} y_l^* y_l^\dagger - \frac{d_2}{c_0} (y_D y_D^\dagger + y_D^* y_D^T) + \ldots . \right) \, . \end{aligned}$$

- m_D **and** M_R **can be simultaneously hierarchical** yet result into (almost) **degenerate spectrum** after the seesaw mechanism.

Quasidegenerate neutrinos in $SO(10)$

ANSATZ

Following this general framework, A quasi-degenerate neutrino spectrum in $SO(10)$ can be obtained by imposing

$$F = a\ H^2$$

$$\mathcal{M}^l_\nu = r_R M_D F^{-1} M_D^T$$

$$M_D = r(H - 3s\ F + t_D\ G)$$

In the diagonal basis of H : $H \to D_H$, $F = H^2 \to D_H\ V^*\ D_H$.

$$\mathcal{M}^l_\nu = \frac{r_R r^2}{a}(V - 6s\ a\ D_H + t_D\ (GD_H^{-1}V - VD_H^{-1}G) + \mathcal{O}(s^2, t_D^2))$$

In the limit $s, t_D \to 0$, **Neutrinos are degenerate**.

(In fact, small s, t_D are required by charged fermion mass spectrum)

ANSATZ

Implications of ansatz $F \sim H^2$:

- In the limit of dominant contribution from H (10-plet Higgs),
  - Correct $b - \tau$ unification is obtained which is favored by the data extrapolated at GUT scale.
  - CKM matrix is unity.
  - Lepton mixing angles are determined from the diagonalization of symmetric unitary matrix V, which are $\theta_{23} = \phi$, $\theta_{12} = \frac{\theta}{2}$ and $\theta_{13} = 0$.

  Thus ansatz can lead to correct description of the quark and leptonic mixing angles to zeroth order.
- Further, the contributions from $\overline{126}$ and 120-plets induce nonzero quark mixing angles & correct mass spectrum of light fermions.

NUMERICAL ANALYSIS: Type-I seesaw

Numerical fits with ansatz $F \sim H^2$:

- We do the χ^2 fitting to check the viability of the model.
- We construct

$$\chi^2(\alpha_j)=\sum_i \left(\frac{X_i(\alpha_j) - O_i}{\sigma_i}\right)^2$$

  Where,
  X_i are the fermion masses and mixing as complex nonlinear functions of parameters α_j calculated from the given model at GUT scale.
  O_i (σ_i) are the input mean values(1σ errors) of respective masses and mixing angles evaluated at $M_{GUT}=2 \times 10^{16}$ GeV for $\tan\beta=10$.
- The effect of RG evolution in neutrino mass matrix is included.
- Then χ^2 is minimized using algorithm based on the numerical nonlinear optimizations.

NUMERICAL ANALYSIS: Type-I seesaw

No.	Observables	Sol. 1 Fitted value	Sol. 1 Pull	Sol. 2 Fitted value	Sol. 2 Pull	Sol. 3 Fitted value	Sol. 3 Pull
1	m_d[MeV]	0.653677	−0.917861	0.678809	−0.856564	0.207819	**-2.00532**
2	m_s[MeV]	17.5885	−0.386821	22.8346	0.622041	21.6923	0.402361
3	m_b[GeV]	1.11131	0.418721	0.94463	**-1.0440**	1.05832	−0.046348
4	m_u[MeV]	0.462718	0.0847896	0.461582	0.0772103	0.450825	0.00549932
5	m_c[GeV]	0.210603	0.0136849	0.212603	0.113153	0.211727	0.0695654
6	m_t[GeV]	63.6891	−0.832404	56.7159	**-1.14208**	67.6155	−0.658038
7	m_e[MeV]	0.358503	0.00969691	0.358516	0.0525082	0.358506	0.0206782
8	m_μ[MeV]	75.6719	0.00734514	75.6765	0.0923818	75.6711	−0.0083064
9	m_τ[GeV]	1.29219	−0.00814429	1.2921	−0.0804718	1.29223	0.0218404
10	$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	0.0303514	0.050109	0.0302197	−0.00862837	0.0303237	0.0377877
11	$\sin\theta^q_{12}$	0.224205	−0.0592102	0.224193	−0.0666865	0.224306	0.00359473
12	$\sin\theta^q_{23}$	0.0351308	0.023704	0.0347491	−0.269936	0.0350426	−0.0441173
13	$\sin\theta^q_{13}$	0.00319336	−0.0132867	0.00322056	0.0411106	0.00315871	−0.0825897
14	$\sin^2\theta^l_{12}$	0.319801	−0.0619079	0.319568	−0.076109	0.321124	0.0187774
15	$\sin^2\theta^l_{23}$	0.481942	0.313909	0.437263	−0.169787	0.436492	−0.178126
16	$\sin^2\theta^l_{13}$	**0.0195266**	–	**0.0463404**	–	**0.00288176**	–
17	$\delta_{CKM}[^\circ]$	67.7227	0.247333	49.8678	−0.422669	56.4935	−0.134071
18	$\delta_{PMNS}[^\circ]$	**53.9786**	–	**-52.7788**	–	**-66.9939**	–
	χ^2		2.038		3.844		4.684

NUMERICAL ANALYSIS: Type-I seesaw

Results :

- Obtained the best fit value of $\chi^2 = 2.04$
- All the 16 observables are fitted within $\lesssim 0.9\sigma$.

NUMERICAL ANALYSIS: Type-I seesaw

Results :

- Obtained the best fit value of $\chi^2 = 2.04$
- All the 16 observables are fitted within $\lesssim 0.9\sigma$.

Predictions from the best fit solution :

- θ_{13} becomes nonzero but remains small $sin^2\theta_{13} = 0.02$.
(However, almost the entire allowed range in θ_{13} is found compatible in other solutions.)
- The best fit solution predicts large CP violating leptonic phases

$$(\delta_{PMNS}, \alpha_1, \alpha_2) \approx (54^o, 147^o, -90^o)$$

- m_0 is determined using the observed value of Δm^2_{atm}.

$$m_0 = 0.31\text{eV} \gg \Delta m^2_{atm}$$

- The m_0 determine the heaviest RH neutrino mass scale

$$M_3 \approx m_t^2/m_0 \approx 1.3 \times 10^{13}\text{GeV}$$

MODEL

Model from Flavor Symmetry to obtain $F \sim H^2$:

Complete Symmetry Group: $G \equiv SO(10) \times O(3) \times U(1)$

Original Fields: $\psi(16, 3, x)$, $\phi_{10}(10, 1, -(x+y))$, $\phi_{\overline{126}}(\overline{126}, 1, -2y)$

Additional Fields: $\Psi_V(16, 3, y)$, $\Psi_{\overline{V}}(\overline{16}, 3, -y)$, $\eta(1, 5, -\frac{1}{2}(x+y))$

MODEL

Model from Flavor Symmetry to obtain $F \sim H^2$:

Complete Symmetry Group: $G \equiv SO(10) \times O(3) \times U(1)$

Original Fields: $\psi(16, 3, x)$, $\phi_{10}(10, 1, -(x+y))$, $\phi_{\overline{126}}(\overline{126}, 1, -2y)$

Additional Fields: $\Psi_V(16, 3, y)$, $\Psi_{\overline{V}}(\overline{16}, 3, -y)$, $\eta(1, 5, -\frac{1}{2}(x+y))$

The general superpotential invariant under G is

$$W = M\Psi_{\overline{V}}\Psi_V + \beta\Psi_V\Psi_V\phi_{\overline{126}} + \gamma\Psi_V\psi\phi_{10} + \frac{\delta}{M_P}\Psi_{\overline{V}}\eta^2\psi + \frac{\delta'}{M_P}Tr\eta^2\Psi_{\overline{V}}\psi + ...$$

The effective theory after integration of heavy vector-like field is

$$W_{eff} \approx \beta\psi\xi^2\psi\phi_{\overline{126}} + \gamma\psi\xi\psi\phi_{10}$$

where,

$$\xi_{ab} \equiv \frac{\delta}{MM_P}(\eta^2_{ab} + \frac{\delta'}{\delta}Tr\eta^2\delta_{ab})$$

MODEL

Model from Flavor Symmetry to obtain $F \sim H^2$:

- The vev of flavon field η breaks the symmetry

$$SO(10) \times O(3) \times U(1) \longrightarrow SO(10)$$

and the Yukawa couplings structure $F \sim H^2$.

- The coupling to the 120 field can be generated by introducing a flavon field χ with the $U(1)$ charge -2x and transforming as a triplet of $O(3)$. This leads to the Yukawa coupling matrix G through the coupling

$$\psi \frac{\chi}{M_P} \psi \phi_{120}$$

Type-II dominated quasi-degenerate neutrino spectrum:

- A degenerate neutrino can be obtained with ansatz $F = c_0 I$.

  [Ioannisian, Valle(1994); Lee, Mohapatra(1994)]

- Further, the departure from degeneracy is induced by type-I seesaw.
- We do the numerical analysis to check the viability of such ansatz with entire fermionic spectrum.

NUMERICAL ANALYSIS: Type-II seesaw

Type-II dominated quasi-degenerate neutrino spectrum:

- A degenerate neutrino can be obtained with ansatz $F = c_0 I$.
  [Ioannisian, Valle(1994); Lee, Mohapatra(1994)]
- Further, the departure from degeneracy is induced by type-I seesaw.
- We do the numerical analysis to check the viability of such ansatz with entire fermionic spectrum.

Results :

Best fit solution: $\chi^2 = 6.0$
(All observables (except m_s, m_b) are fitted within 1σ)
Predictions: $m_o = 0.36$ eV, $sin^2\theta_{13} = 0.036$, CPV Phases=$(-25^o, 138^o, -33^o)$

NUMERICAL ANALYSIS: Type-II seesaw

Type-II dominated quasi-degenerate neutrino spectrum:

- A degenerate neutrino can be obtained with ansatz $F = c_0 I$.
  [Ioannisian, Valle(1994); Lee, Mohapatra(1994)]
- Further, the departure from degeneracy is induced by type-I seesaw.
- We do the numerical analysis to check the viability of such ansatz with entire fermionic spectrum.

Results :

Best fit solution: $\chi^2 = 6.0$

(All observables (except m_s, m_b) are fitted within 1σ)

Predictions: $m_o = 0.36$ eV, $sin^2\theta_{13} = 0.036$, CPV Phases=$(-25^o, 138^o, -33^o)$

The obtained fit in the type-II case is however not as good as in the case of pure type-I seesaw.

CONCLUSION

- Obtaining a unified description of vastly different patterns of quark and lepton spectra is challenging. This becomes more so if neutrinos are quasi-degenerate.
- It is possible to obtain such description in general $SO(10)$ model with
    1. dominant type-I seesaw mechanism supplemented with proper ansatz.
    2. mixture of type-II and type-I seesaw mechanisms.
- Such ansatz are capable of explaining the entire fermionic spectrum and not just quasi-degenerate neutrinos.
- The origin of large lepton mixing angles is linked to the quasi-degenerate structure of neutrinos providing yet another reason why quark and leptonic mixing angles are so different in spite of underlying unified mass structure.