Beta Beams,
EUROnu WP4

SEVENTH FRAMEWORK PROGRAMME

Beta Beam Challenges: Collective Effects

Christian Hansen

2010/10/08

Many thanks to: E. Benedetto, A. Chancé, K. Li, E. Metral, N. Mounet, G. Rumolo, B. Salvant & E. Wildner

Outline

- Motivation
- Collective Effect Studies
  - ➡ Laslett's Tune Shifts
  - ➡ Wakefield Instabilities
    - ✦ HEADTAIL & MOSES
    - ✦ Intensity Thresholds
- Conclusion

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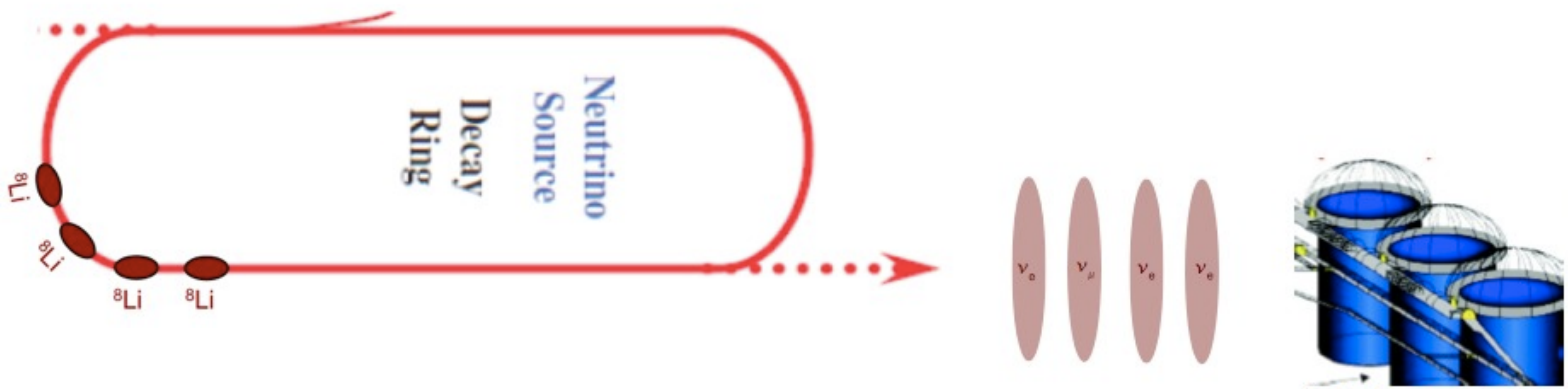
Duty Factor

MOTIVATION

- **To suppress atmospheric background detectors can only be open short time periods**
  - **<u>Suppression Factor</u>, SF = opened time ratio of the detector**
- **The DR will be filled only with short bunches so that neutrinos are send only when the detector is opened**
  - **<u>Duty Factor</u>, DF = filled ratio of the Decay Ring**

Duty Factor = Suppression Factor



Only "opened" when neutrinos arrive

Duty Factor

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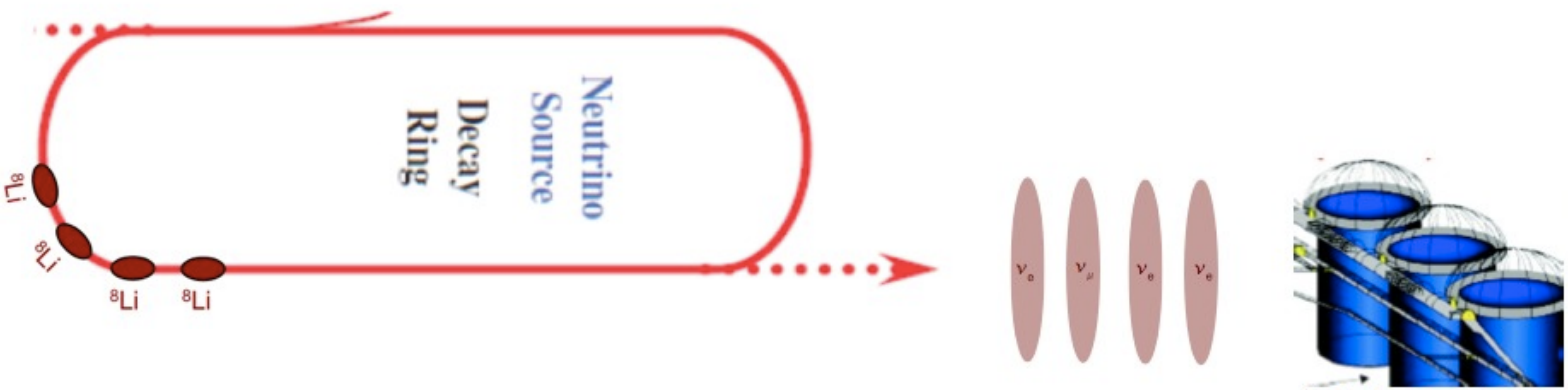
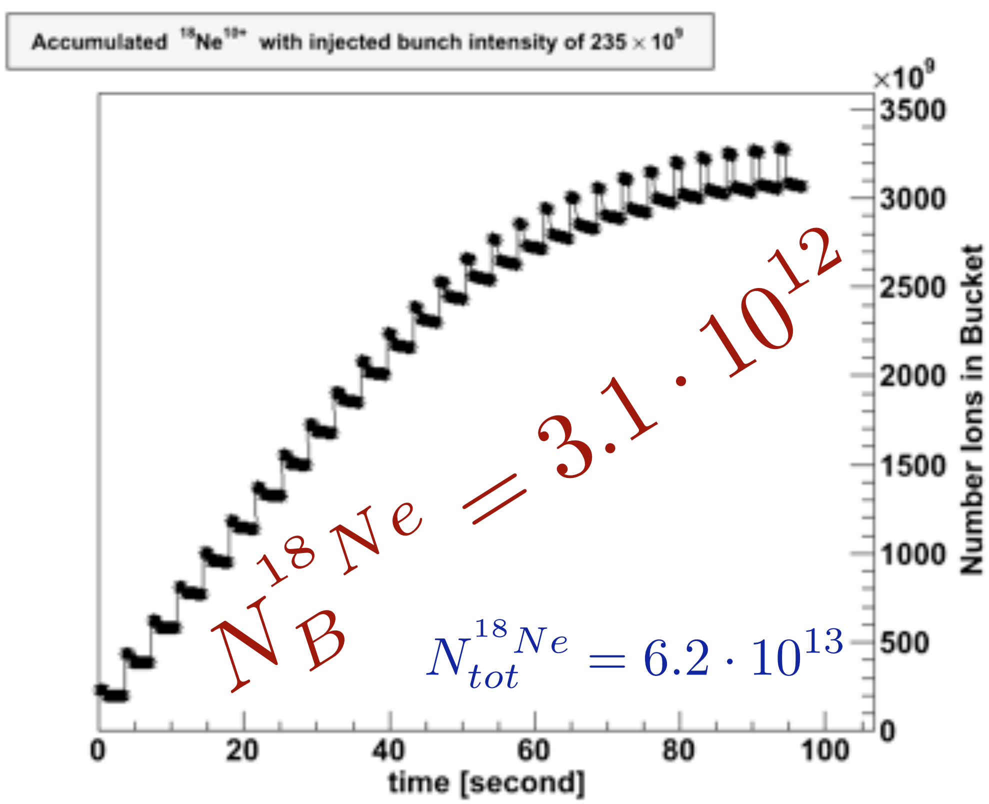
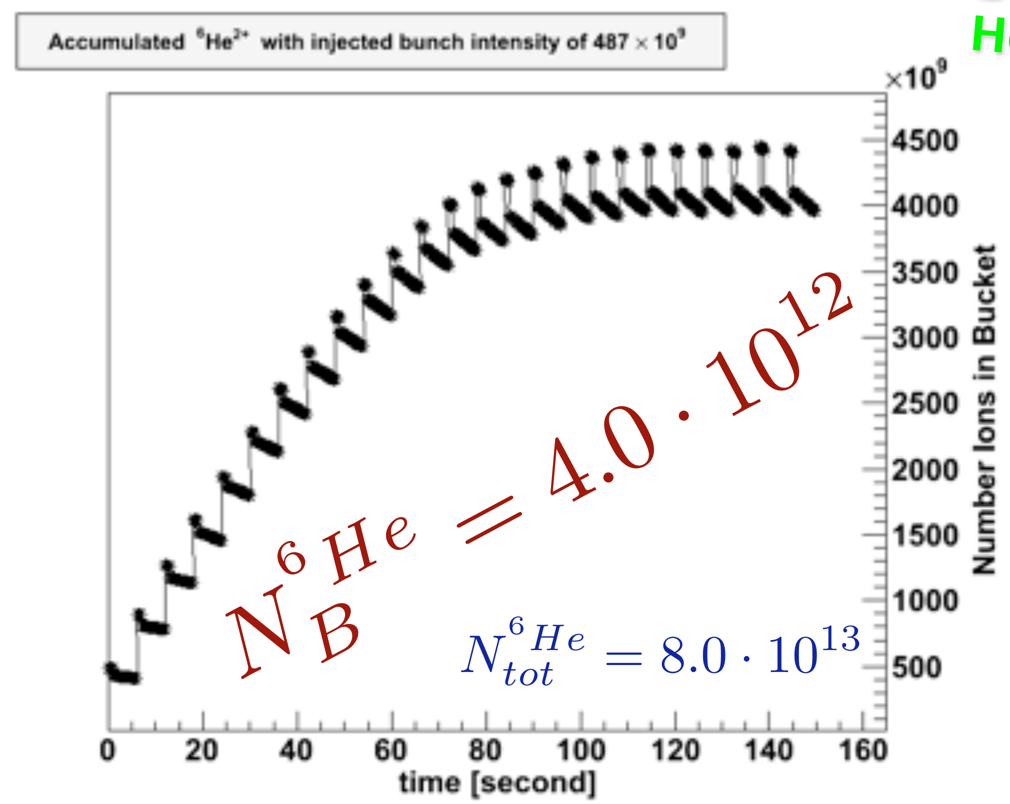


Accumulation

MOTIVATION

- **Assume about 2e13 ^{18}Ne and ^{6}He ions/sec can be produced**
- **Then about 2.4e11 ^{18}Ne and 4.9e11 ^{6}He are injected into the DR per bunch**
- **Due to collimation and radioactive decay the number of ions per bunch saturates to 3.1e12 ^{18}Ne and 4.0e12 ^{6}He ions per bunch:**

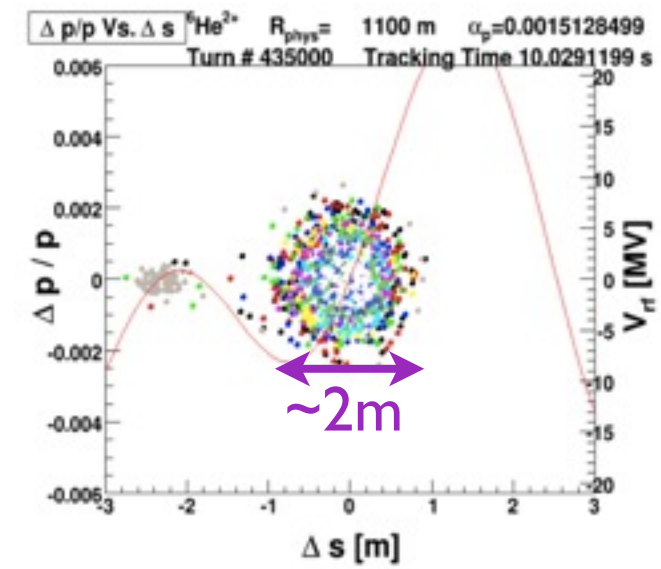




Daniel C. Heinrich

Sensitivity

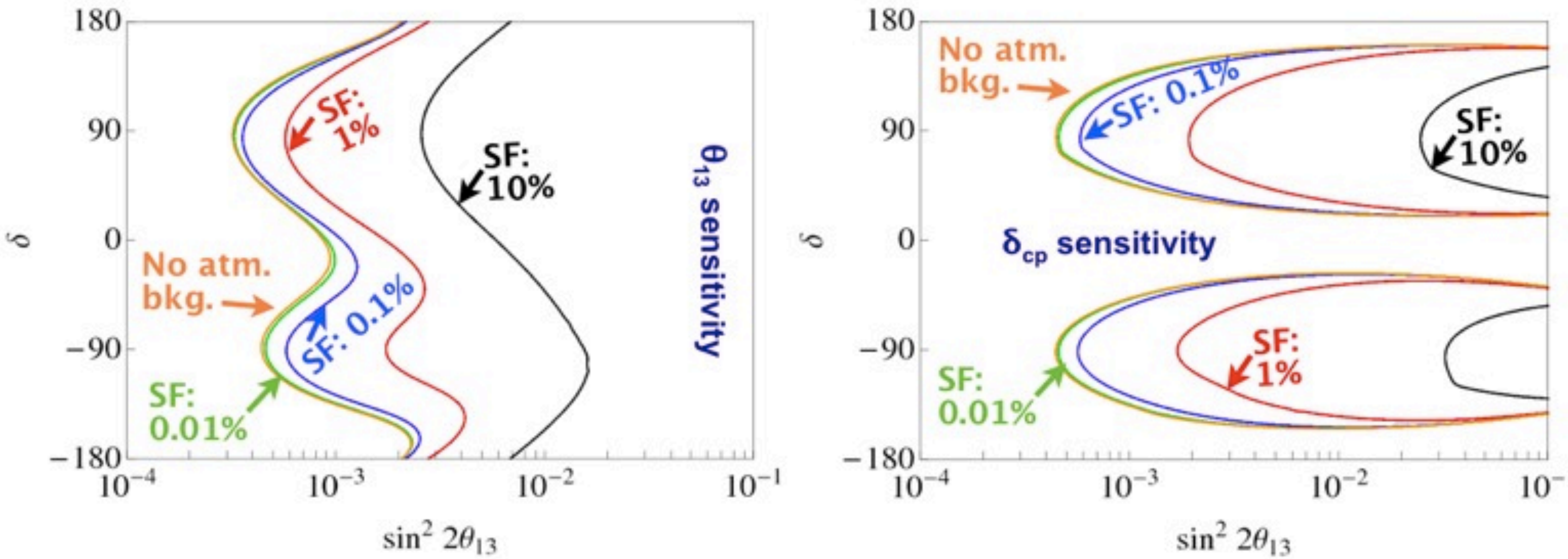
- “Merging” gives ~2m long bunches
  → ε_l = 43.3 (14.5) eVs for ^{18}Ne (^{6}He)



- 20 bunches from SPS to DR
  → **SF = 20 · 2m / 6911m = 0.58%**

- With intensities shown in previous slide there are OK sensitivities between 0.1% and 1%:

Assumed Fluxes: 1.1e18 ν/year from ^{18}Ne and 2.9e18 anti-ν/year from ^{6}He



Enrique Fernandez

- **Good, BUT: What about Collective Effects?**

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COLLECTIVE

BB Collective Effects Studies

- Instability studies are a crucial part of the Beta Beam project, since
  - ➡ High intensity ion beams are foreseen
  - ➡ Collective Effects could limit the final performance
- Will study all ions and all machines
  - ➡ So far only ^{18}Ne and ^{6}He in the DR
- Will study all possible reasons for instabilities
  - ➡ So far only
    - ✦ Laslett's tune shifts and
    - ✦ Transverse Resonance Broad Band Impedance

BB Collective Effects Studies

COLLECTIVE

- So far all studies based on EURISOL FP6 parameters

TABLE 2. Input parameters from previous Beta Beam Decay Ring design report [10].

Parameters	Description	DR ^{18}Ne	DR ^{6}He
Z	*Charge Number*	10	2
A	*Mass Number*	18	6
h	*Harmonic Number*	924	924
C [m]	*Circumference*	6911.6	6911.6
ρ [m]	*Magnetic Radius*	155.6	155.6
γ_{tr}	*Gamma at Transition*	27.00	27.00
V_{RF} [MV]	*Voltage*	1.196e+01	2.000e+01
dB/dt [T/s]	*Magnetic Ramp*	0.00	0.00
γ	*Relativistic Gamma*	100.0	100.0
δ_{max}	*Maximum Momentum Spread*	2.50e-03	2.50e-03
E_{rest} [MeV]	*Rest Energy*	16767.10	5605.54
M	*Number Bunches per Batch*	20	20
L_b [m]	*Full Bunch Length*	1.970	1.970
N_b	*Number Ions per Injected Bunch*	2.35e+11	4.87e+11
N_B	*Average Number Ions per Bunch*	3.10e+12	4.00e+12
m_r	*Merges Ratio*	20	15
$t_{1/2}$ [s]	*Half Life at Rest*	1.67	0.81
T_c [s]	*Revolution Time*	3.60	6.00
Q_x	*Horizontal Tune*	22.23	22.23
Q_y	*Vertical Tune*	12.16	12.16
$\langle\beta\rangle_x$ [m]	*Average Horizontal Betatron Function*	148.25	148.25
$\langle\beta\rangle_y$ [m]	*Average Vertical Betatron Function*	173.64	173.64
$\langle D\rangle_x$ [m]	*Average Dispersion*	-0.60	-0.60
ξ_x	*Horizontal Chromaticity*	0.0	0.0
ξ_y	*Vertical Chromaticity*	0.0	0.0
$\varepsilon_{N_x}(1\sigma)$ [πm·rad]	*Normalized Horizontal Emittance*	1.48e-05	1.48e-05
$\varepsilon_{N_y}(1\sigma)$ [πm·rad]	*Normalized Vertical Emittance*	7.90e-06	7.90e-06
ε_l (full) [eVs]	*Full Longitudinal Emittance*	42.89	14.36
b_x [cm]	*Horizontal Beam Pipe Size*	16.0	16.0
b_y [cm]	*Vertical Beam Pipe Size*	16.0	16.0
ρ_{res} [Ω m]	*Resistivity*	1.0e-07	1.0e-07

TABLE 3. Assumed impedance input parameters.

Parameters	Description	DR ^{18}Ne	DR ^{6}He
$Q_{\parallel}$	*Longitudinal Quality Factor*	1.00	1.00
$\omega_{r,\parallel}$ [GHz]	*Longitudinal Angular Resonance Frequency*	6.28	6.28
$\left\vert Z_{\parallel}/n\right\vert$ [Ω] $= \lim_{\omega\to 0}\frac{\vert Z(\omega)\vert}{\omega/\omega_{rev}}$		10.00	10.00
$R_{s,\parallel}$ [MΩ] $= \frac{\vert Z/n\vert Q\omega_r}{\omega_{rev}}$	*Longitudinal Shunt Impedance*	0.231	0.231
$Q_{\perp}$	*Transverse Quality Factor*	1.00	1.00
$\omega_{r,\perp}$ [GHz]	*Transverse Angular Resonance Frequency*	6.28	6.28
$R_{s,\perp}$ [$M\Omega/m$]	*Transverse Shunt Impedance*	20.00	20.00

Data Base:
http://j2eeps.cern.ch/beta-beam-parameters/

TABLE 4. Calculated values.

		DR ^{18}Ne	DR ^{6}He
r_0 [m] $= r_p Z^2/A$	*Ion Radius*	8.53e-18	1.02e-18
E_{tot} [GeV] $= \gamma\cdot E_{rest}$	*Total Energy*	1676.71	560.55
$\beta = \sqrt{1-1/\gamma^2}$	*Relativistic Beta*	1.00	1.00
$\eta = (1/\gamma_{tr})^2 - (1/\gamma)^2$	*Phase Slip Factor*	1.27e-03	1.27e-03
T_{rev} [μ s] $= C/(\beta c)$	*Revolution Time*	23.0558	23.0558
R [m] $= C/2\pi$	*Machine Radius*	1100.02	1100.02
ω_{rev} [MHz] $= 2\pi/T_{rev}$	*Angular Revolution Frequency*	0.27	0.27
$\sigma_\delta = \delta_{max}/2$	*1 Sigma Momentum Spread*	1.25e-03	1.25e-03
τ_b [ns] $= L_b/(\beta c)$	*Full Bunch Length*	6.57	6.57
$\hat{I}$ [A] $= ZeN_B/\tau_b$	*Peak Current*	755.80	195.04
I_b [A] $= ZeN_B/T_{rev}$	*Beam Current*	0.22	0.06
$\varepsilon_l^{2\sigma}$ [eVs] $= \frac{\pi}{2}\beta^2 E_{tot}\tau_b\delta_{max}$	*2 Sigma Longitudinal Emittance*	43.27	14.46
$Q_s = \sqrt{\frac{hZeV\vert\eta\cos\phi_s\vert}{2\pi\beta^2 E_{tot}}}$	*Synchrotron Tune*	0.00	0.00
ω_s [kHz] $= Q_s\cdot\omega_{rev}$	*Synchrotron Angular Frequency*	1.00	1.00
ω_x [MHz] $= Q_x\cdot\omega_{rev}$	*Horizontal Betatron Angular Frequency*	6.06	6.06
ω_y [MHz] $= Q_y\cdot\omega_{rev}$	*Vertical Betatron Angular Frequency*	6.06	6.06
ω_c [GHz] $= \beta c/b_{min(x,y)}$	*Cut-Off Angular Frequency*	1.87	1.87
$\Delta Q_{\xi_x} = \xi_x\delta_{max}Q_x$	*Horizontal Tune Shift due to Chromaticity*	0.0	0.0
$\Delta Q_{\xi_y} = \xi_y\delta_{max}Q_y$	*Vertical Tune Shift due to Chromaticity*	0.0	0.0
ω_{ξ_x} [MHz] $= \xi_x Q_x\omega_{rev}/\eta$	*Horizontal Chromatic Angular Frequency*	2.38e+02	2.38e+02
ω_{ξ_y} [MHz] $= \xi_y Q_y\omega_{rev}/\eta$	*Vertical Chromatic Angular Frequency*	1.30e+02	1.30e+02

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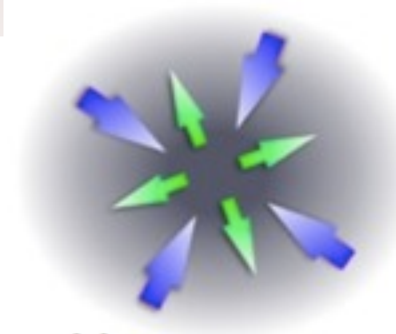
Laslett's Tune Shifts

LASLETT'S



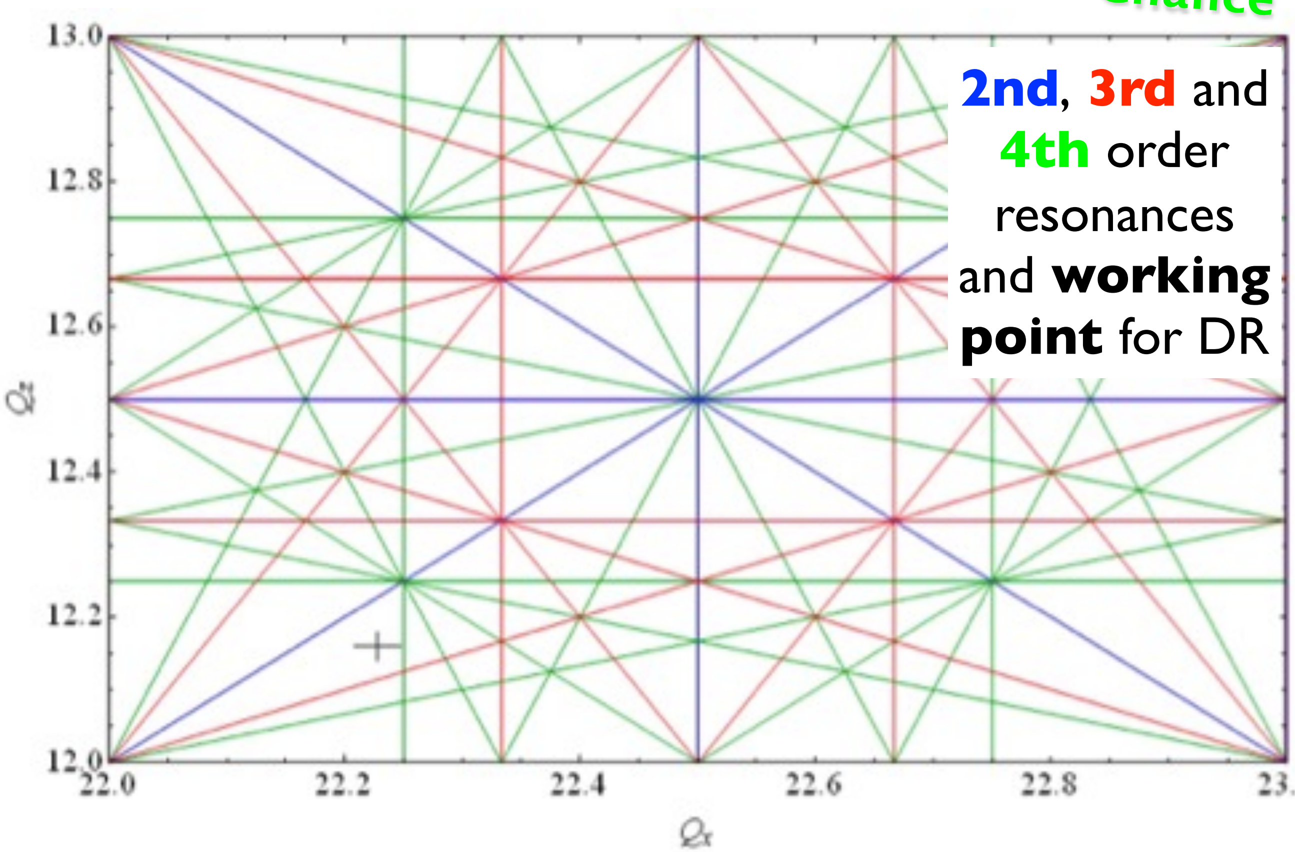
- Coulomb Forces
  - within the bunch; "**Direct Space Charge**"
  - between bunch and pipe; "**Image Field**"

(Assuming perfect conductive beam pipe, for resistive beam pipe Resistive Wall Impedance studies necessary)

can cause change in number betatron oscillations per turn

- These Tune Shifts, ΔQ, could in-stabilize the beam if they cross resonances
- A "rule of thumb for synchrotrons with short cycles":
  if | ΔQ | < 0.2
  → normally no severe instability

Antoine Chancé



2nd, **3rd** and **4th** order resonances and **working point** for DR

LASLETT'S

Laslett's Tune Shifts

- Tune shifts due to **Direct Space Charge** and **Image Fields** are described by "Laslett's Equations"

DR →

SC	DR ^{18}Ne	DR ^{6}He
ΔQ_{dsc_x}	-0.030	-0.005
ΔQ_{dsc_y}	-0.069	-0.011
ΔQ_x^{incoh}	-2.97e-02	-4.59e-03
ΔQ_y^{incoh}	-6.86e-02	-1.06e-02
$\Delta Q_x^{coh\ p}$	-1.27e-04	-1.96e-05
$\Delta Q_y^{coh\ p}$	-2.32e-04	-3.59e-05
$\Delta Q_x^{coh\ np}$	-4.55e-05	-7.05e-06
$\Delta Q_y^{coh\ np}$	-8.32e-05	-1.29e-05

➡ For DR (γ=100)

$|\Delta Q_{DSC}| << 0.2$ ☺

- DR not a short cycle (ions could stay ~1min) → "rule of thumb" maybe not applicable → Might need a deeper DSC study
- Image Fields turned out to have even less effects

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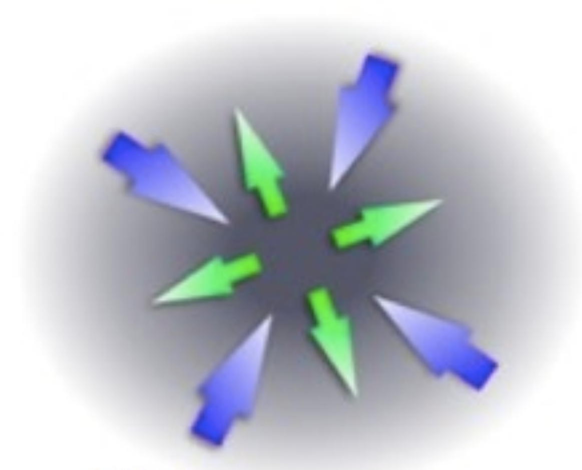
- *Note also;* $\Delta Q_{DSC_{x,y}} \propto 1/\gamma^2$ since for relativistic beams the repulsive **E forces** are cancelled by the contracting **B forces**



➡ For PS (low γ) ΔQ_{DSC} could be crucial
(to be investigated)

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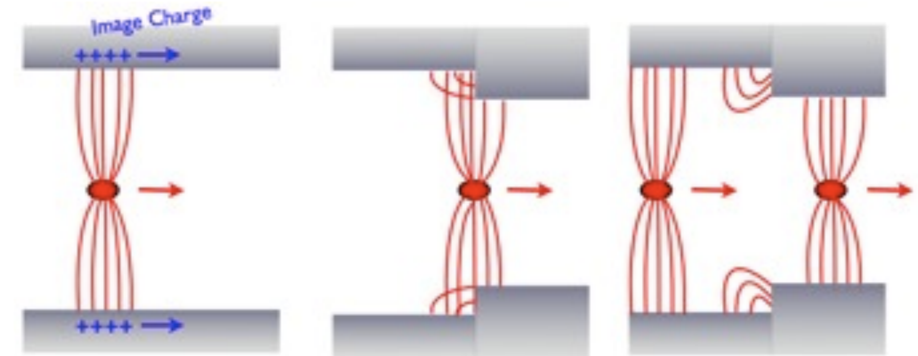
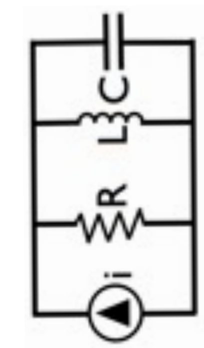
Resonance Impedance

WAKE FIELDS

- Wake Fields *(time domain; W(t))* can
  - be trapped in pipe cavities
  - cause "**Resonance Impedance**"



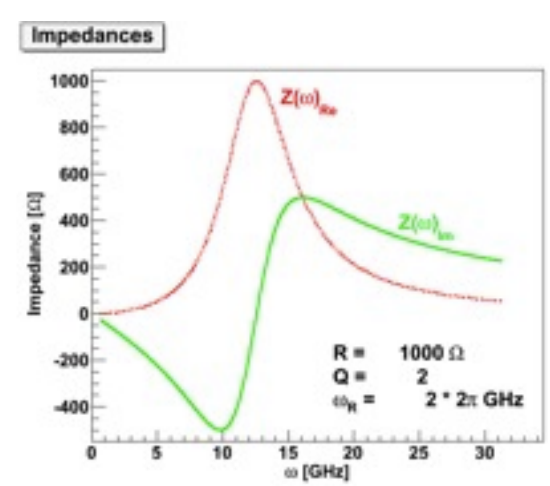
- Resonance Impedance *(frequency domain; $Z(\omega)=\mathcal{F}[W(t)]$)*,
  - in the Transverse plane can be modeled by an RLC circuit as:

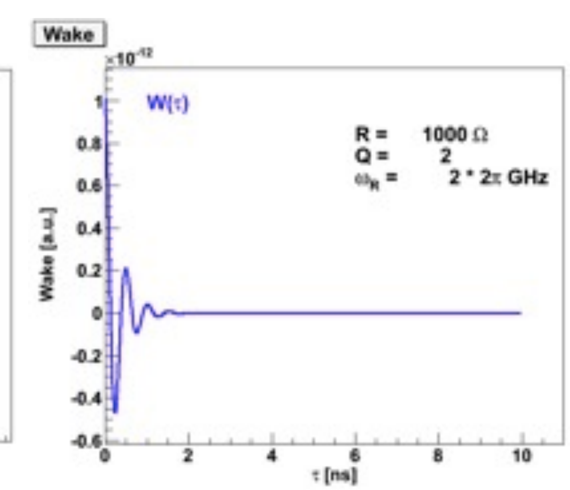


$$Z_\perp(\omega) = \frac{R_\perp \frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Q = "**Quality Factor**"
ω_r = "**Resonance Angular Frequency**"
$R_\perp$ = "**Shunt Impedance**"

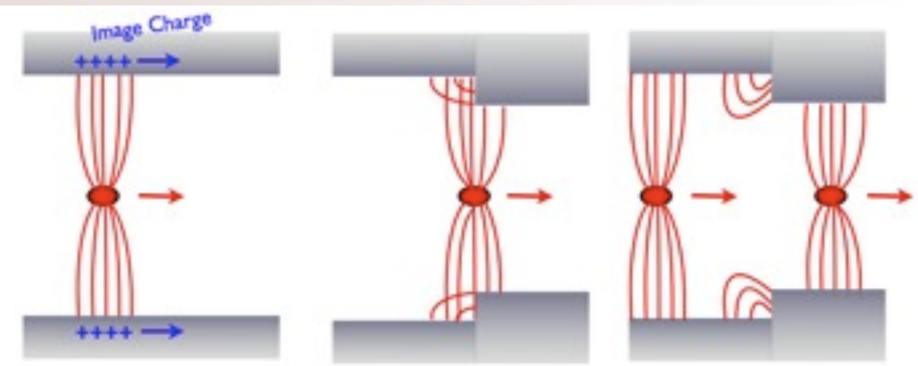
  - For low Quality Factor ($Q\approx 1$) the Wake Field is short lived and the impedance is "Broad Band"





Resonance Impedance

WAKE FIELDS



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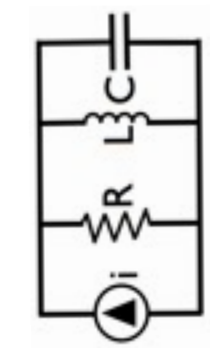

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$$Z_{\perp}(\omega) = \frac{R_{\perp}\frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Q = **"Quality Factor"** = **1**

ω_r = **"Resonance Angular Frequency"** ≈ **$\omega_c = \beta c/b_y$**

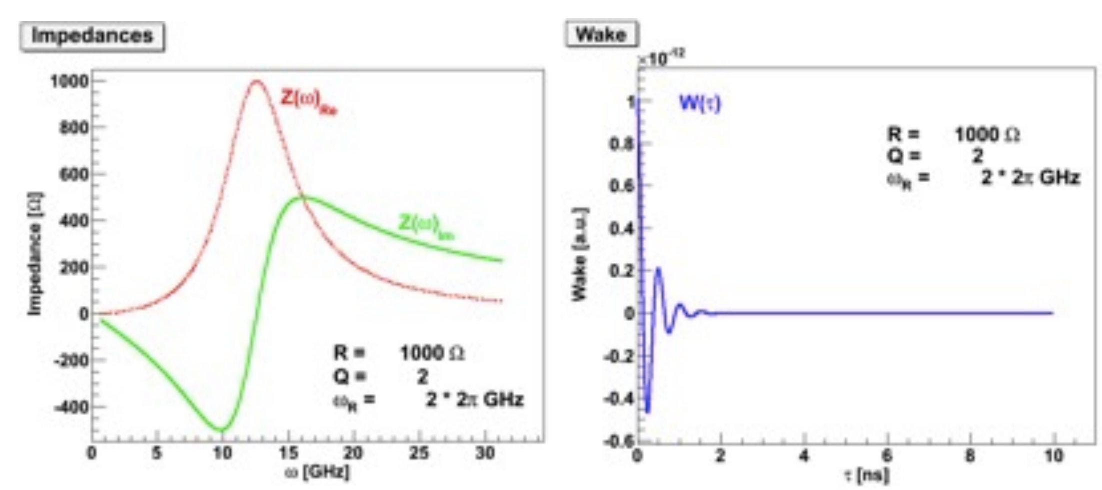
$R_{\perp}$ = **"Shunt Impedance"**

(see next slide)

  - For low Quality Factor (Q≈1) the Wake Field is short lived and the impedance is "Broad Band"

- Will show results from

  "Transverse Resonance Broad Band Impedance"



Shunt Impedance

- The "Shunt Impedance", $R_\perp$, is the main parameter in the RLC model of the Resonance Impedance

$$Z_\perp(\omega) = \frac{R_\perp \frac{\omega_r}{\omega}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$

Private discussions with E. Métral

- Modeling existing machines the same way we get

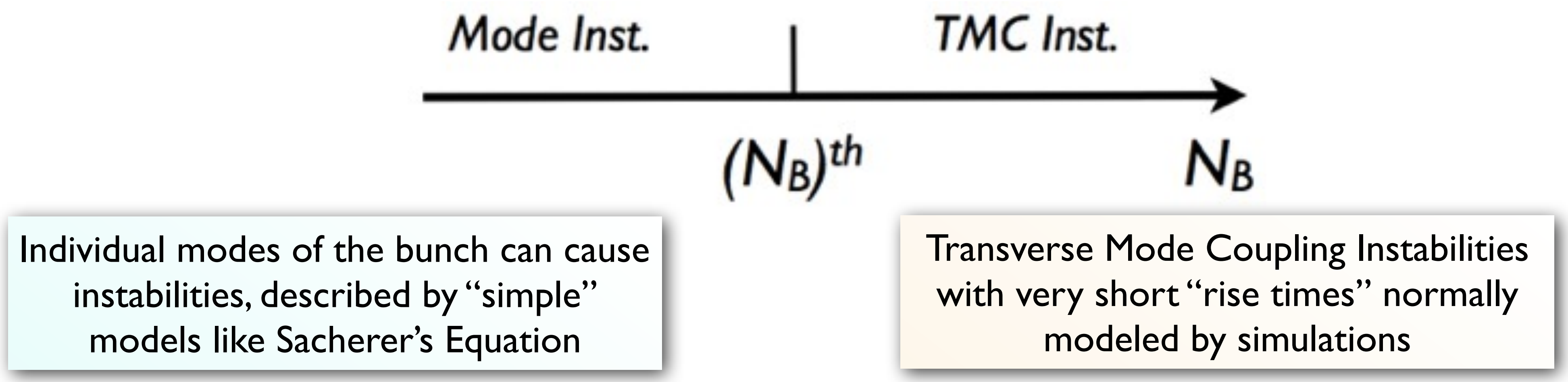
	PS	SPS	LHC	LHC (no collimators)
$R_\perp$ [MΩ/m]	3	20	30	2

- Will use these two as examples

Intensity Threshold

- Instabilities caused by $Z_\perp(\omega)$ are described by different theories depending on the intensity regime

$\perp$



- Important to find $\boldsymbol{N_b^{th}}$ since that is *absolute* maximum number ions we can have per bunch:
  - Will define $\boldsymbol{N_b^{th}}$ as the intensity that gives instabilities with *very* short rice times (*optimistic approach*)
    (i.e. when we have **strong** Transverse Mode Coupling)
  - But also for longer rice times (*pessimistic approach*)
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Intensity Threshold

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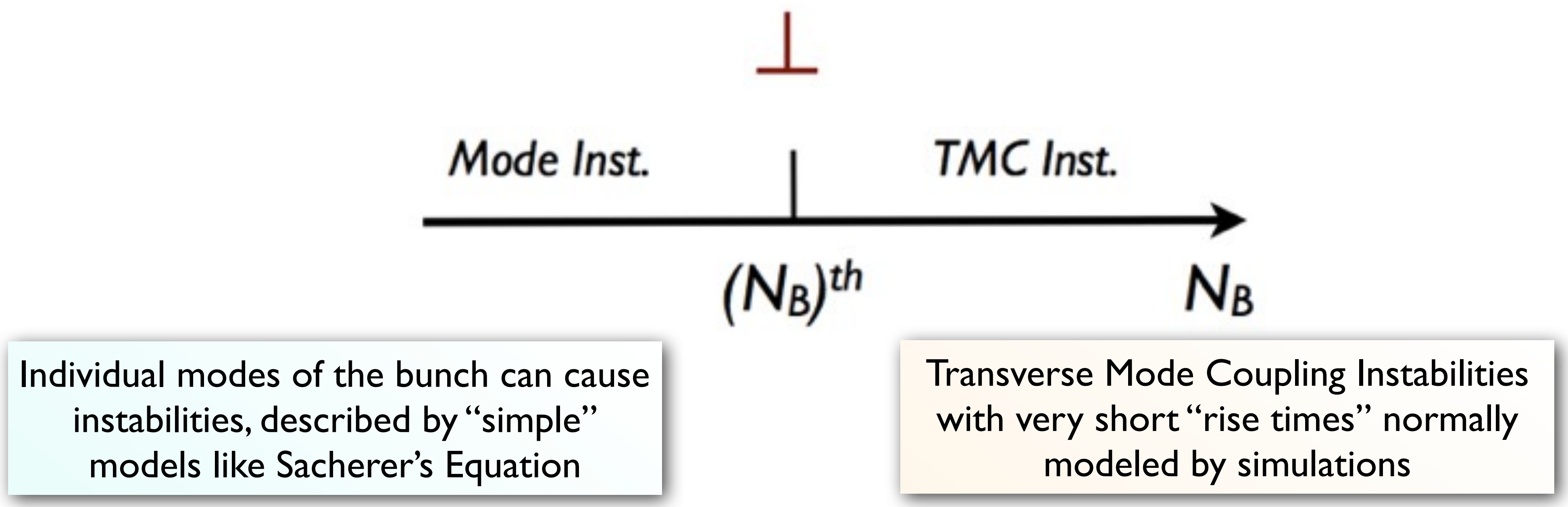

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    $(1/T)^{th} = 400\text{Hz}$
  - But also for longer rice times (*pessimistic approach*)
    (i.e. when we have not so strong Transverse Mode Coupling)
    $(1/T)^{th} = 20\text{Hz}$

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3 Tools

- Three different ways to find N_b^{th}:
  - By using the peak current values as inputs to a formula for coasting beams one obtains a theoretical equation for the bunch intensity limit, which we will call the "Coasting Beam Equation":

$$N^{th}_{B_{x,y}} = \frac{32}{3\sqrt{2}\pi} \frac{R|\eta|\epsilon_l^{2\sigma}\omega_r}{\langle\beta\rangle_{x,y} Z^2\beta^2 c} \frac{1+\omega_{\xi_{x,y}}/\omega_r}{\Re\left[Z^{BB}_{\perp_{x,y}}\right]_{max}}$$

E. Métral, CERN, Overview of Single-Beam Coherent Instabilities in Circular Accelerators

  - A theoretical program, "MOSES"
  - A multi-particle tracking program, "HEADTAIL"

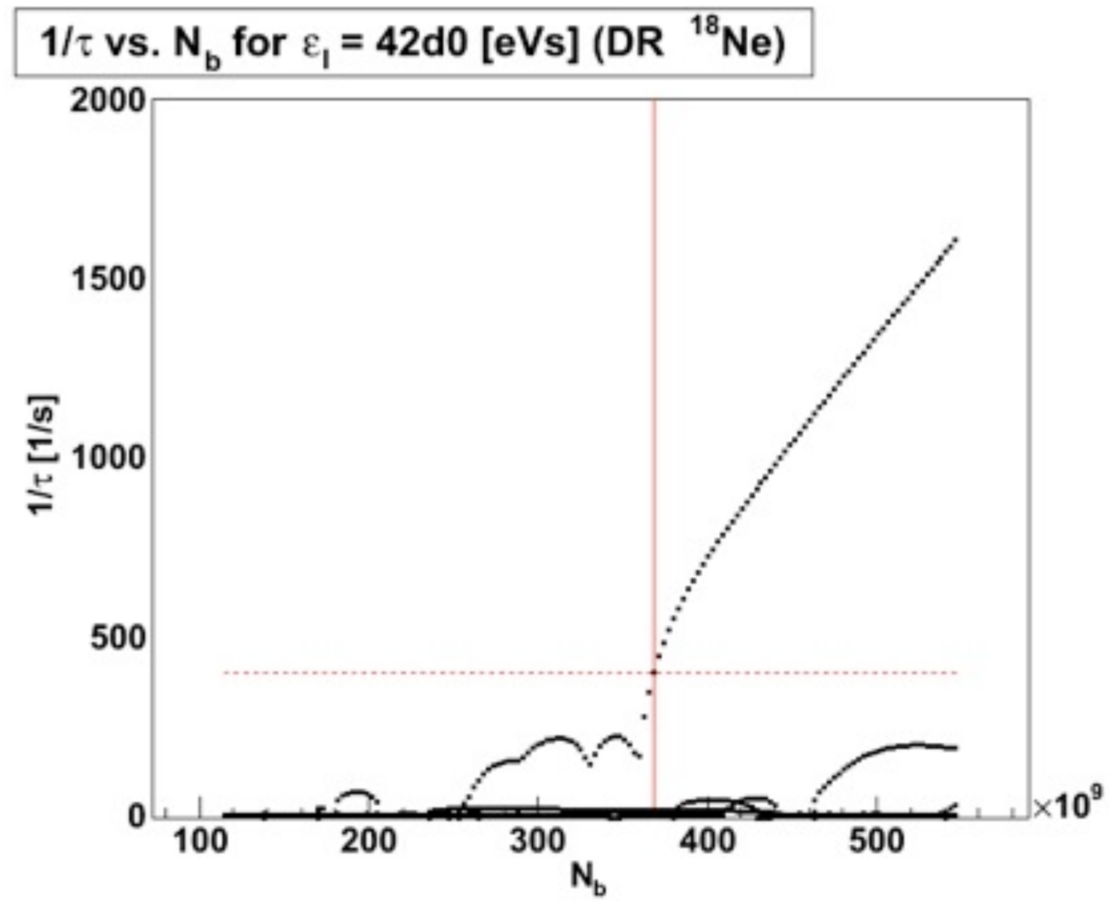
"Next Slides"

MOSES

- MOSES is a theoretical program
  - ➡ It solves a dispersion integral equation

Y.H.Chin CERN-LEP-TH/88-05

- It gives the growth rate (inverse of the rise time) for different "bunch modes"



- Intensity limit, $\boldsymbol{N_b^{th}}$, depends on optimistic ($\boldsymbol{(1/\tau)^{th}}$ = 400Hz) or pessimistic ($\boldsymbol{(1/\tau)^{th}}$ = 20Hz) approach

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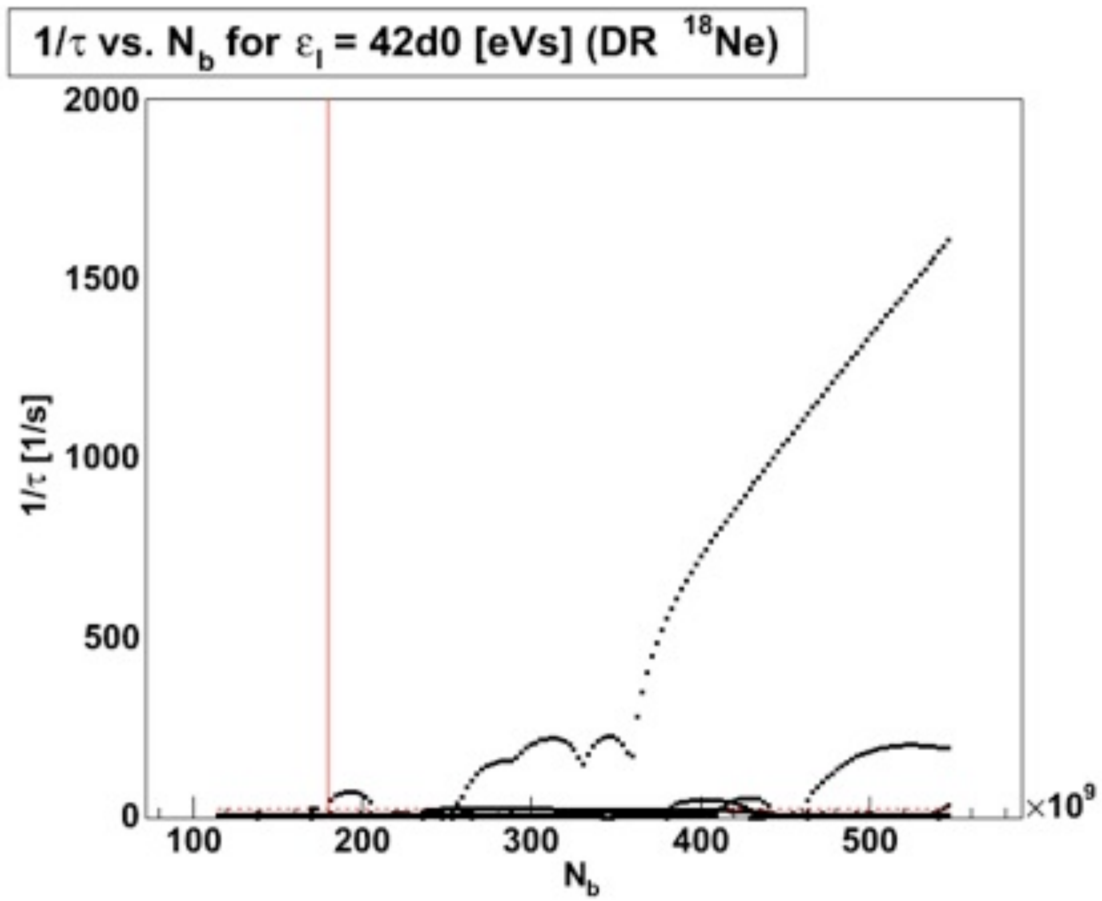

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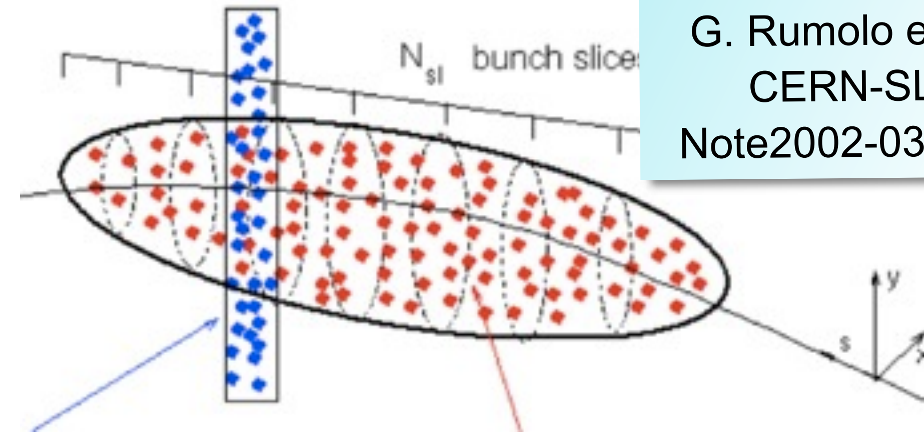
HEADTAIL

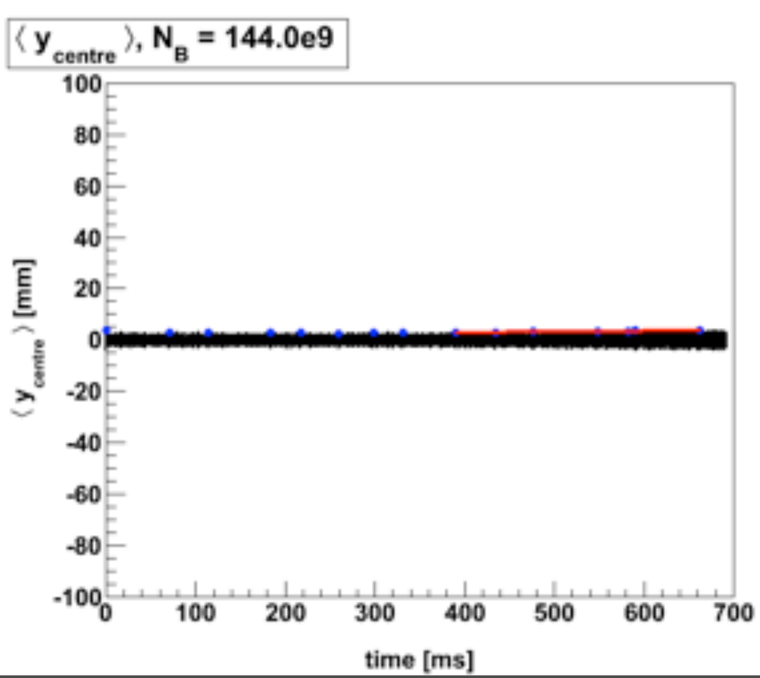
HEADTAIL / MODES

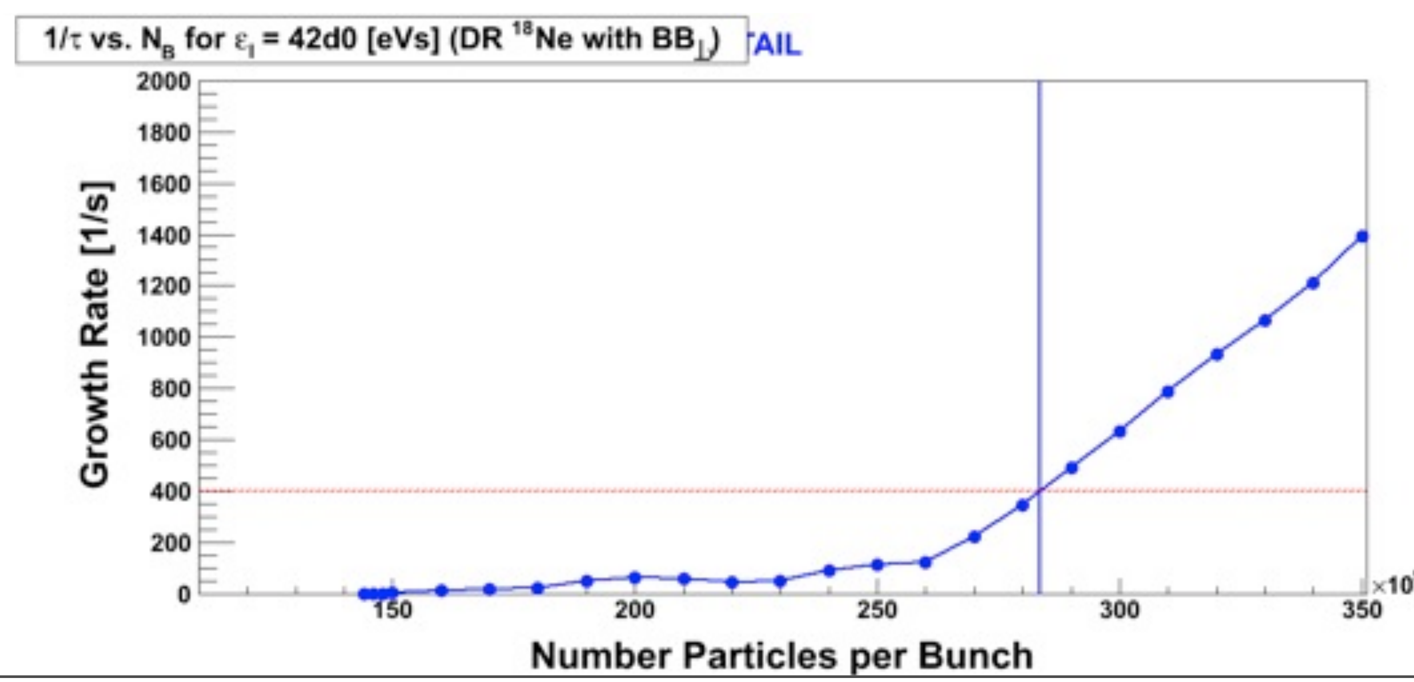
- In HEADTAIL a bunch of multi-particle is tracked
  - ➡ It is sliced longitudinally
  - ➡ At each impedance location each slice leaves a wake field behind and gets a kick by the field generated by the preceding slices
  - ➡ The bunch is then transferred to the next impedance location via a transport matrix



G. Rumolo et ali, CERN-SL-Note2002-036-AP

- N_b^{th} is given by the growth of the bunch oscillation

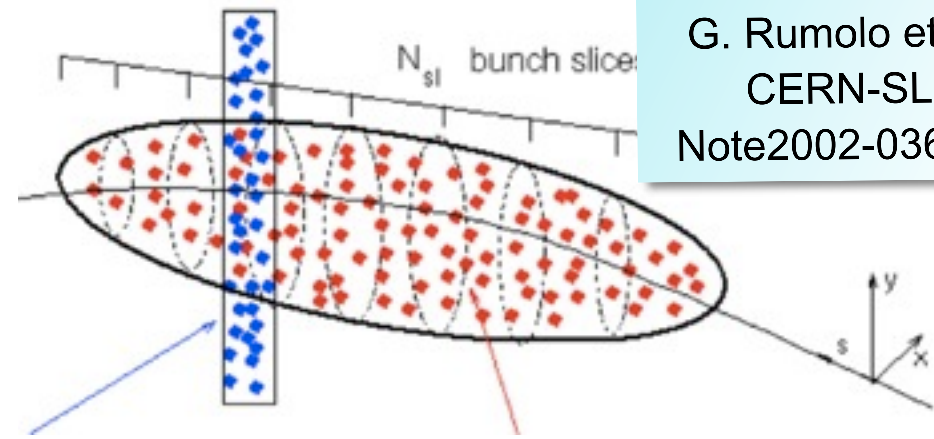




$(1/\tau)^{th}$ = 400Hz

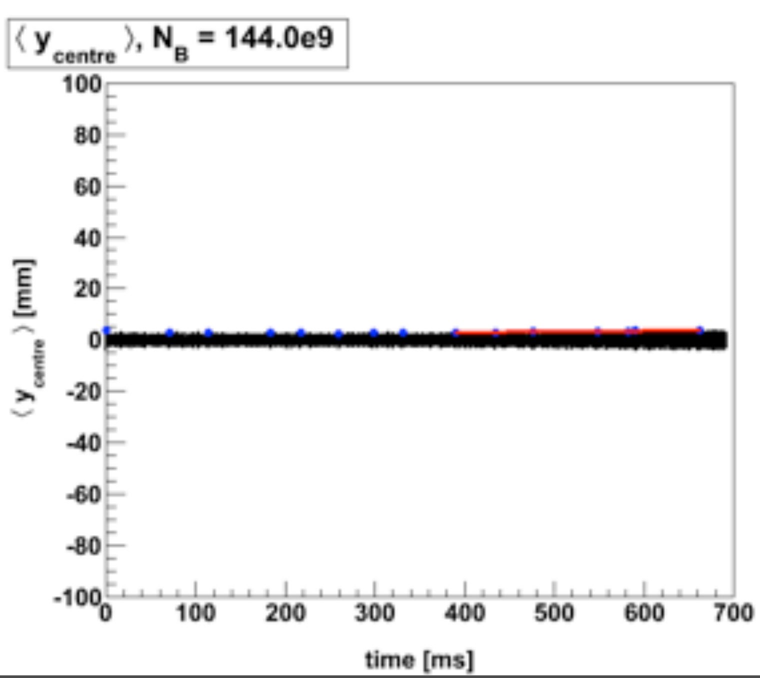
HEADTAIL

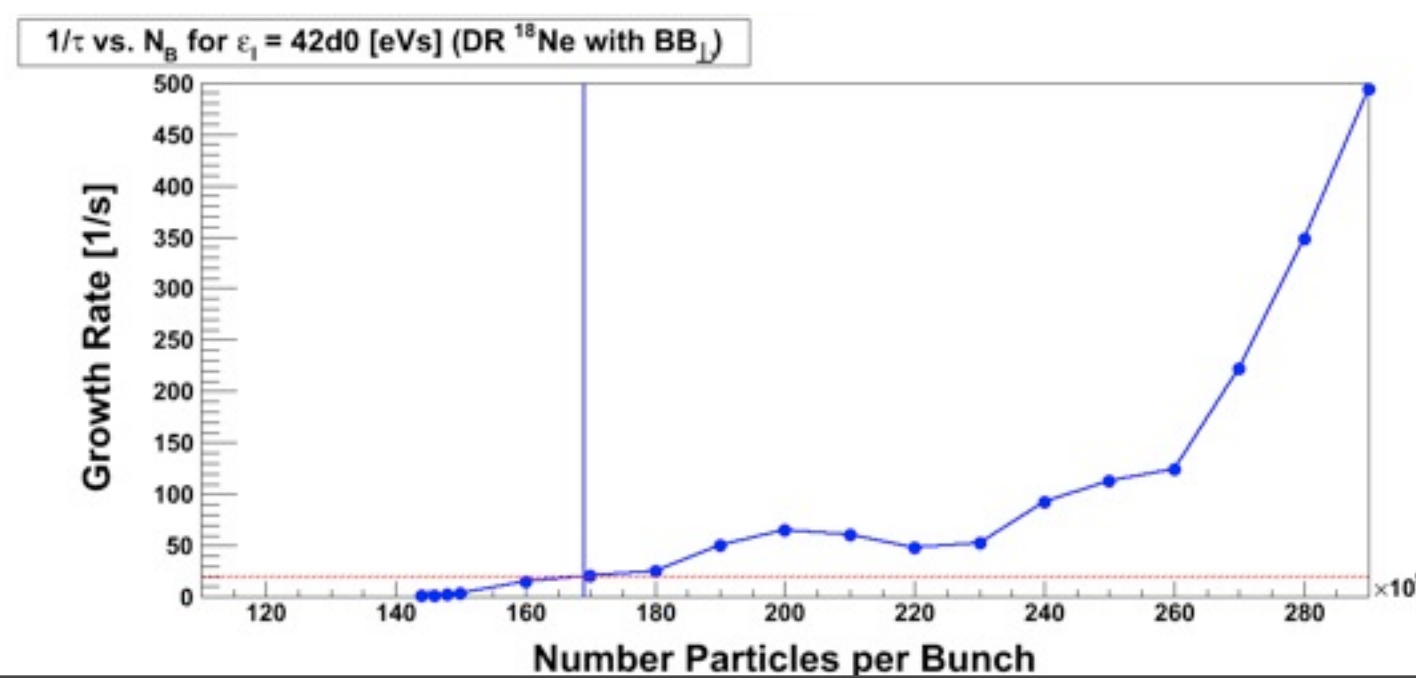
- In HEADTAIL a bunch of multi-particle is tracked
  - ➡ It is sliced longitudinally
  - ➡ At each impedance location each slice leaves a wake field behind and gets a kick by the field generated by the preceding slices
  - ➡ The bunch is then transferred to the next impedance location via a transport matrix



G. Rumolo et ali, CERN-SL-Note2002-036-AP

- N_b^{th} is given by the growth of the bunch oscillation



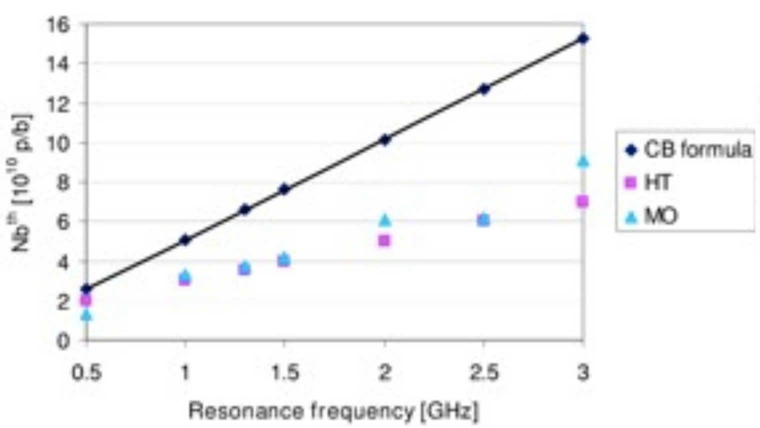


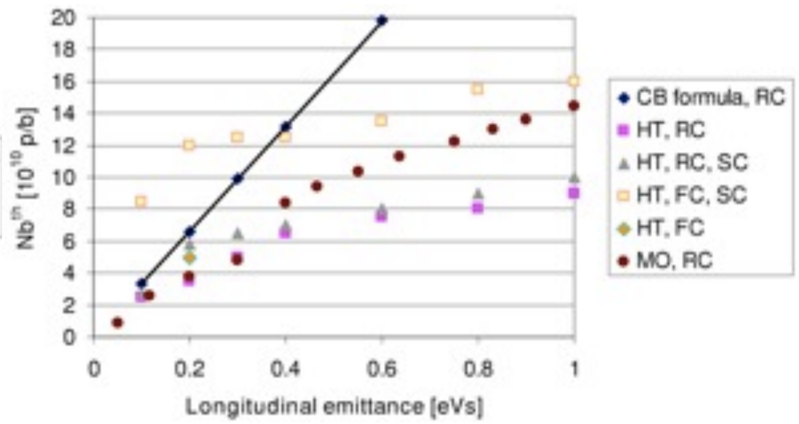
$(1/\tau)^{th}$ = 20Hz

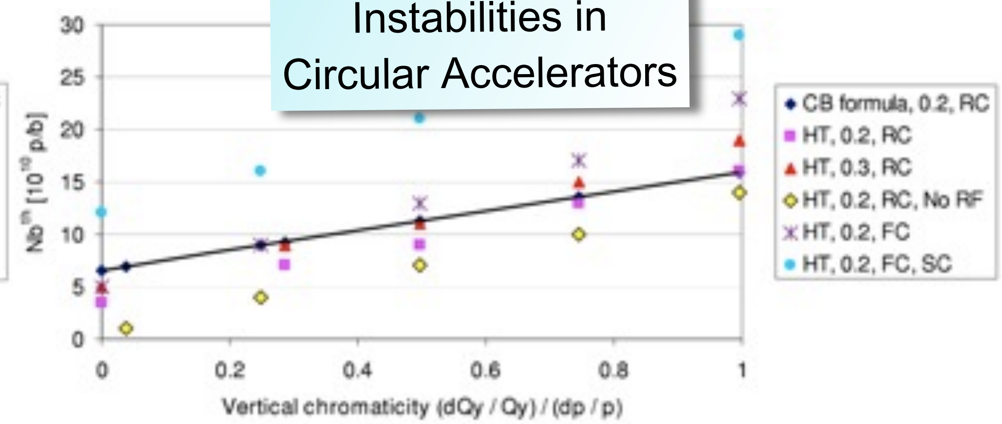
MOSES & HEADTAIL

- Despite two totally different approaches “MOSES” and “HEADTAIL” has been successfully benchmarked both with each other and with data
- E.g. scans over ε_l, f_r and ξ;

E. Métral, CERN, Overview of Single-Beam Coherent Instabilities in Circular Accelerators



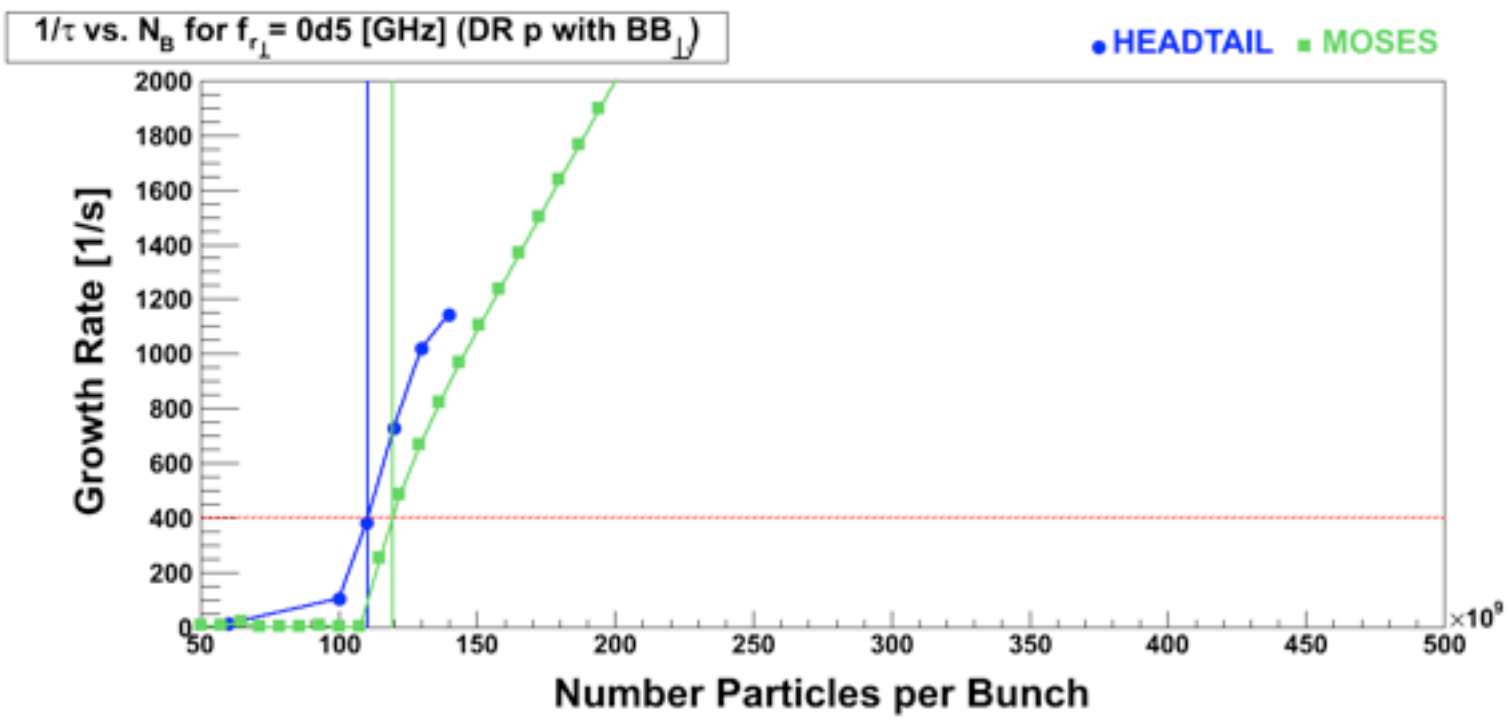


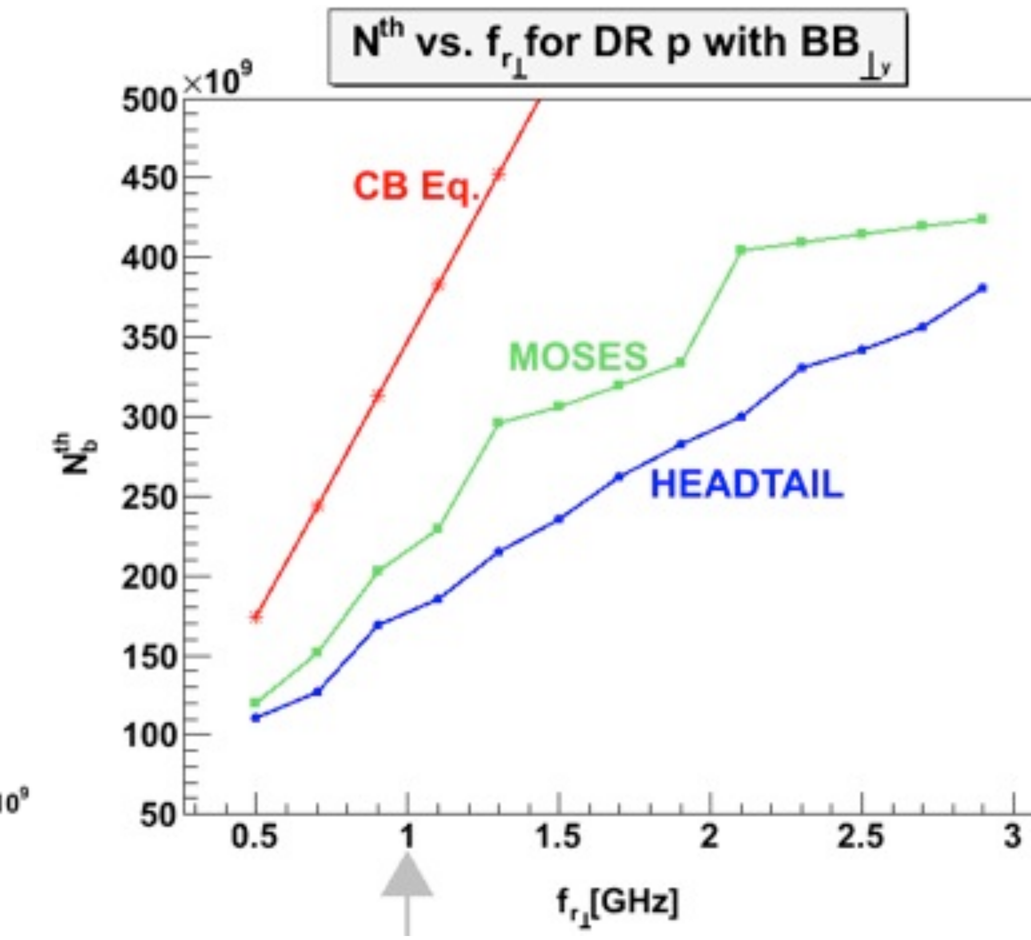


- However, both have mostly been used with Protons
- So before we use HEADTAIL and MOSES for the Ions in the Beta Beams, let’s start with Protons…

f_r Scan for Protons in DR

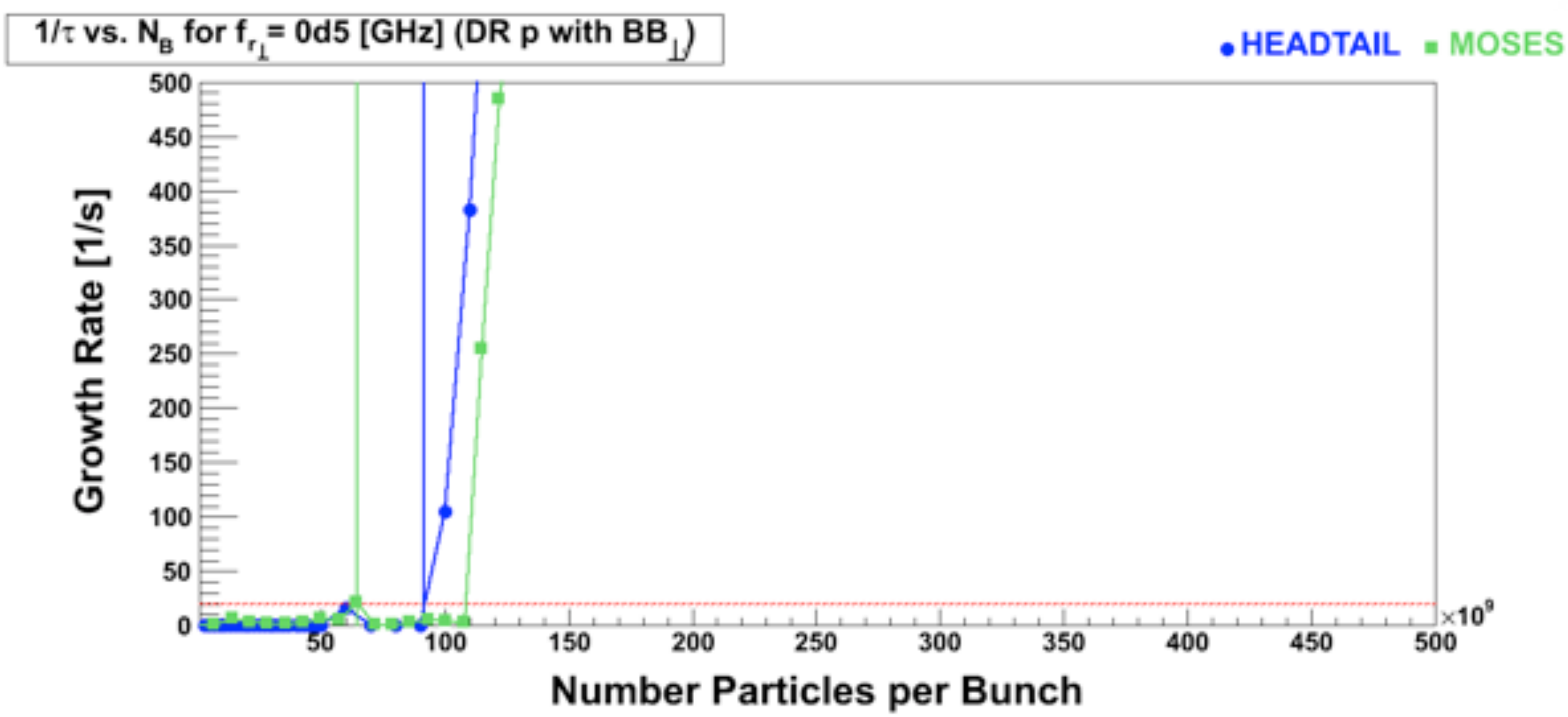
- Strong instabilities, $1/\tau > 400$Hz, starts at about same $N_b{}^{th}$ for MOSES and HEADTAIL

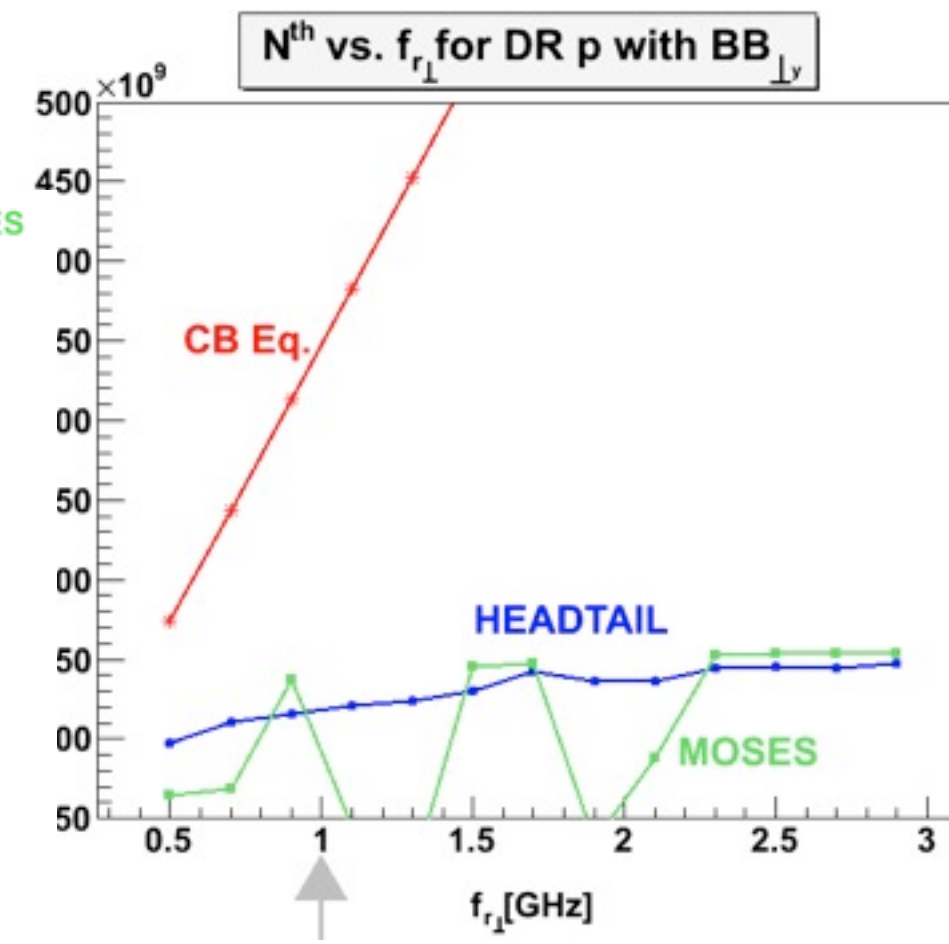




f_r Scan for Protons in DR

- Strong instabilities, $1/\tau$ > 400Hz, starts at about same N_b^{th} for MOSES and HEADTAIL





- Weaker instabilities, $1/\tau$ > 20Hz, could show up for low N_b^{th} according to MOSES → discrepancies
  - ➡ Due to weak mode coupling and decoupling
  - ➡ Should be seen by HEADTAIL also → under investigation

Outline

- Beta Beam Overview
- Collective Effects
  - ➡ Laslett's Tune Shifts
  - ➡ Wakefield Instabilities
    - ✦ HEADTAIL & MOSES
    - ✦ Intensity Thresholds
- Conclusion

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Back to Ions

THRESHOLD

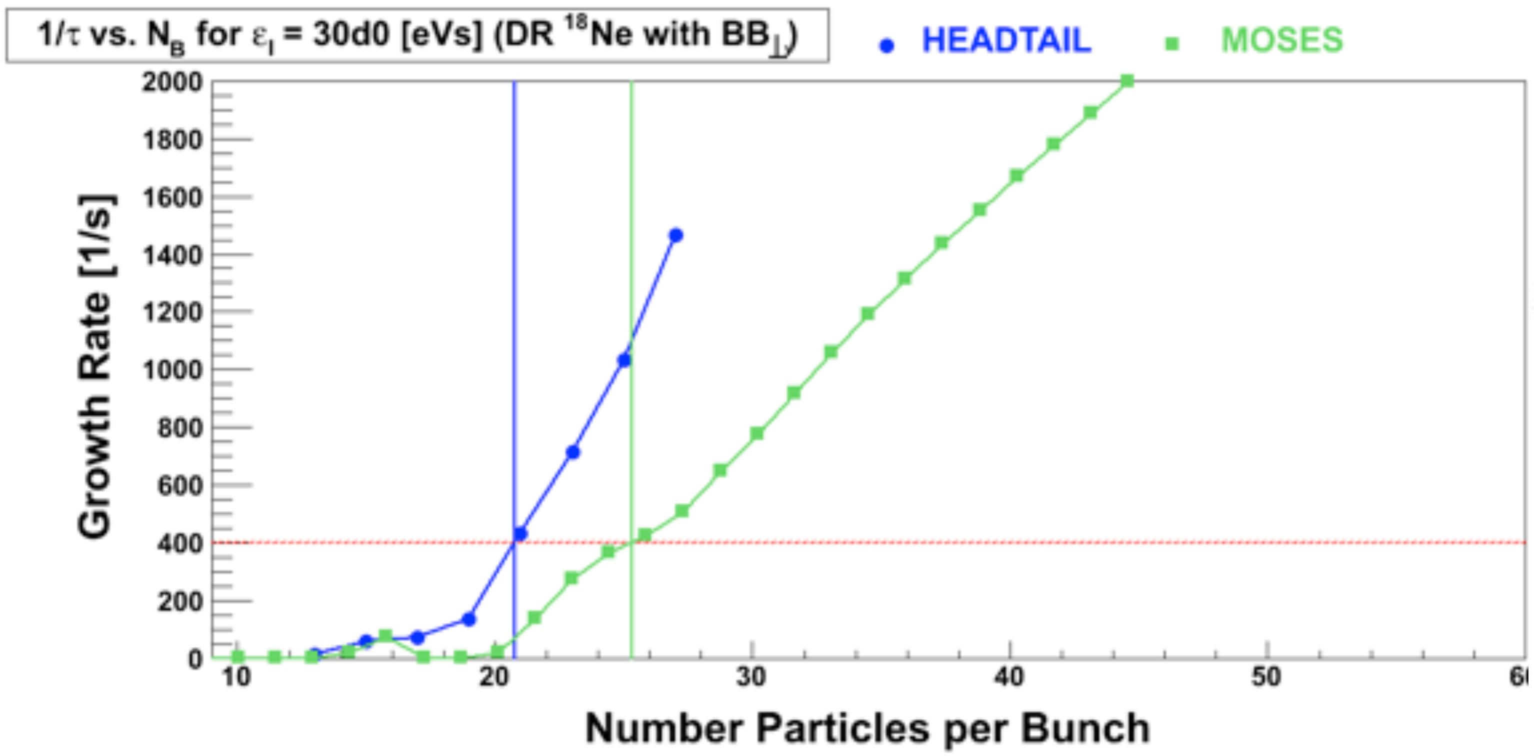
- Let's apply HEADTAIL and MOSES to ^{6}He and ^{18}Ne
- Will see if the required $\boldsymbol{N_b^{th}}$ can be achieved for different longitudinal emittances, $\boldsymbol{\varepsilon_l}$, of the bunch and different shunt impedances, $\mathbf{R}_\perp$, of the machine

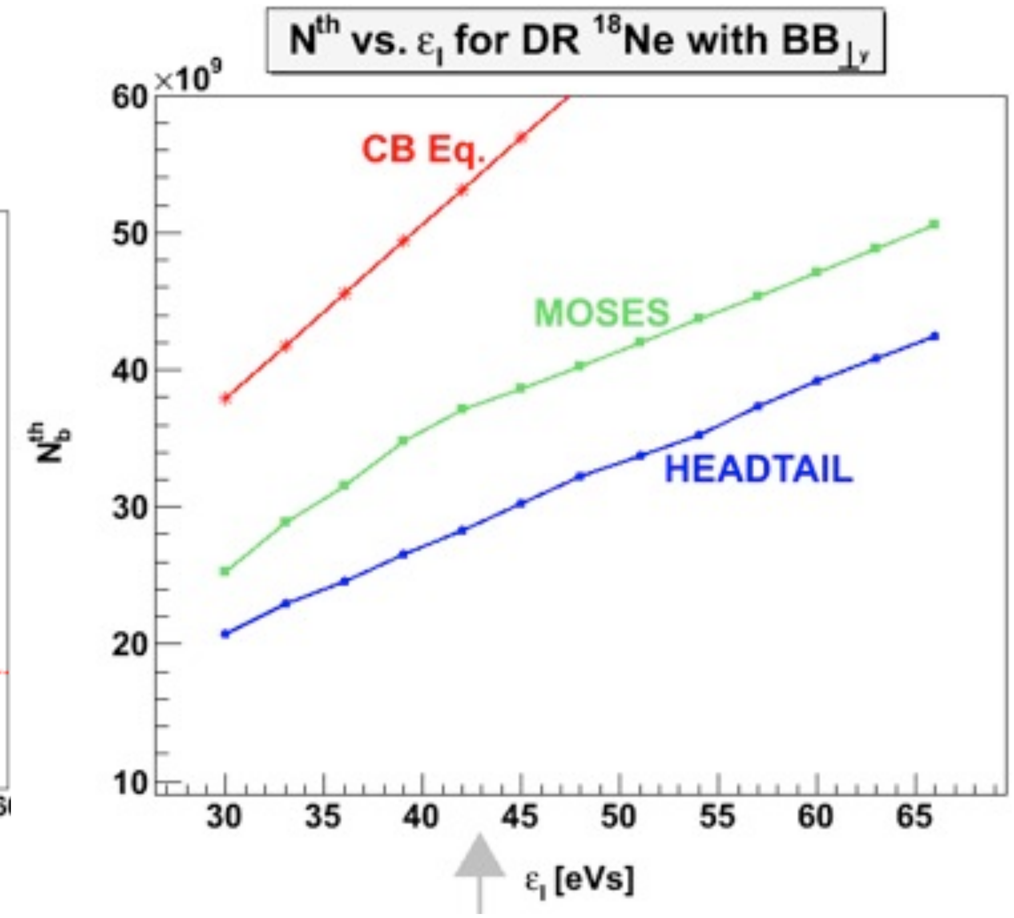
- These two programs have however never been used like this for ions (as far as we know) so development of new procedures (and thoroughly tests of these) necessary:
  - ➡ Possibility of bunches with ^{18}Ne and ^{6}He was added to HEADTAIL
  - ➡ We get the ion equivalent threshold from MOSES by
    $\boldsymbol{N_b^{th}} = N_b^{th} / Z$ *(see back-up slide)*

ε_l Scan for ^{18}Ne $(1/\tau)^{th}$ = 400Hz

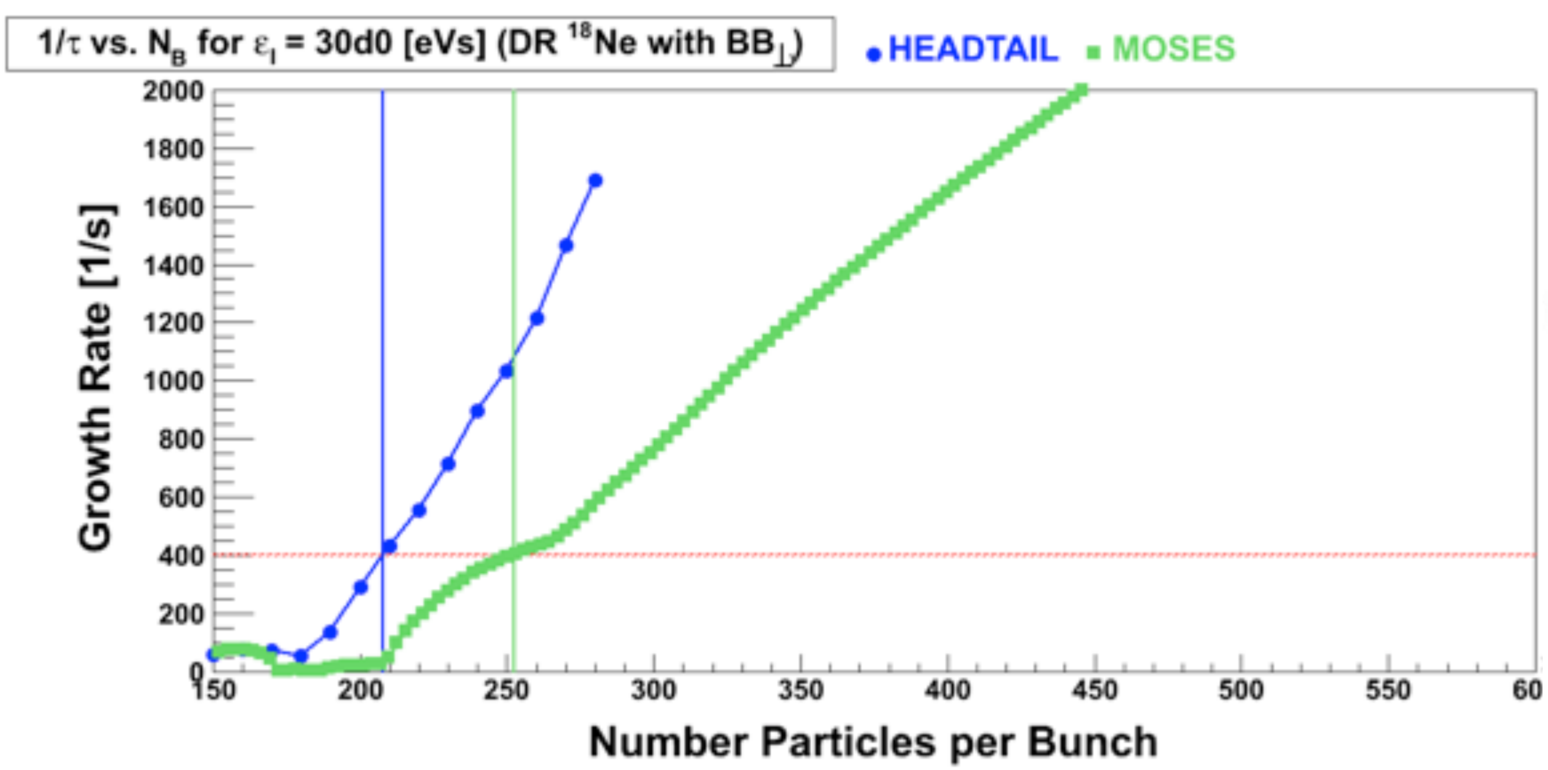
T H R E S H O L D

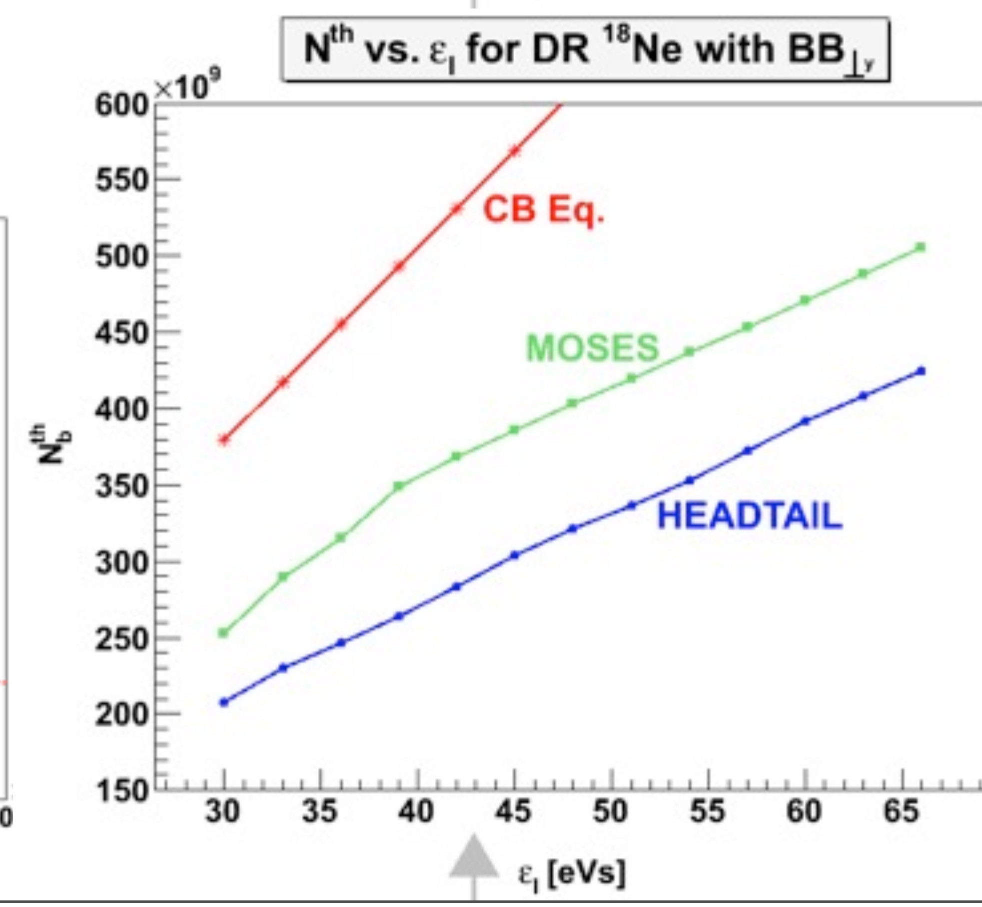
R$_\perp$ = 20 MΩ/m (SPS)





R$_\perp$ = 2 MΩ/m (LHC no col.)

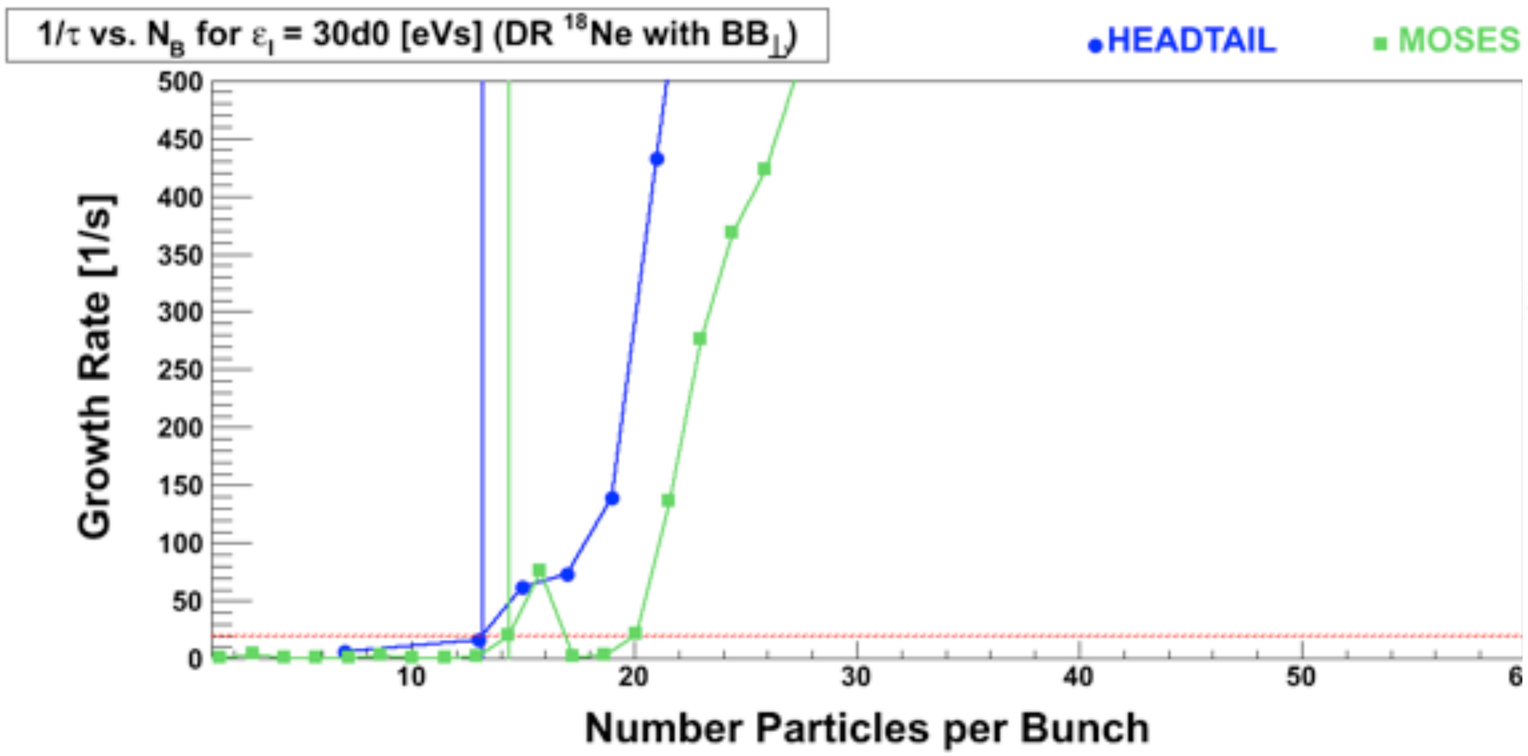


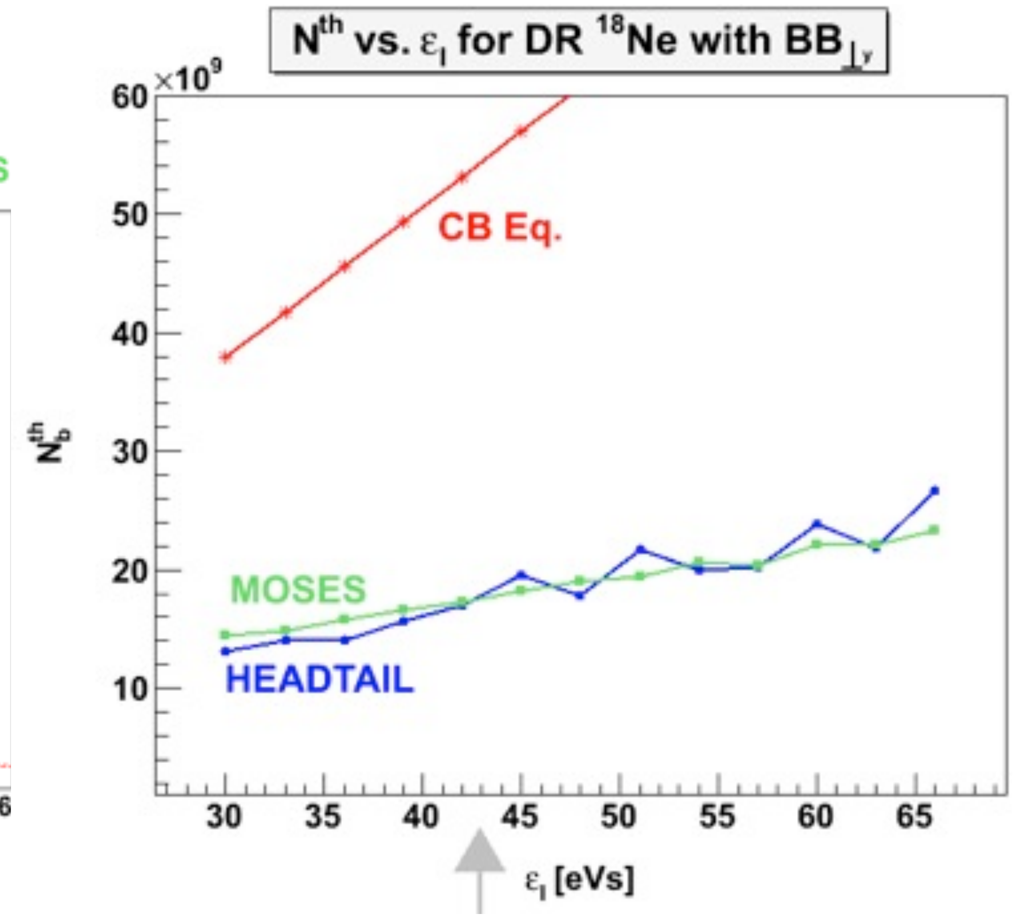


ε_l Scan for ^{18}Ne $(1/\tau)^{th}$ = 20Hz

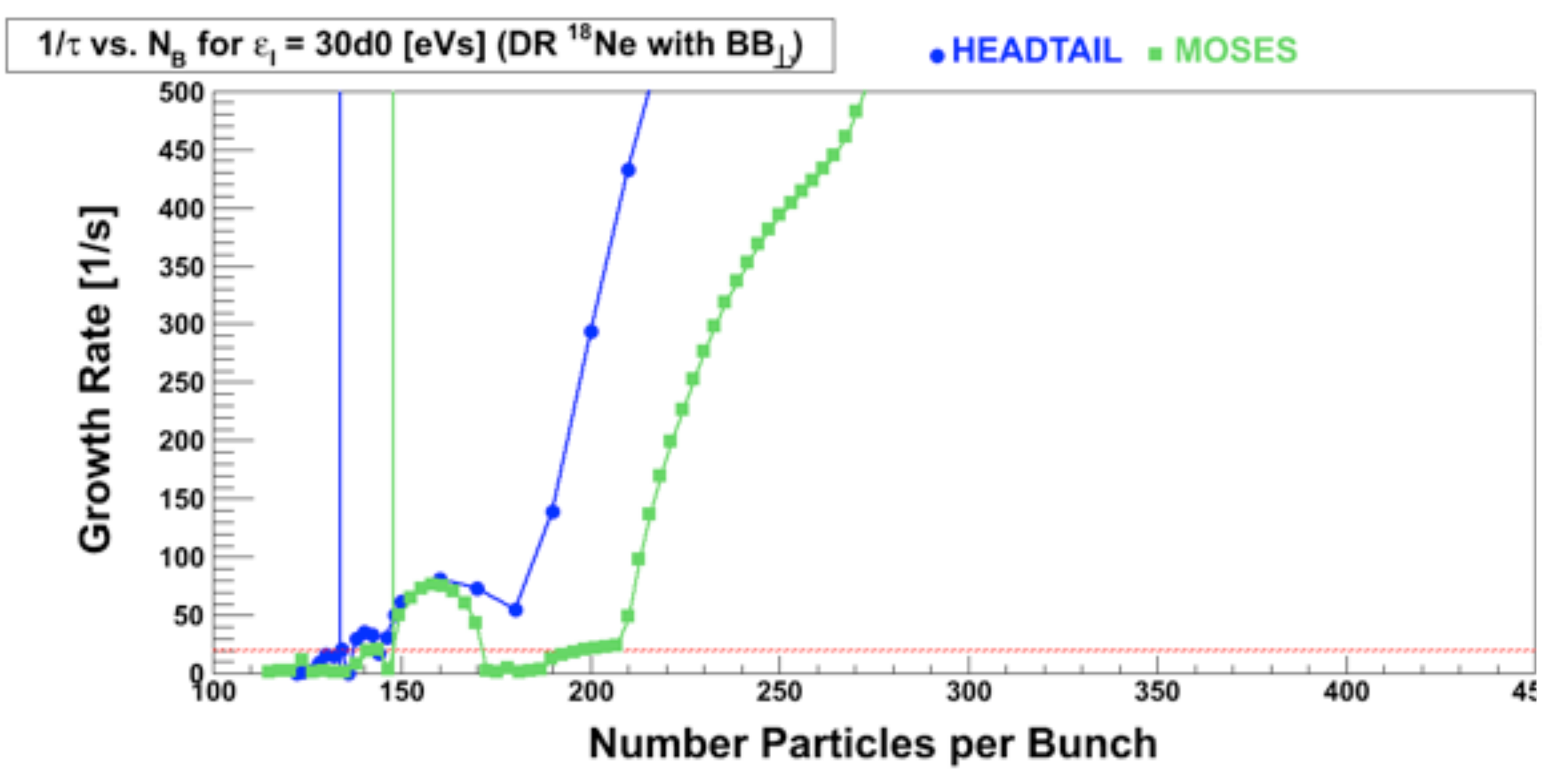
THRESHOLD

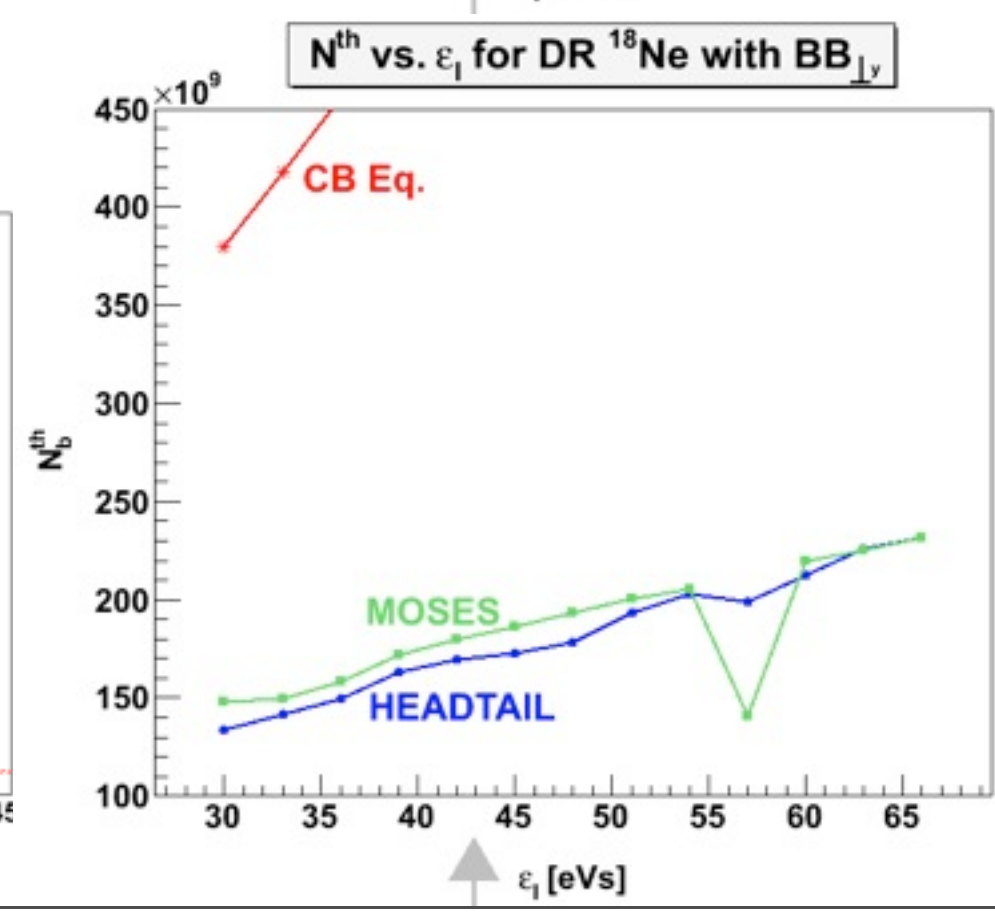
R$_\perp$ = 20 MΩ/m (SPS)





R$_\perp$ = 2 MΩ/m (LHC no col.)



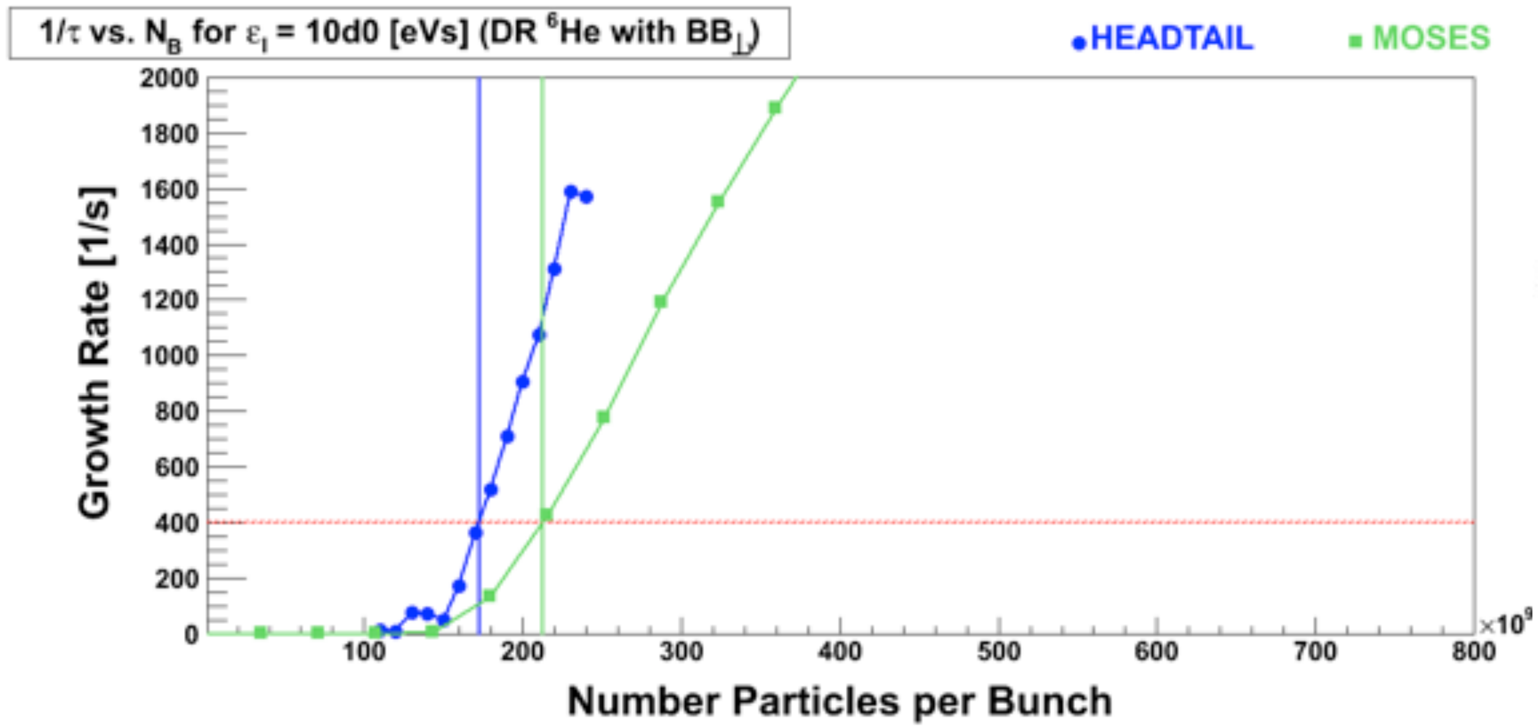


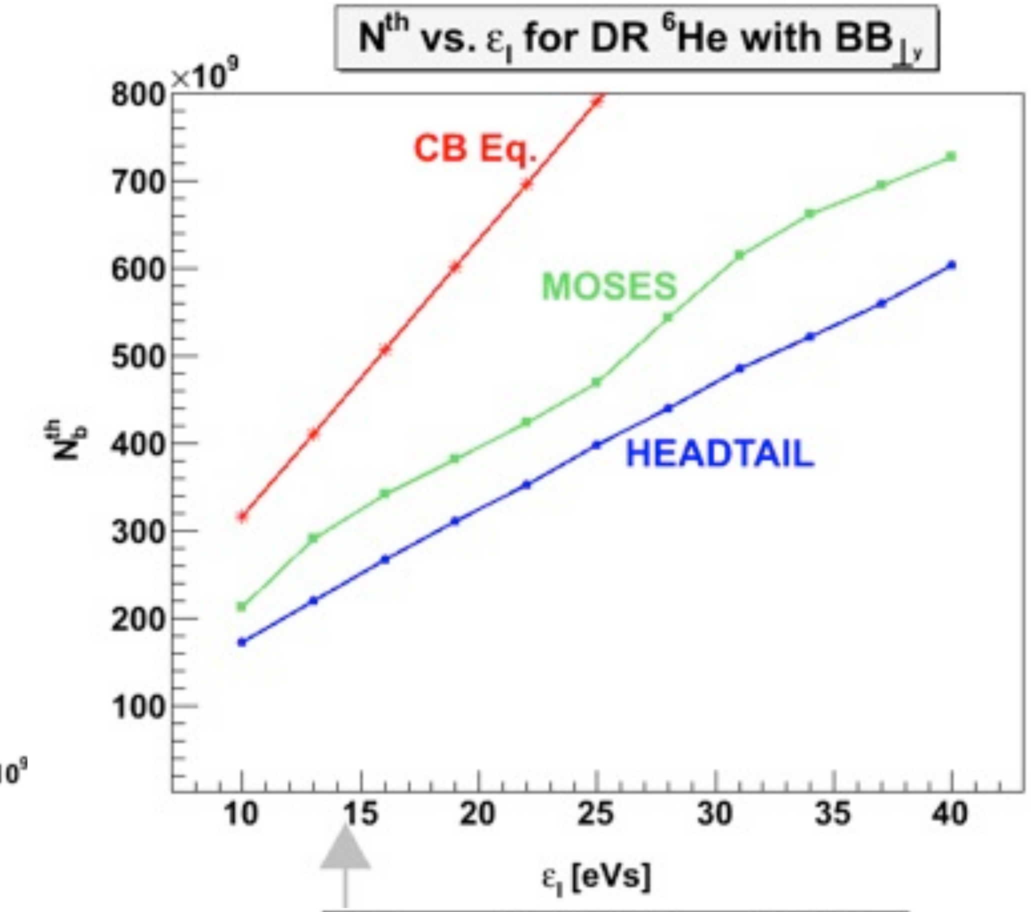
ε_l Scan for ^{6}He

$(1/\tau)^{th} = 400$Hz

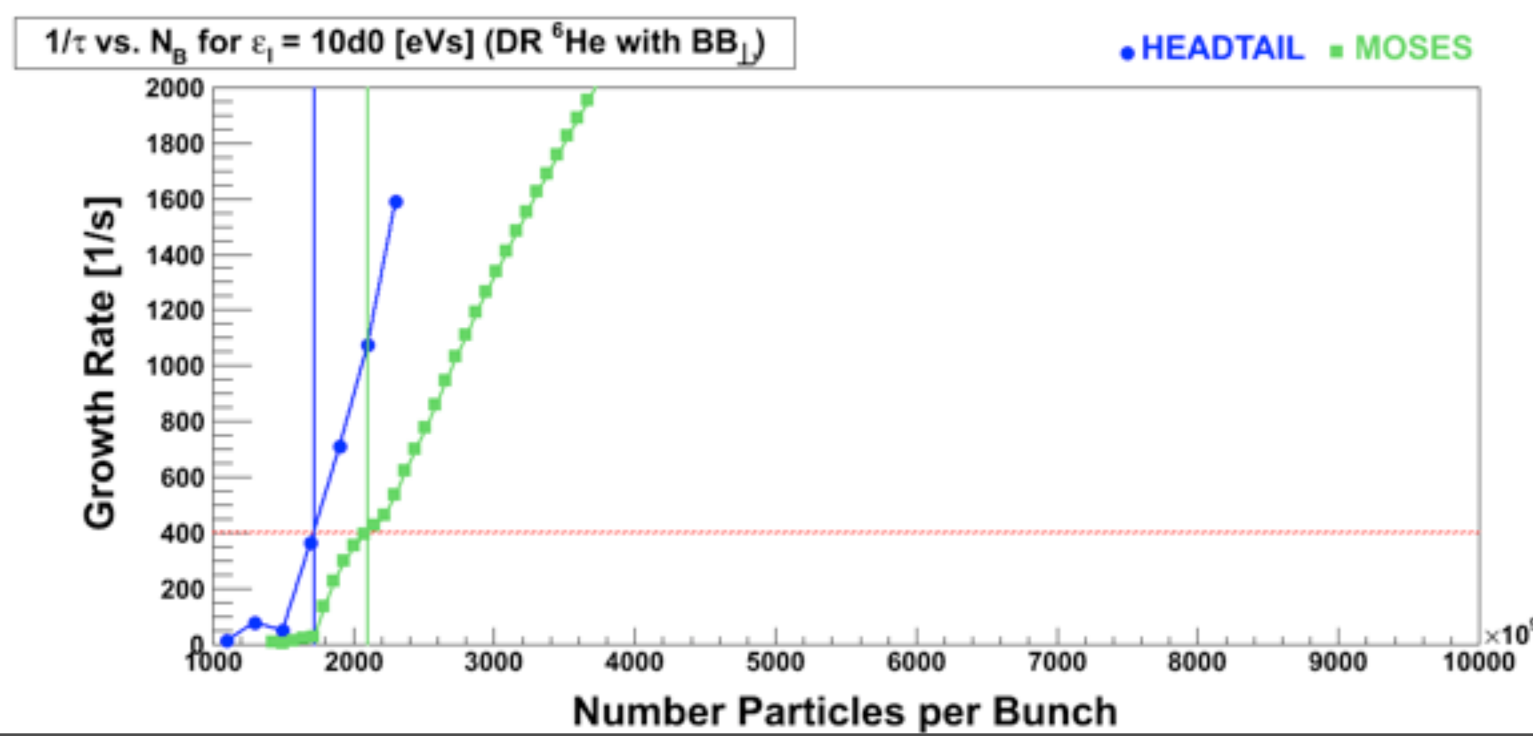
THRESHOLD

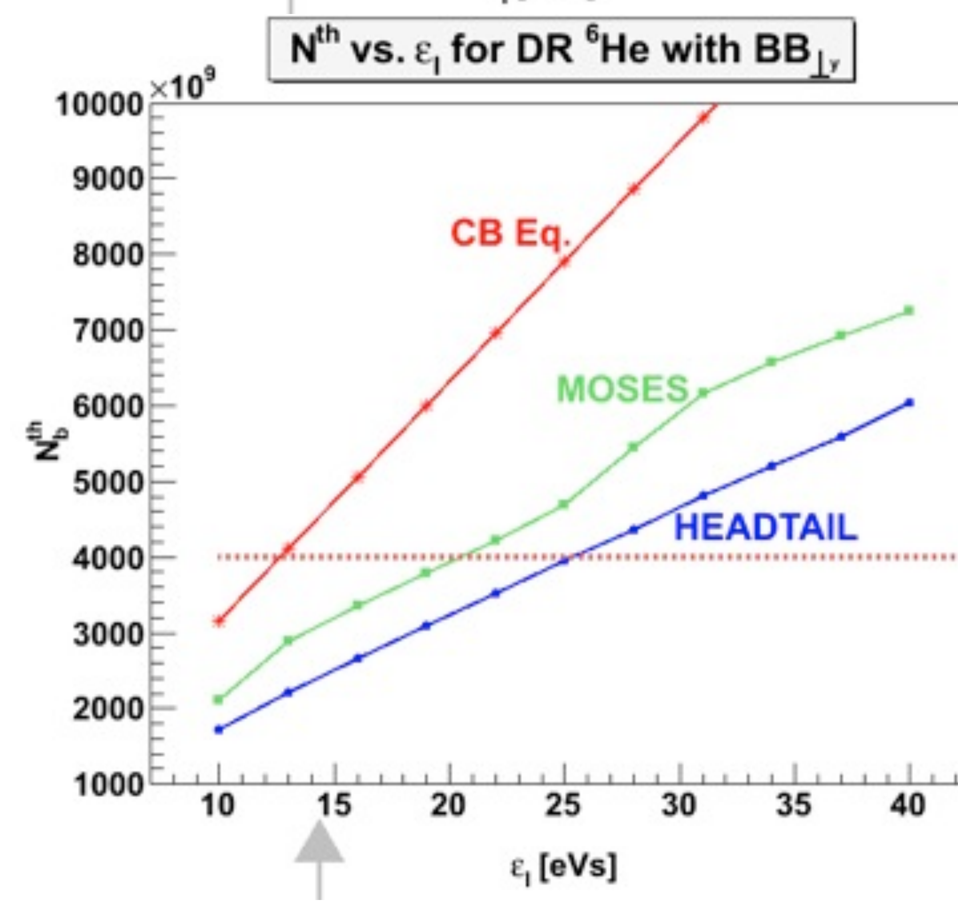
$\mathbf{R}_\perp$ = 20 MΩ/m (SPS)





$\mathbf{R}_\perp$ = 2 MΩ/m (LHC no col.)

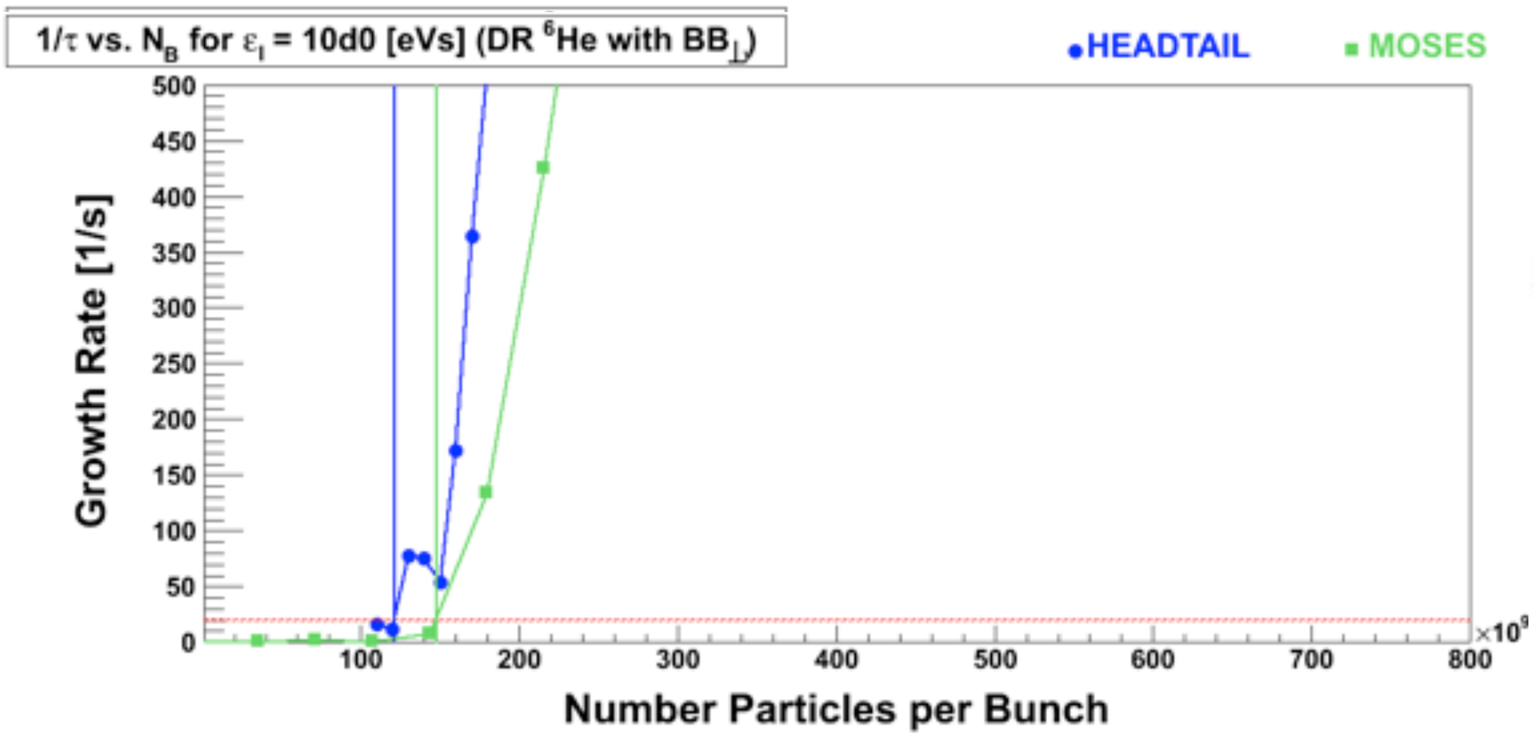


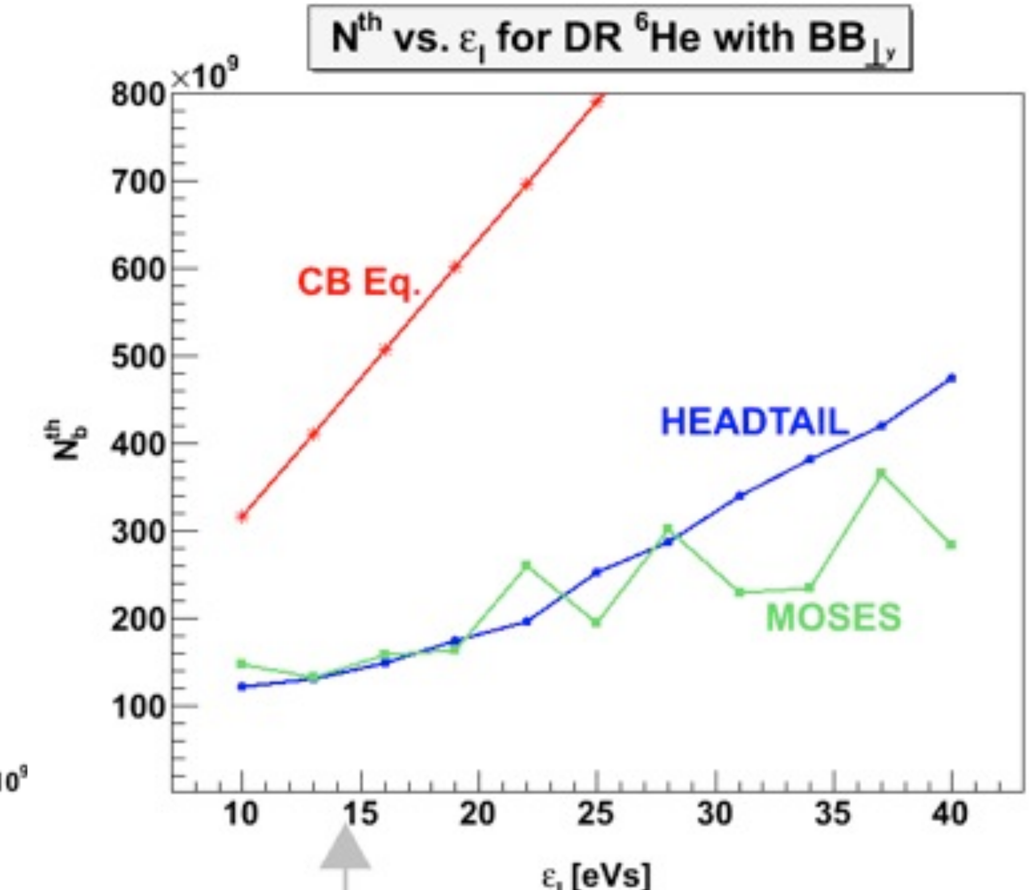


ε_l Scan for ^{6}He $(1/\tau)^{th}$ = 20Hz

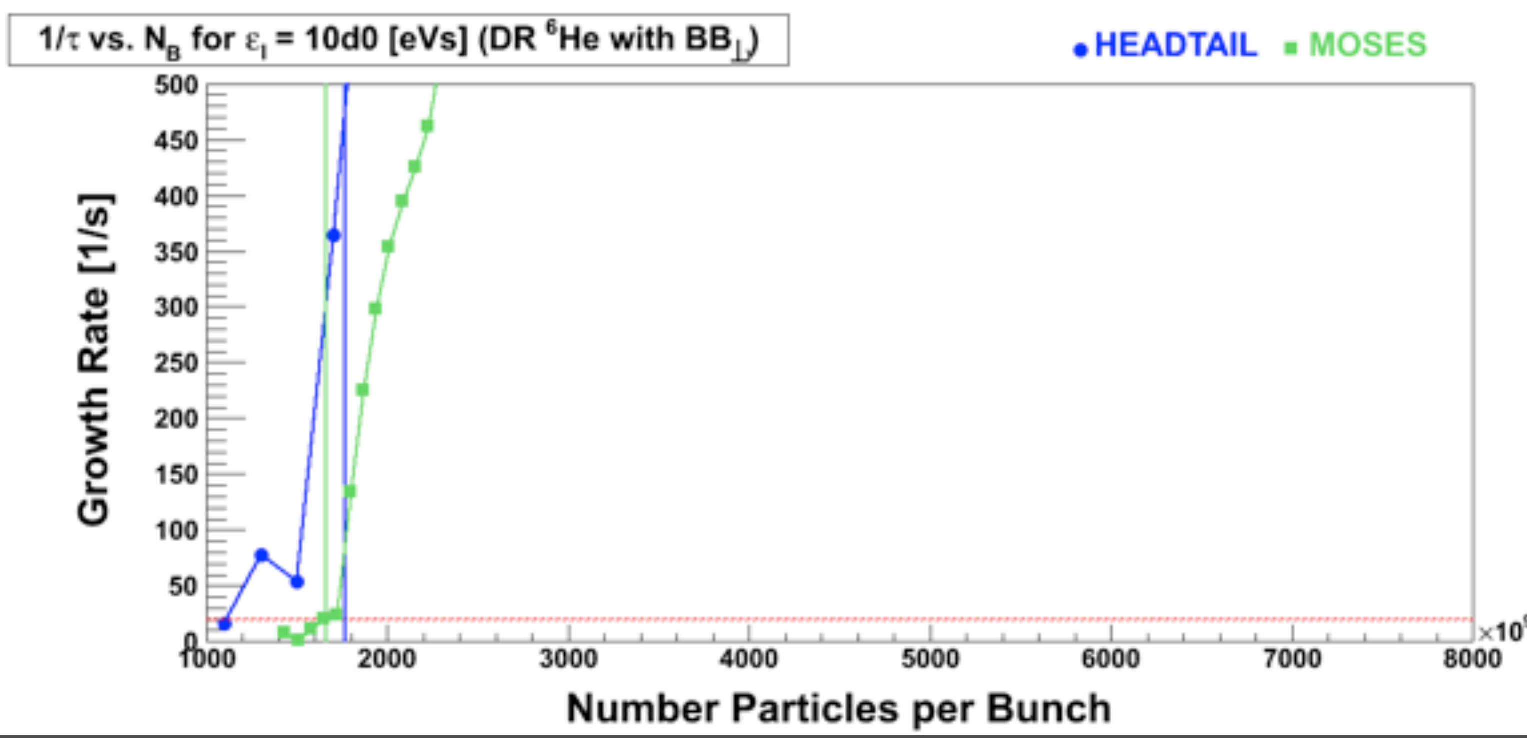
THRESHOLD

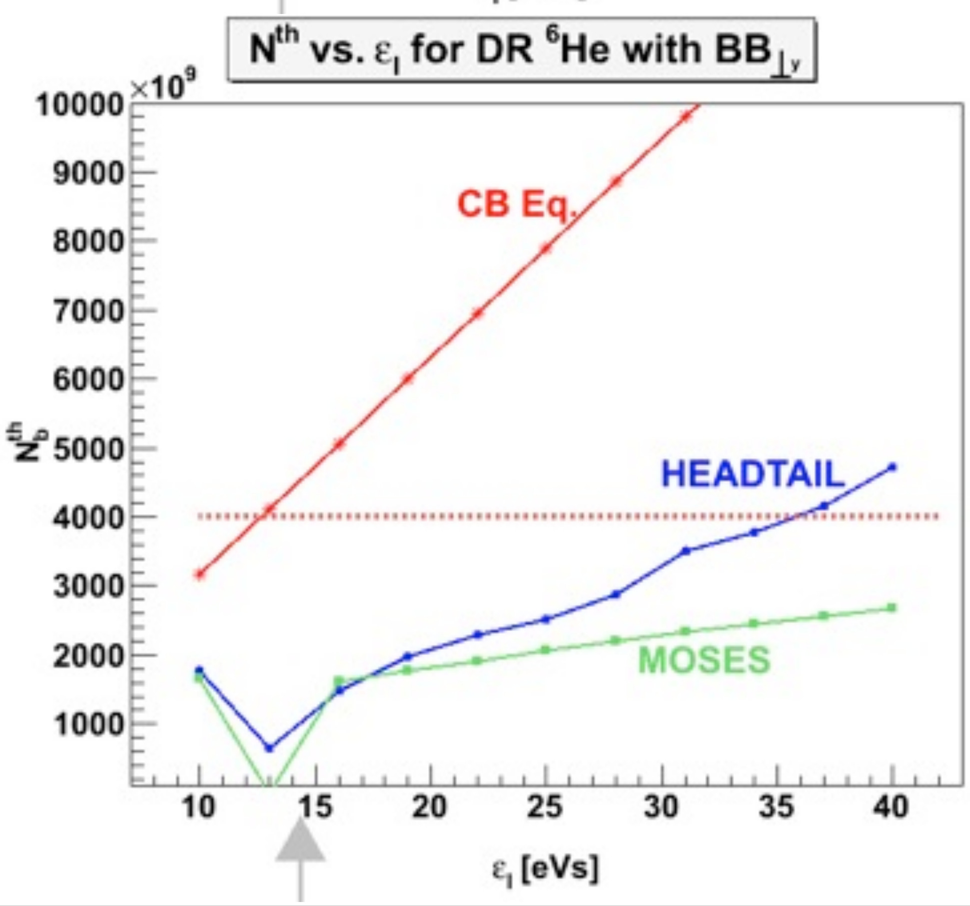
R$_\perp$ = 20 MΩ/m (SPS)





R$_\perp$ = 2 MΩ/m (LHC no col.)



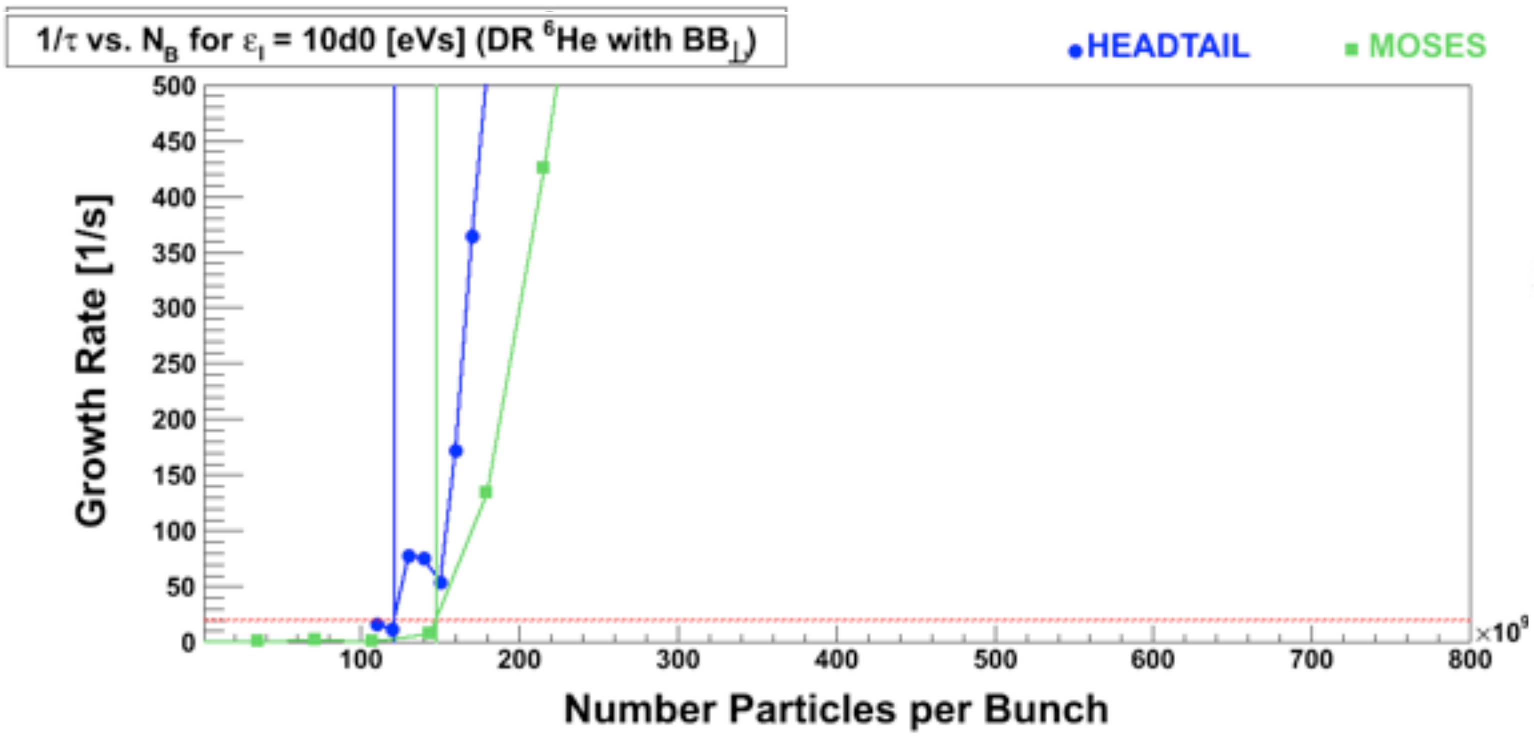


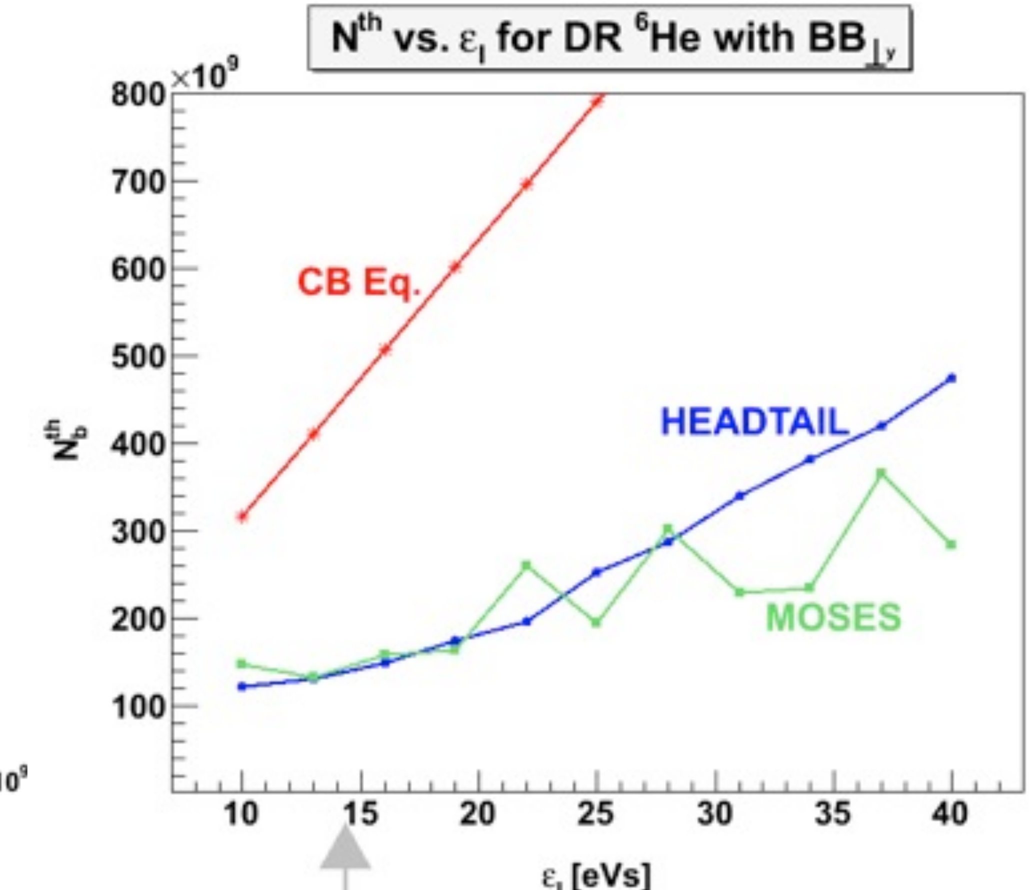
ε_l Scan for ^{6}He

$(1/\tau)^{th}$ = 20Hz

THRESHOLD

R$_\perp$ = 20 MΩ/m (SPS)





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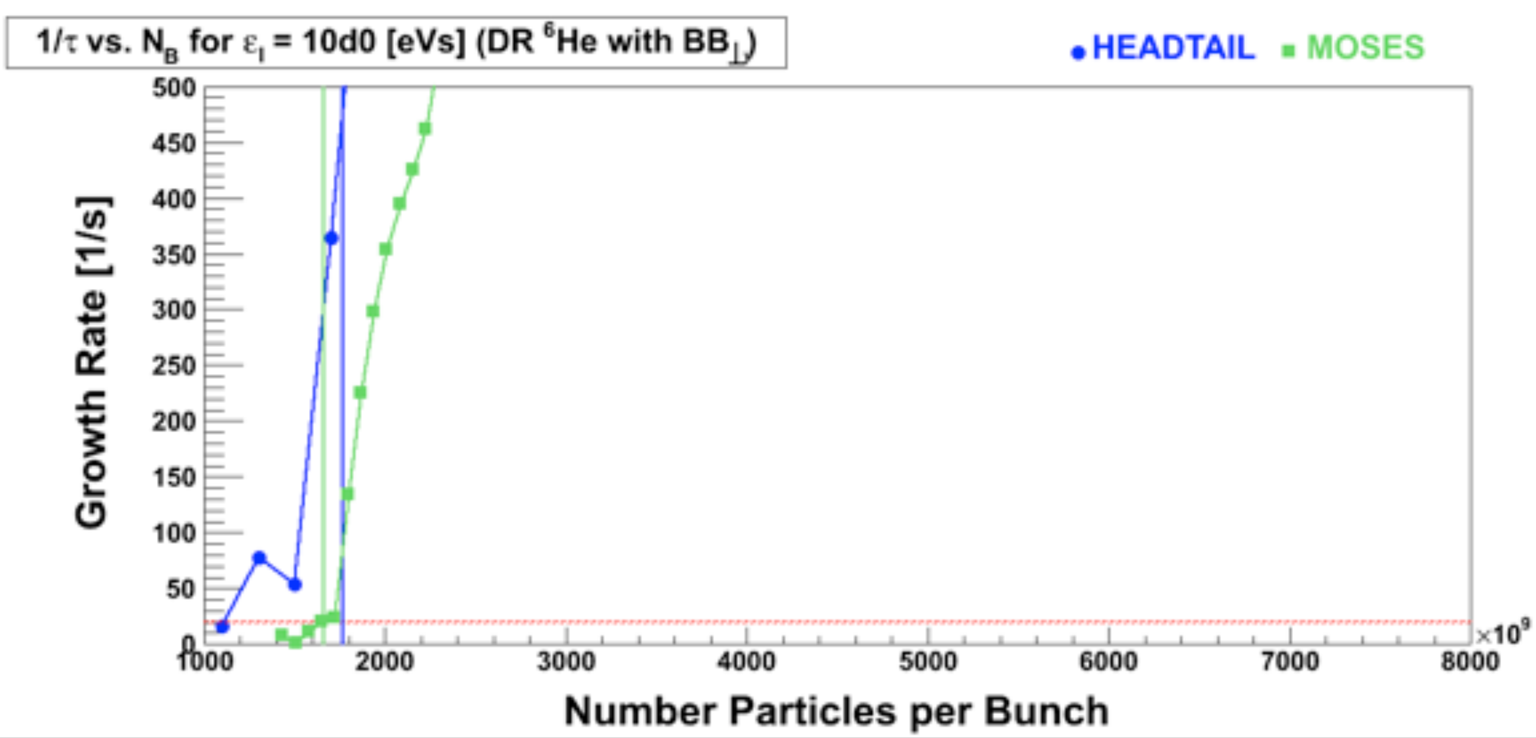
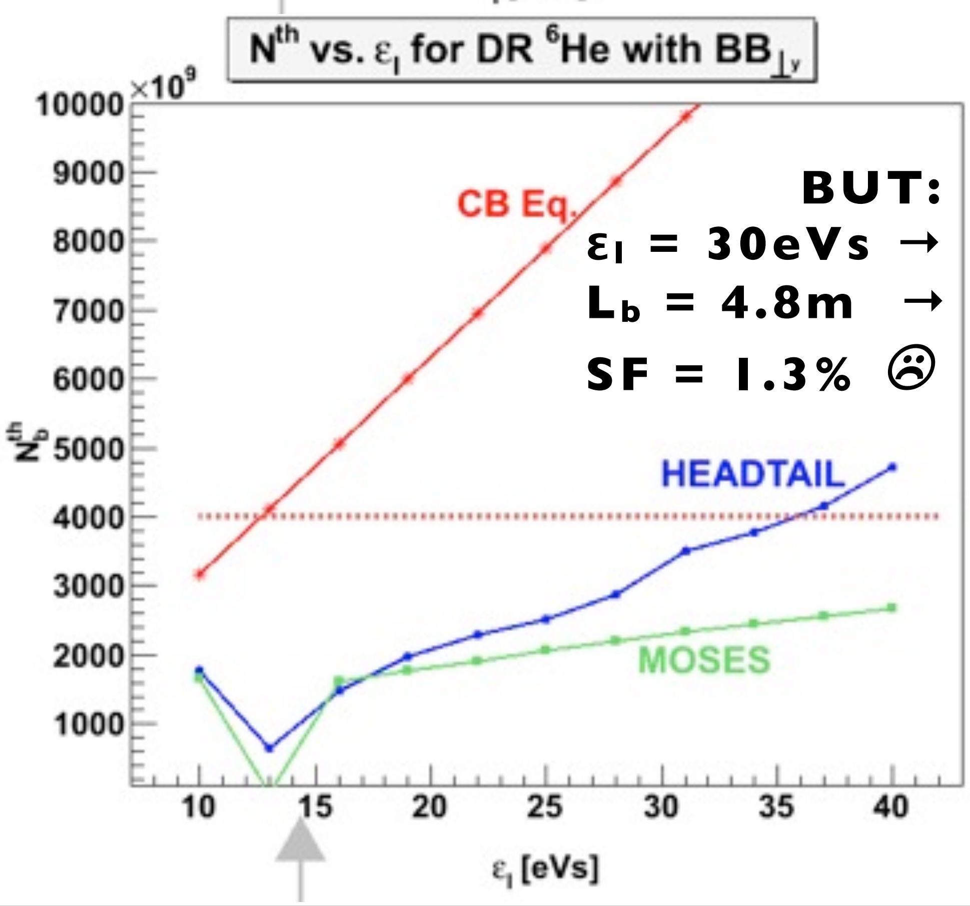
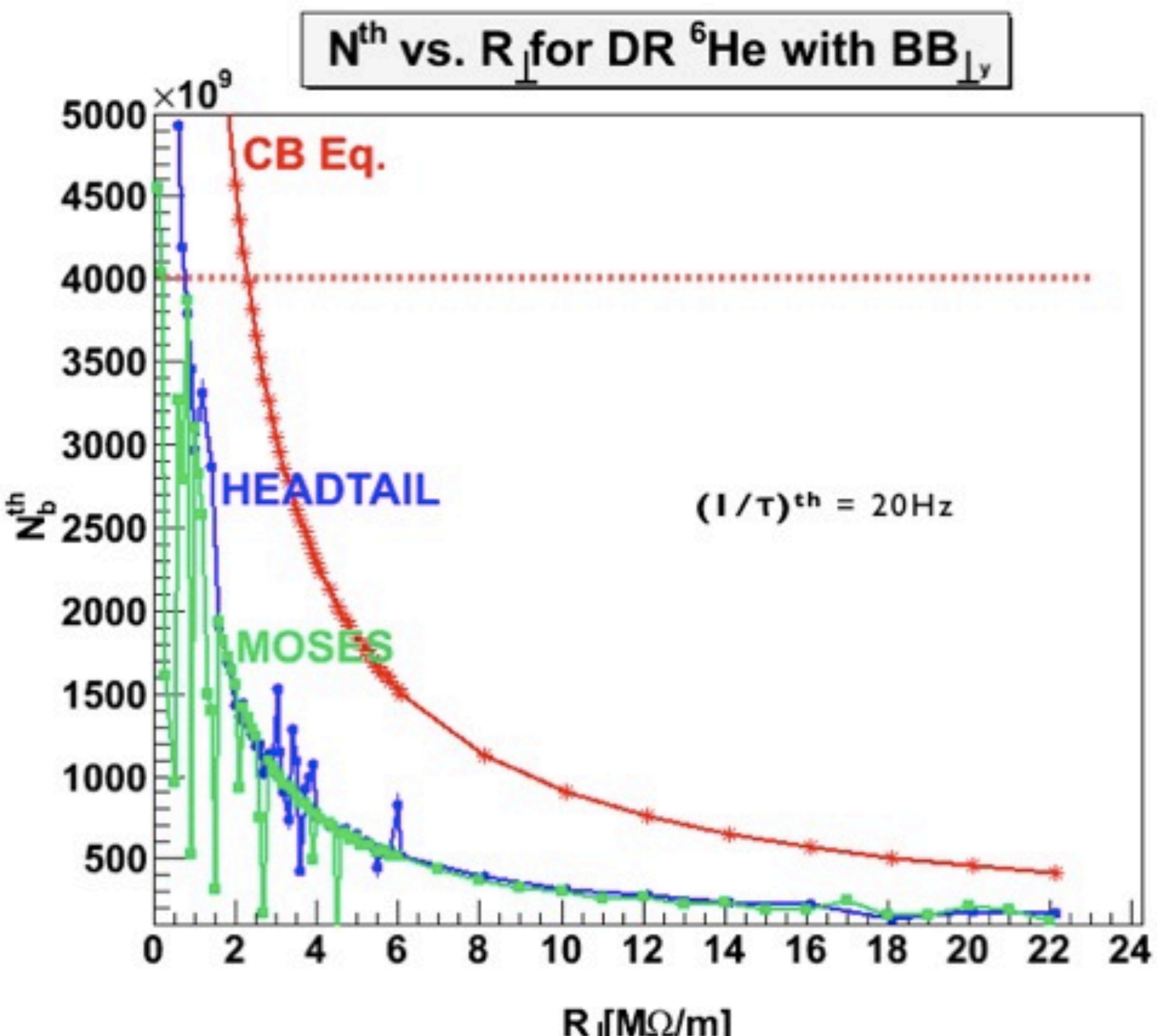


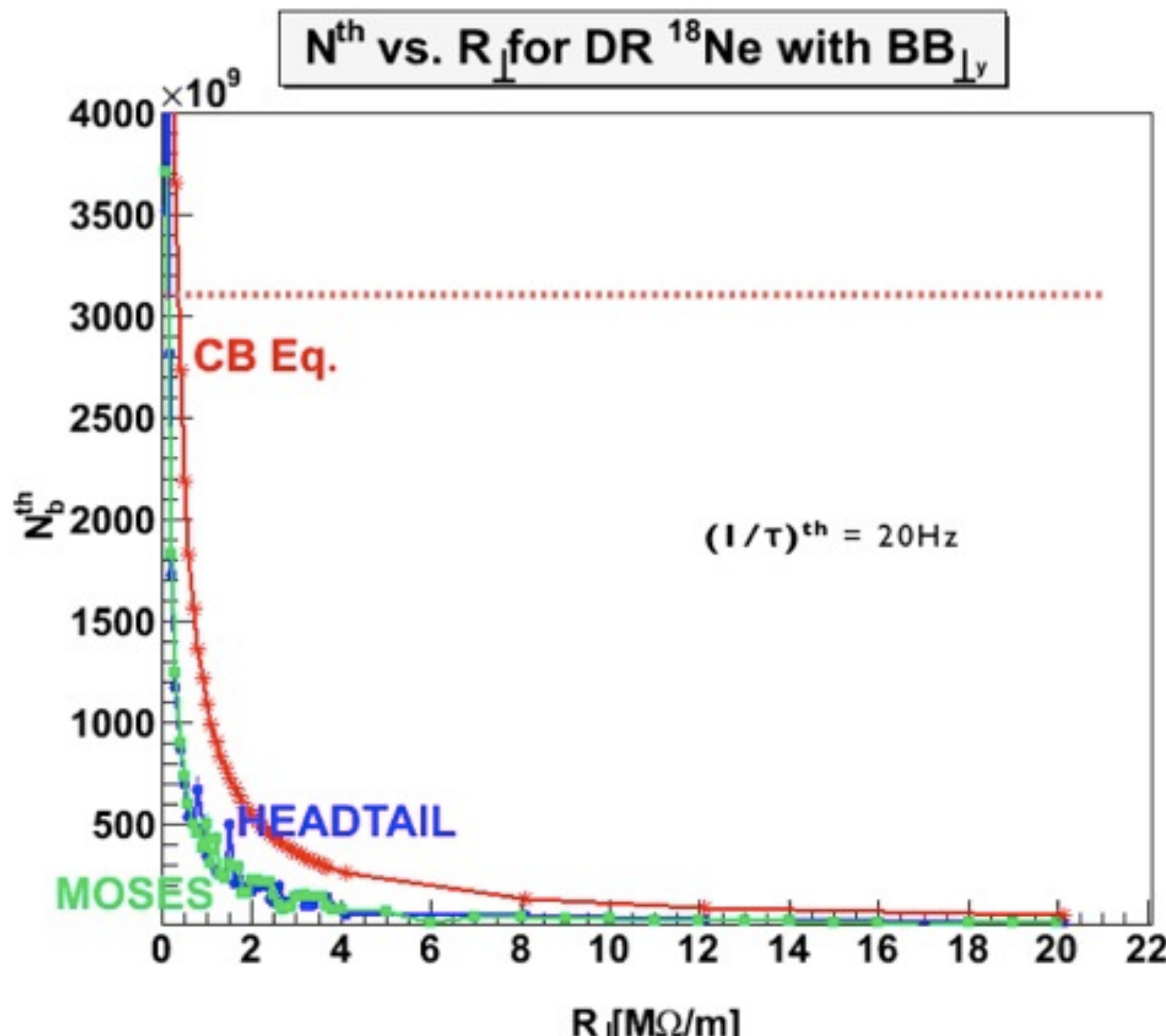


N_b^{th} vs. $R_\perp$ in DR

THRESHOLD

- **^{6}He: Even if $R_\perp$ is 2MΩ/m N_B^{th} = 4.0e12 could not be reached by changing ε_l, due to the SF < 1%**
- **^{18}Ne: N_B^{th} = 3.1e12 seems beyond the horizon …**
- **Let's find required $R_\perp$ to allow N_B^{th}**





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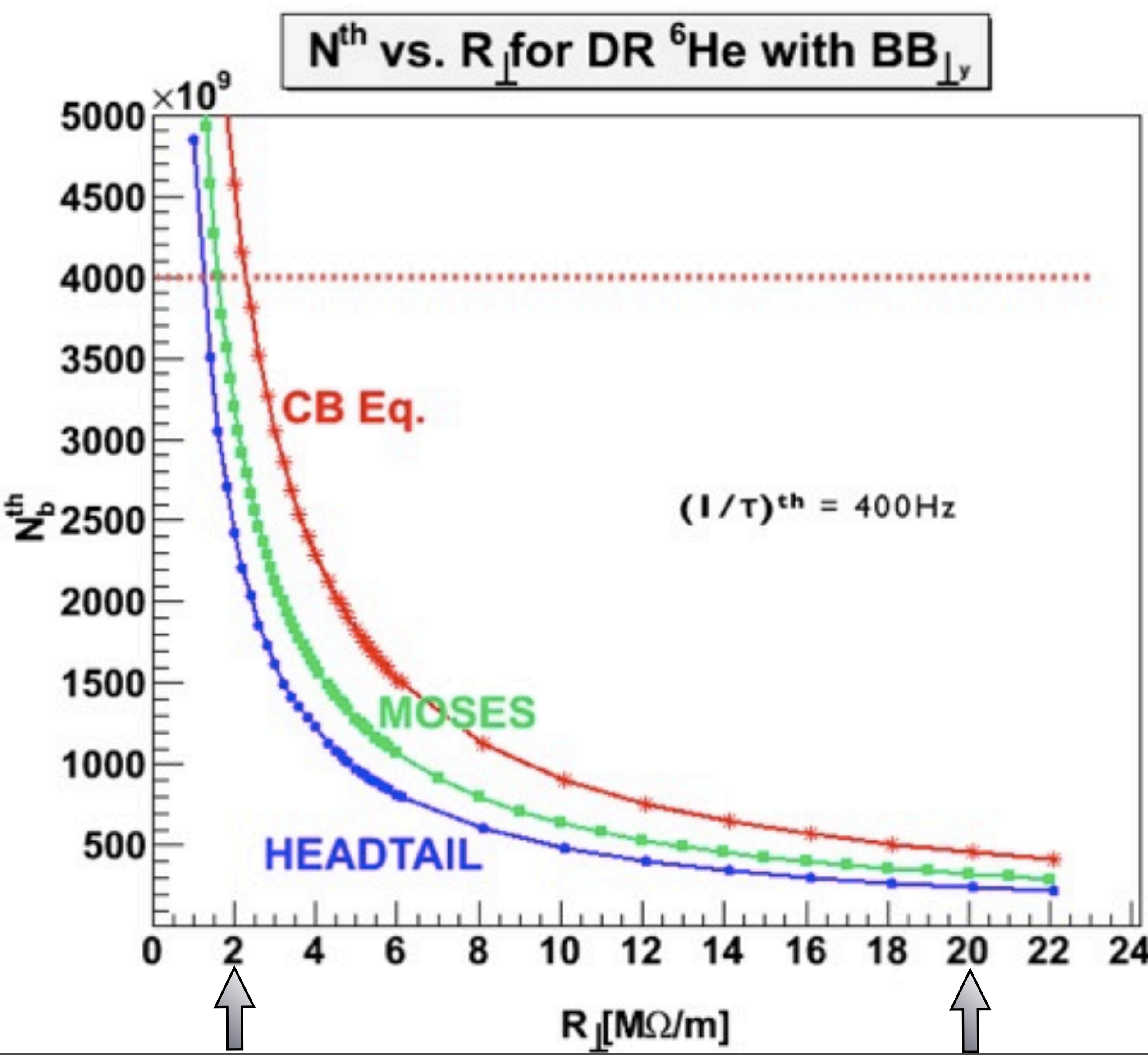

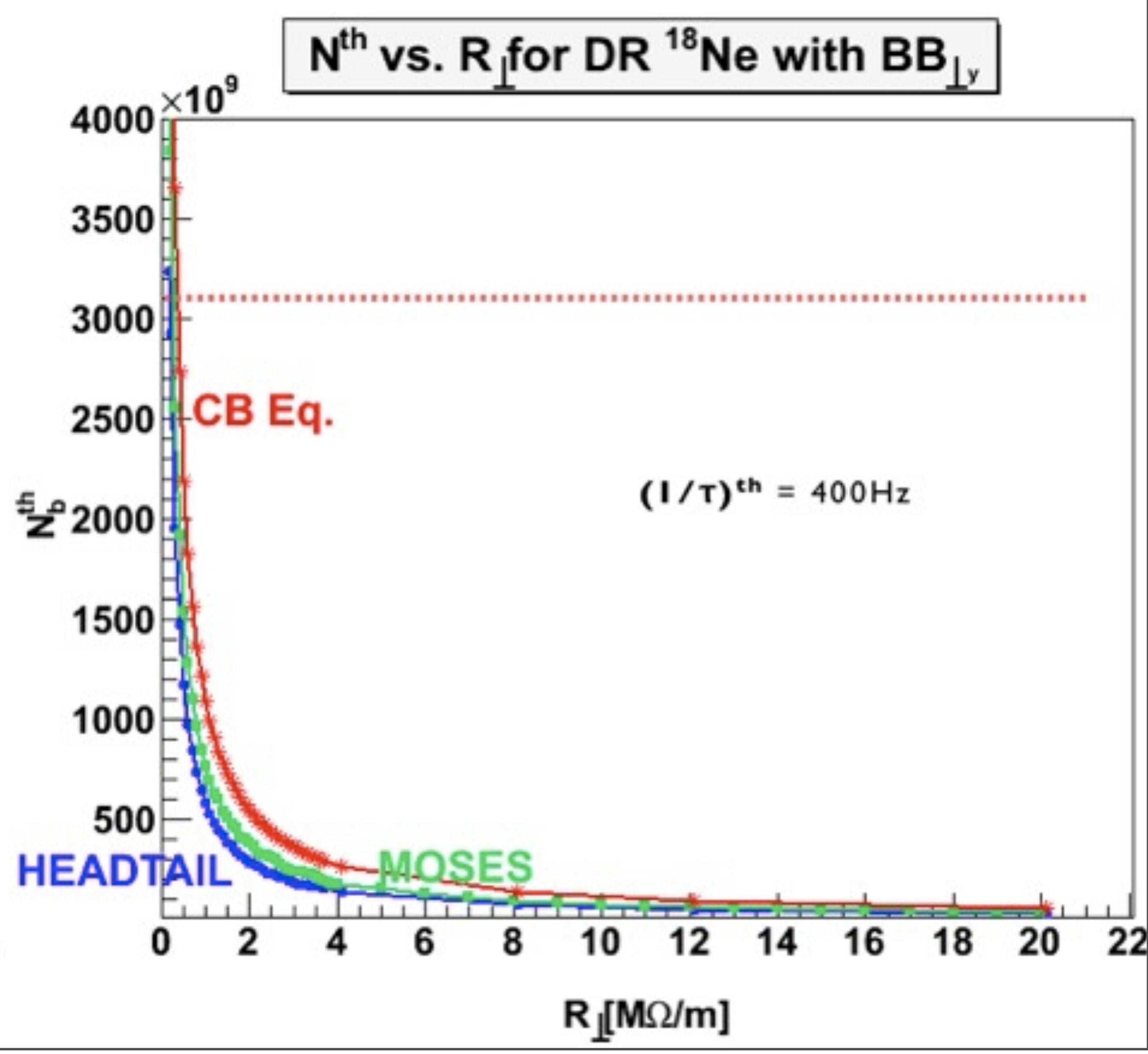

N_b^{th} vs. $R_\perp$ in DR

THRESHOLD

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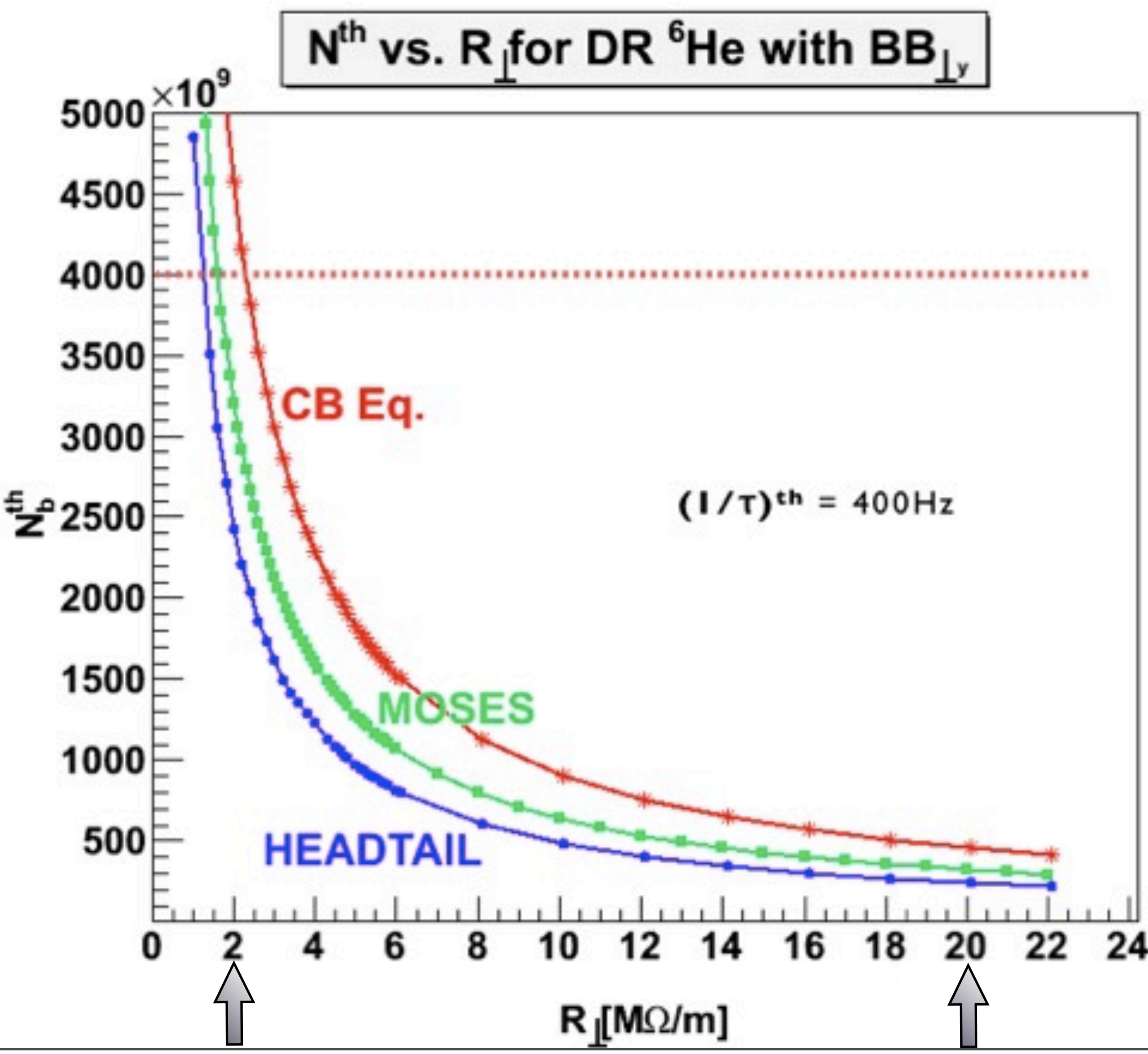

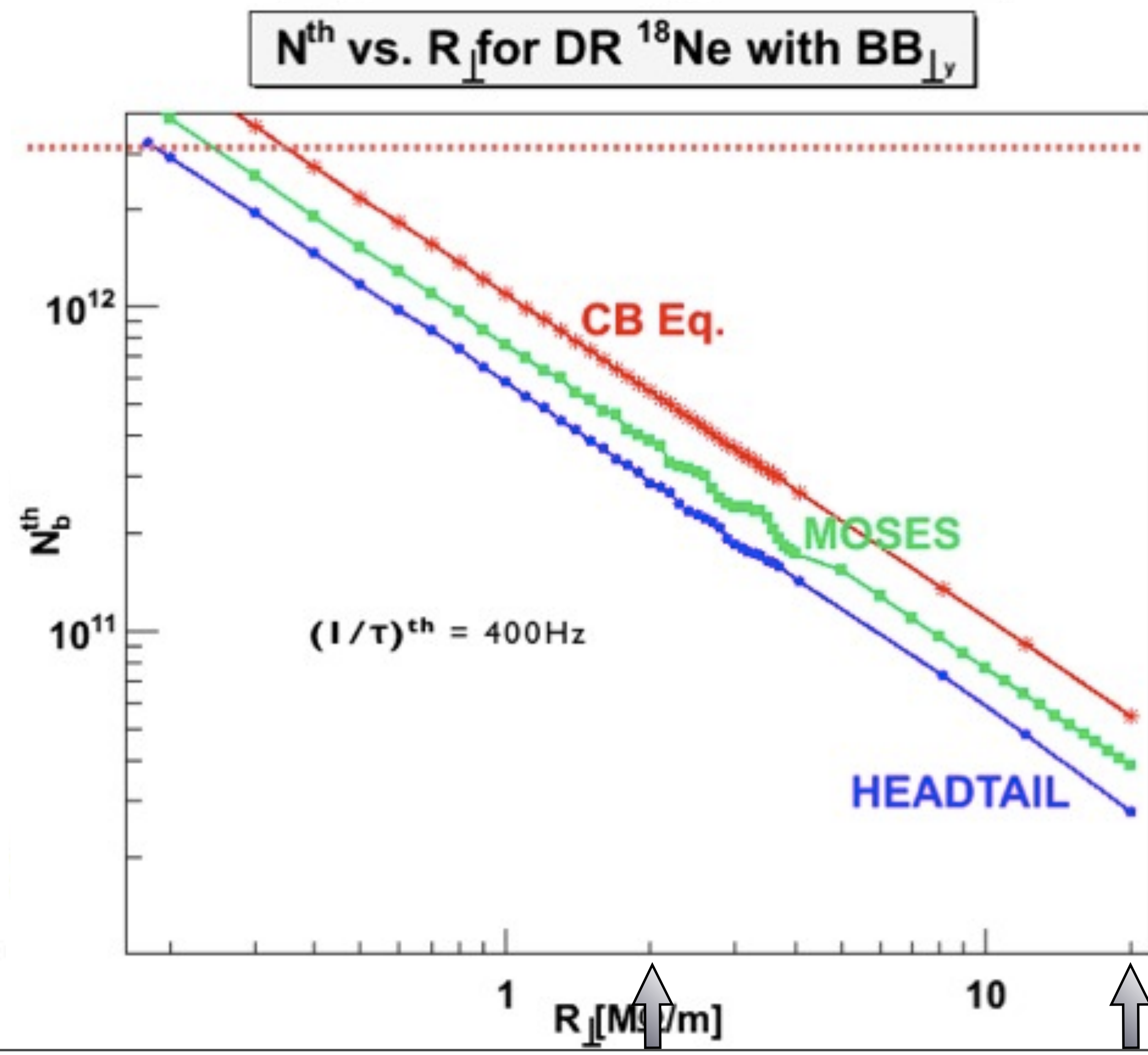

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Conclusion

CONCLUSION

- Direct Space Charge effect will not limit the performance of the Decay Ring (Laslett's Equations)

- We have a very challenging upper limit of the DR's Transversal Shunt Impedance, $R_{\perp}$:

➡ 10 times smaller than LHC (without collimators) for ^{18}Ne

… based on HEADTAIL and MOSES studies

- This study, that was completely based on parameters from "FP6", suggests a re-optimization of the Beta Beam design

Note under preparation:
http://chansen.web.cern.ch/chansen/PUBLICATIONS/bbCollective.pdf
SVN: http://svnweb.cern.ch/world/wsvn/bbcollective

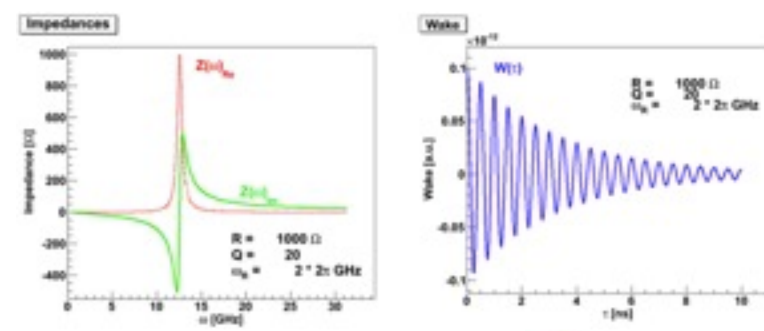
To Do

CONCLUSION

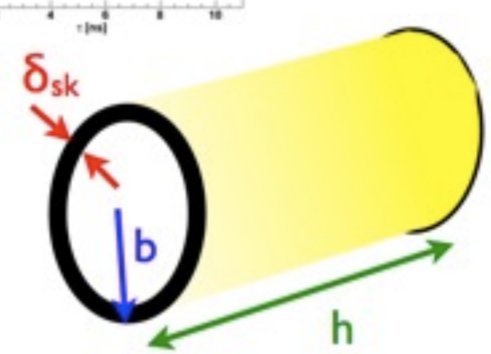
- Study Beta Beam "Cocktails" (suggested by A. Donini, WP6)
  - ➡ Specially $\phi_{Ne}/5$ & $\phi_{He}*2$

SETUP	γ	Ions	Fluxes $[10^{18}]$	Years	$(\sin^2 2\theta_{13})_{min}$	NH,$(\sin^2 2\theta_{13})_{min}$
CERN-Frèjus, 3	100	^6He	$\bar{\Phi}_0 \times 2$	2	1×10^{-3}	No Sensitivity
Ref. [1]		^{18}Ne	$\Phi_0/5$	8		

- Same study in longitudinal plane $Z_{||}(\omega) = \dfrac{R_{||}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$
  - ➡ Ongoing HEADTAIL simulations, but can't use MOSES since only for $\perp$



- Same with Narrow Band

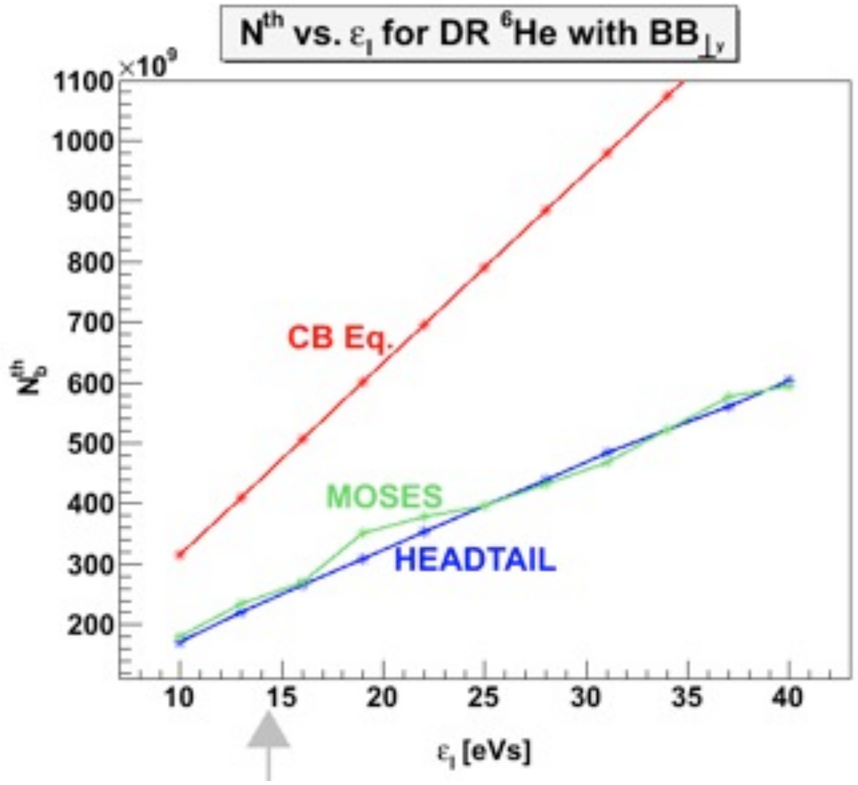


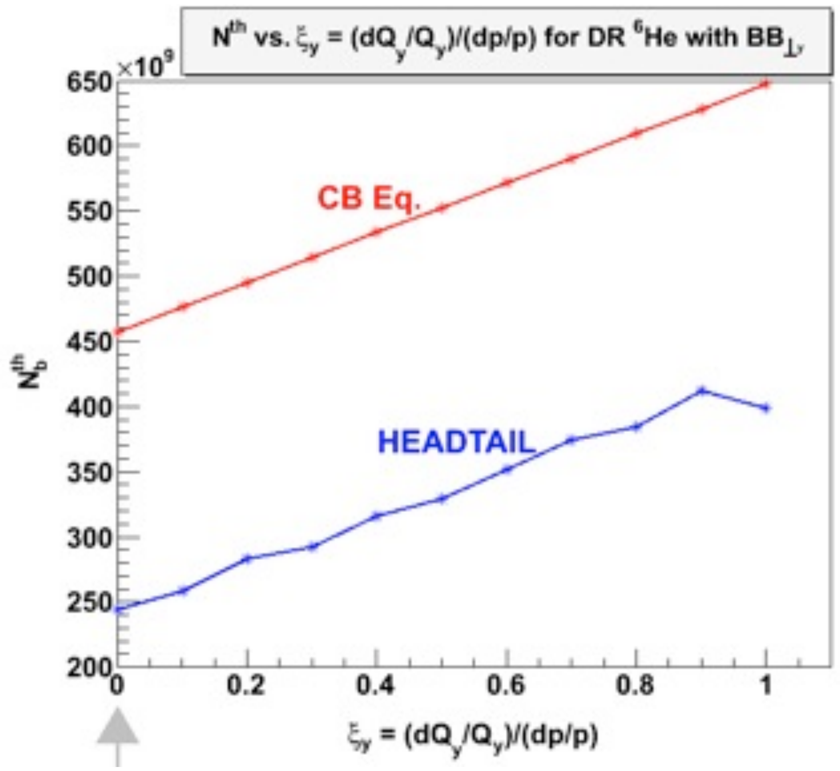
- Same with Resistive Wall Impedance
- Same with SPS, PS and ^8Li & ^8B

Backup Slides

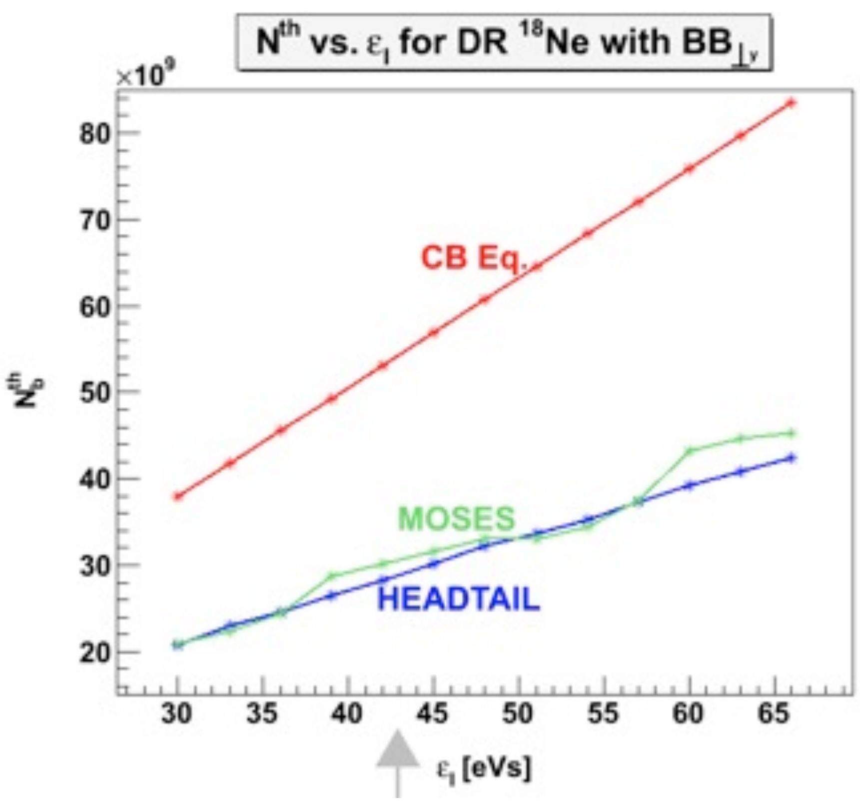
ε_l and ξ_y Scans

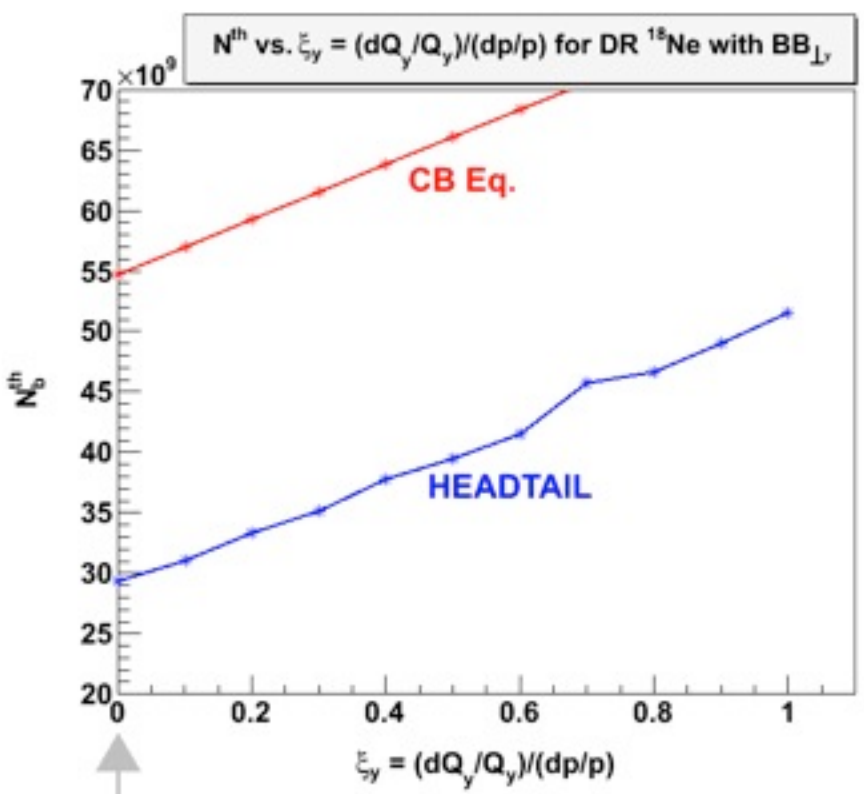
- Scans over Longitudinal Emittance and Chromaticity for
  - ^{6}He





  - ^{18}Ne





Input Parameters for Protons

BACKUP

TABLE 8. Input parameters from previous Beta Beam design report.

Parameters	Description	DR p
Z	*Charge Number*	1
A	*Mass Number*	1
h	*Harmonic Number*	924
C [m]	*Circumference*	6911.6
ρ [m]	*Magnetic Radius*	155.6
γ_{tr}	*Gamma at Transition*	27.00
V_{RF} [MV]	*Voltage*	2.000e+01
dB/dt [T/s]	*Magnetic Ramp*	0.00
γ	*Relativistic Gamma*	100.0
δ_{max}	*Maximum Momentum Spread*	3.50e-03
E_{rest} [MeV]	*Rest Energy*	938.27
M	*Number Bunches per Batch*	1
L_b [m]	*Full Bunch Length*	1.600
N_b	*Number Ions per Injected Bunch*	4.87e+11
N_B	*Average Number Ions per Bunch*	4.00e+12
m_r	*Merges Ratio*	15
$t_{1/2}$ [s]	*Half Life at Rest*	9999999999.90
T_c [s]	*Revolution Time*	6.00
Q_x	*Horizontal Tune*	22.23
Q_y	*Vertical Tune*	12.16
$\langle\beta\rangle_x$ [m]	*Average Horizontal Betatron Function*	148.25
$\langle\beta\rangle_y$ [m]	*Average Vertical Betatron Function*	173.64
$\langle D\rangle_x$ [m]	*Average Dispersion*	-0.60
ξ_x	*Horizontal Chromaticity*	0.00
ξ_y	*Vertical Chromaticity*	0.00
$\varepsilon_{N_x}(1\sigma)$ [πm·rad]	*Normalized Horizontal Emittance*	1.48e-05
$\varepsilon_{N_y}(1\sigma)$ [πm·rad]	*Normalized Vertical Emittance*	7.90e-06
ε_l (full) [eVs]	*Full Longitudinal Emittance*	14.36
b_x [cm]	*Horizontal Beam Pipe Size*	16.0
b_y [cm]	*Vertical Beam Pipe Size*	16.0
ρ_{res} [Ω m]	*Resistivity*	1.0e-07

TABLE 9. Assumed input parameters.

Parameters	Description	DR p
$Q_{\|\|}$	*Longitudinal Quality Factor*	1.00
$\omega_{r,\|\|}$ [GHz]	*Longitudinal Angular Resonance Frequency*	6.28
$\left\|Z_{\|\|}/n\right\|$ [Ω]		10.00
$R_{s,\|\|}$ [MΩ] = $\frac{\|Z/n\| Q\omega_r}{\omega_{rev}}$	*Longitudinal Shunt Impedance*	0.231
$Q_{\perp}$	*Transverse Quality Factor*	1.00
$\omega_{r,\perp}$ [GHz]	*Transverse Angular Resonance Frequency*	6.28
$R_{s,\perp}[M\Omega/m]$	*Transverse Shunt Impedance*	20.00

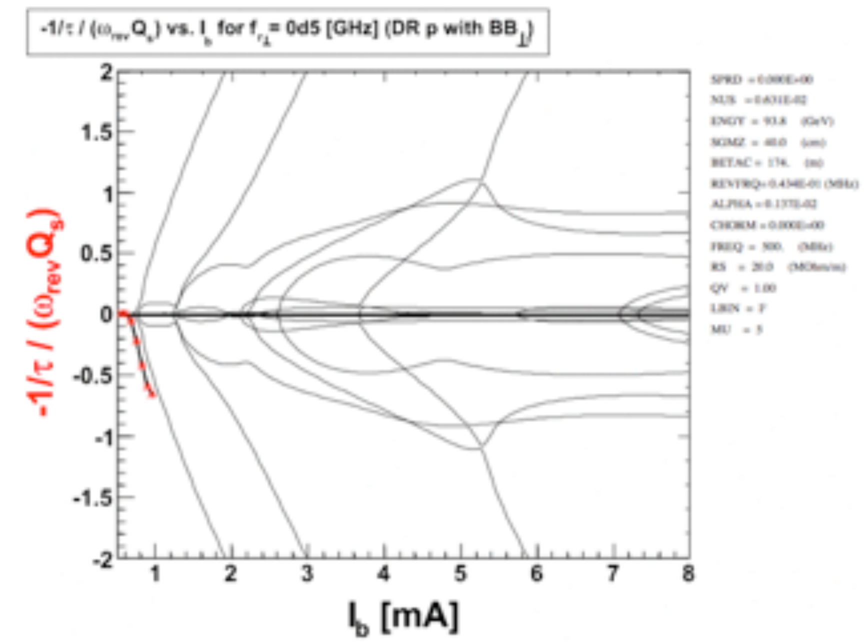
TABLE 10. Calculated values.

		DR p
r_0 [m] $= r_p Z^2/A$	*Ion Radius*	1.53e-18
E_{tot} [GeV] $= \gamma \cdot E_{rest}$	*Total Energy*	93.83
$\beta = \sqrt{1-1/\gamma^2}$	*Relativistic Beta*	1.00
$\eta = (1/\gamma_{tr})^2 - (1/\gamma)^2$	*Phase Slip Factor*	1.27e-03
T_{rev} [μ s] $= C/(\beta c)$	*Revolution Time*	23.0558
R [m] $= C/2\pi$	*Machine Radius*	1100.02
ω_{rev} [MHz] $= 2\pi/T_{rev}$	*Angular Revolution Frequency*	0.27
$\sigma_\delta = \delta_{max}/2$	*1 Sigma Momentum Spread*	1.75e-03
τ_b [ns] $= L_b/(\beta c)$	*Full Bunch Length*	5.34
$\hat{I}$ [A] $= ZeN_B/\tau_b$	*Peak Current*	120.07
I_b [A] $= ZeN_B/T_{rev}$	*Beam Current*	0.03
$\varepsilon_l^{2\sigma}$ [eVs] $= \frac{\pi}{2}\beta^2 E_{tot}\tau_b\delta_{max}$	*2 Sigma Longitudinal Emittance*	2.75
$Q_s = \sqrt{\frac{hZeV\|\eta\cos\phi_s\|}{2\pi\beta^2 E_{tot}}}$	*Synchrotron Tune*	0.0063
ω_s [kHz] $= Q_s \cdot \omega_{rev}$	*Synchrotron Angular Frequency*	1.72
ω_x [MHz] $= Q_x \cdot \omega_{rev}$	*Horizontal Betatron Angular Frequency*	6.06
ω_y [MHz] $= Q_y \cdot \omega_{rev}$	*Vertical Betatron Angular Frequency*	6.06
ω_c [GHz] $= \beta c/b_{min(x,y)}$	*Cut-Off Angular Frequency*	1.87
$\Delta Q_{\xi_x} = \xi_x\delta_{max}Q_x$	*Horizontal Tune Shift due to Chromaticity*	0.00e+00
$\Delta Q_{\xi_y} = \xi_y\delta_{max}Q_y$	*Vertical Tune Shift due to Chromaticity*	0.00e+00
ω_{ξ_x} [MHz] $= \xi_x Q_x \omega_{rev}/\eta$	*Horizontal Chromatic Angular Frequency*	0.00e+00
ω_{ξ_y} [MHz] $= \xi_y Q_y \omega_{rev}/\eta$	*Vertical Chromatic Angular Frequency*	0.00e+00

MOSES modes

- Maximum number modes are used for MOSES:
  - Head Tail Modes: [-10, 3]
  - Number Radial Modes: 13

  but still might not always find the crucial modes (those that couple)



- Could explain the discrepancy
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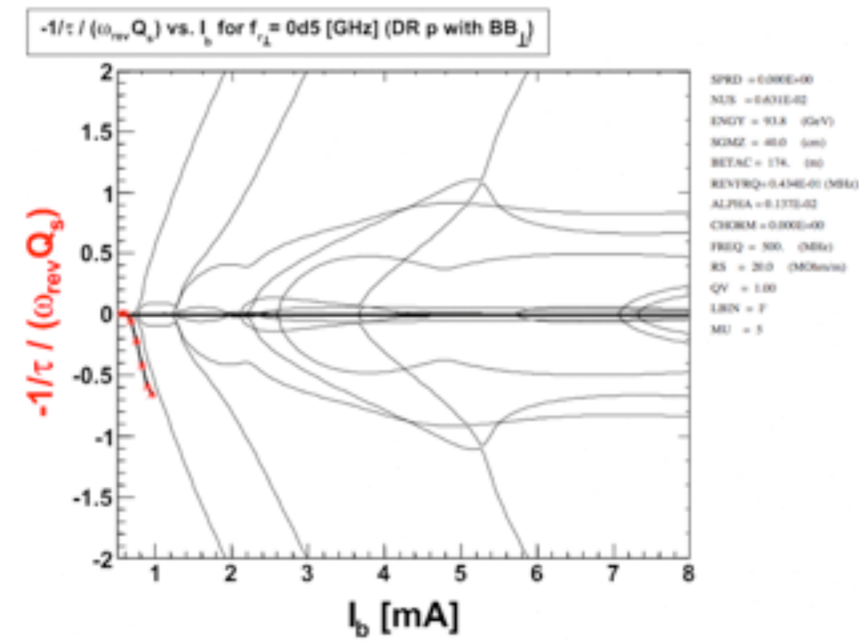

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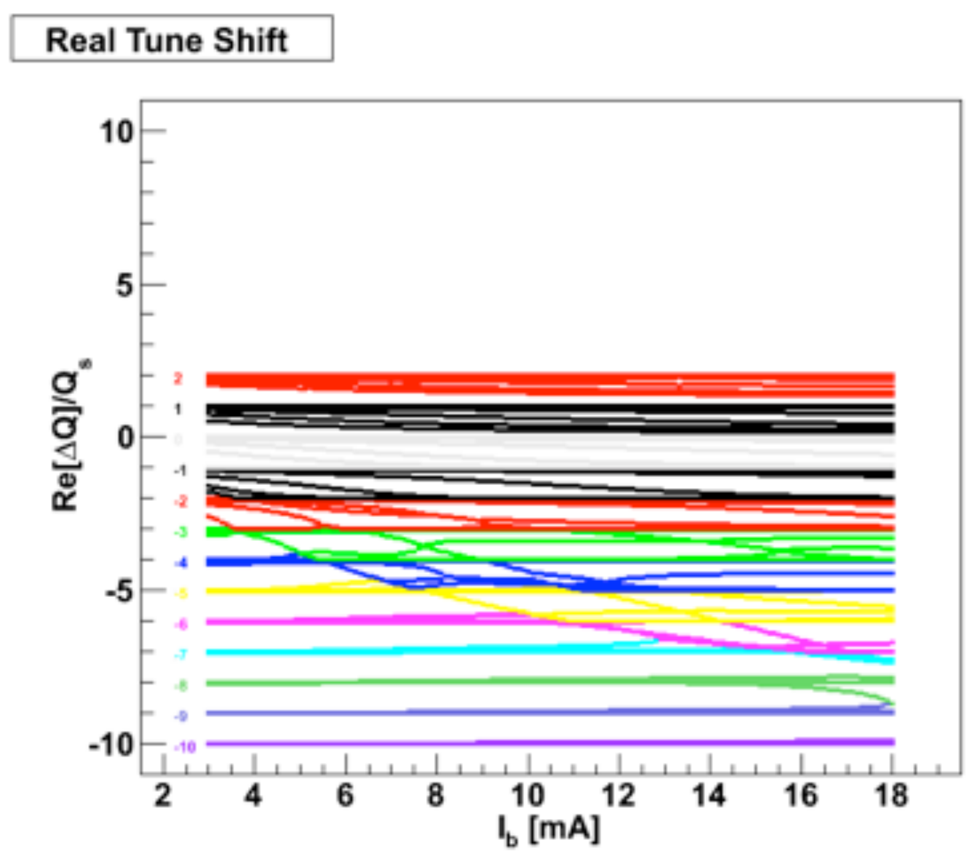

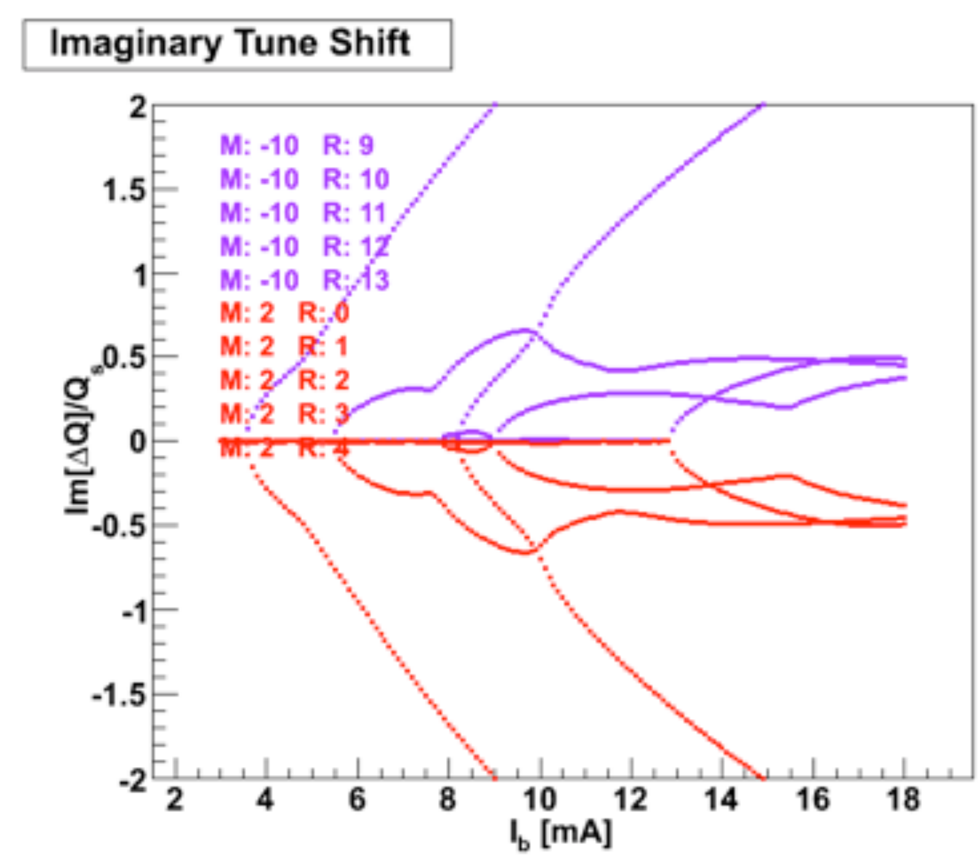

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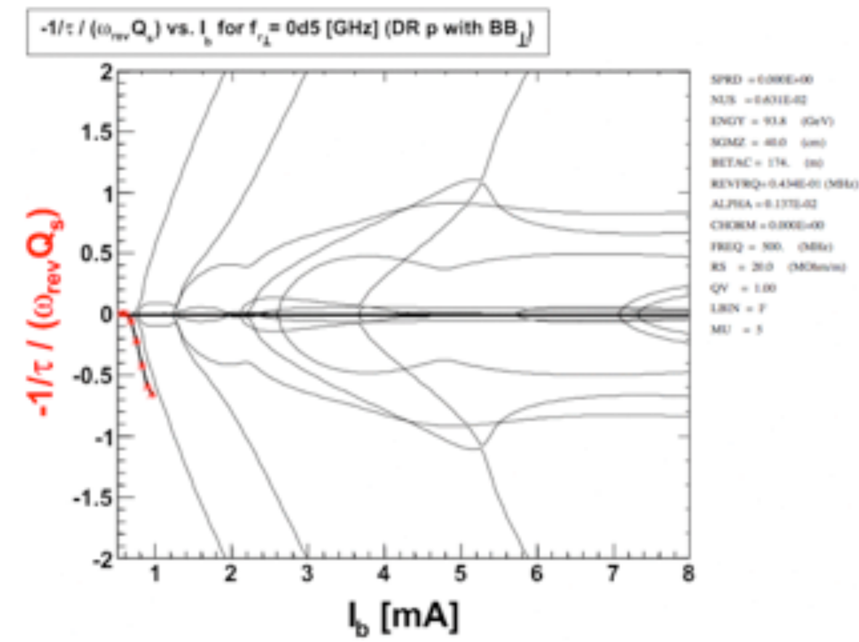

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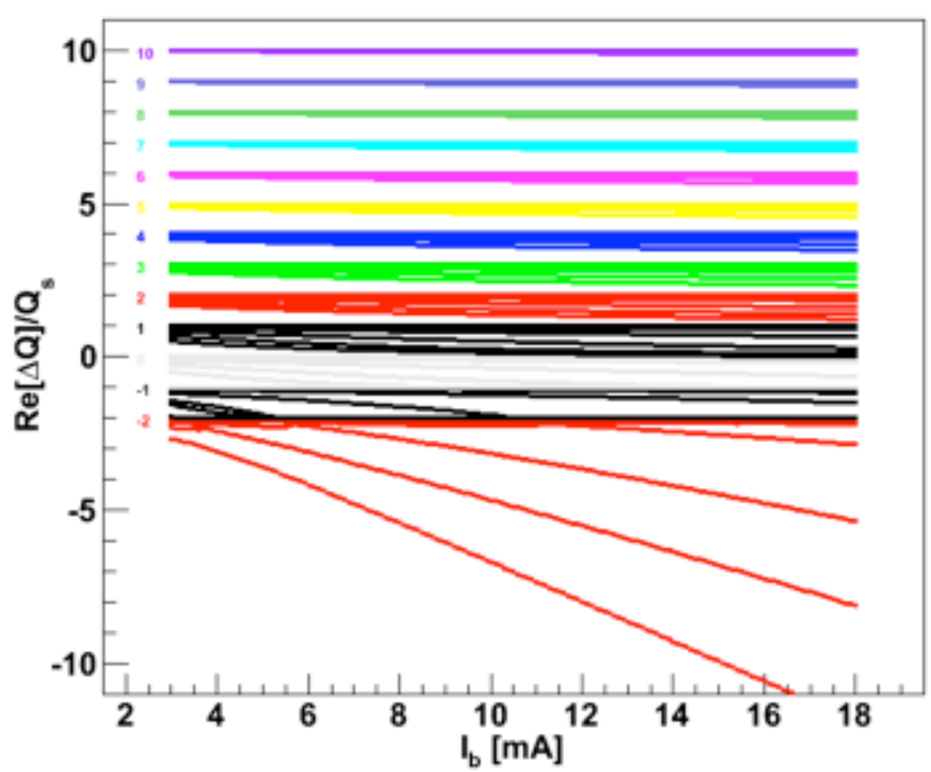
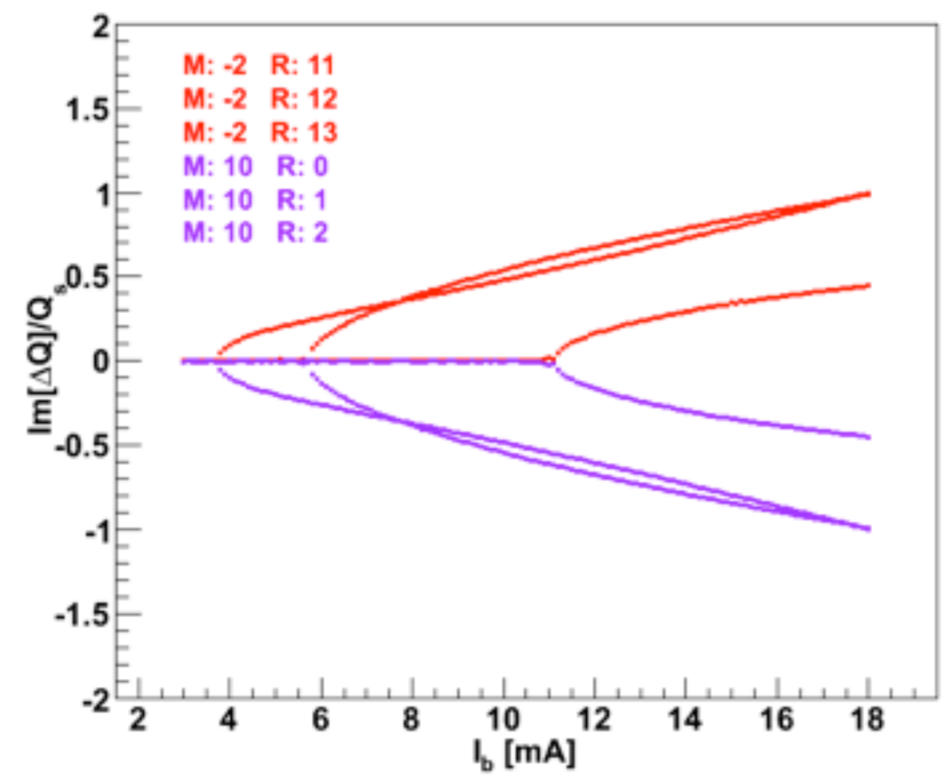
Real Tune Shift



Imaginary Tune Shift

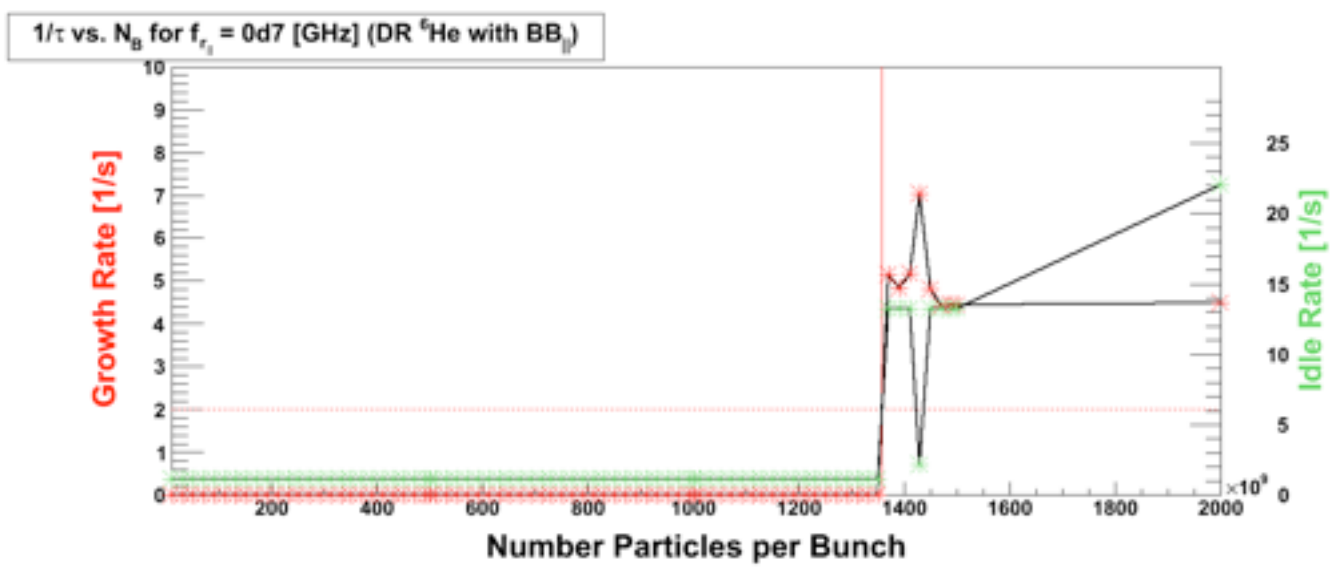


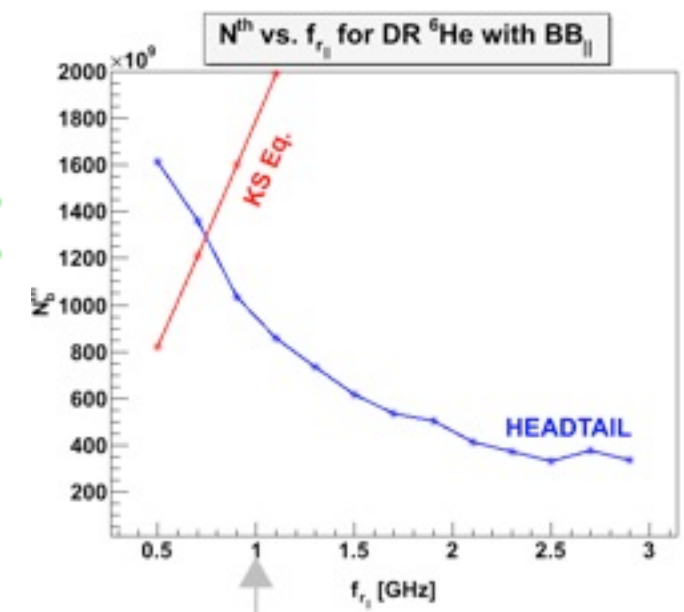
CONCLUSION

To Do

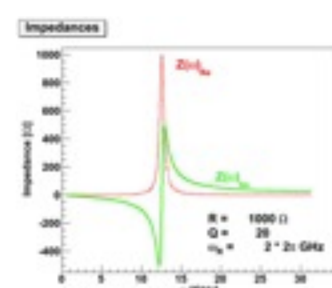
- Same study in longitudinal plane
  - ➡ Ongoing, but
    - ✦ MOSES is only for Transversal
    - ✦ Keil Schnell does not seem to hold in our regime

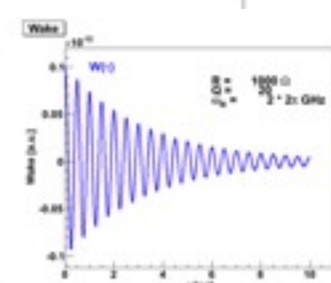
$$Z_{||}(\omega) = \frac{R_{||}}{1 + iQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)}$$



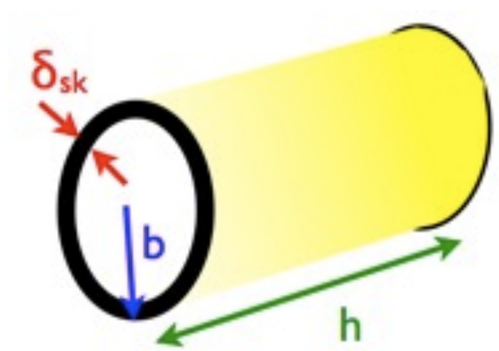


- Same with Narrow Band





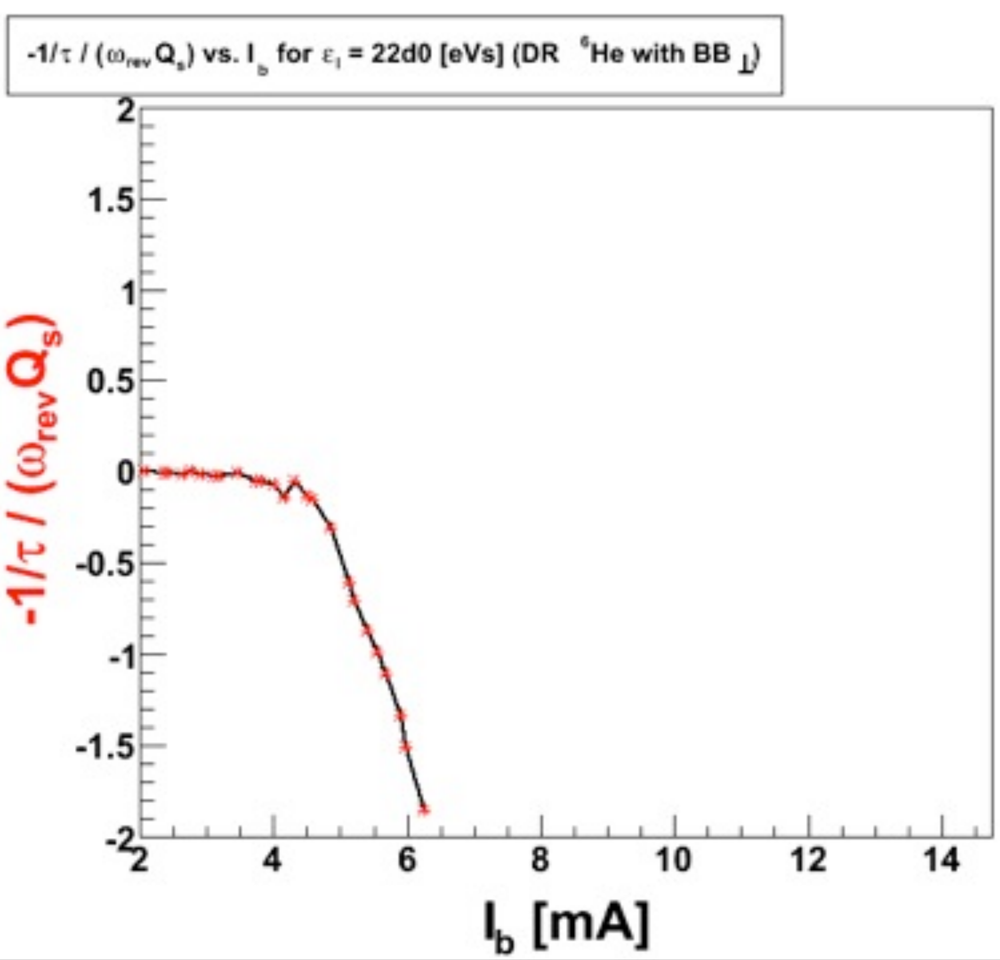
- Same with Resistive Wall Impedance
- Same with SPS, PS and ^{8}Li & ^{8}B

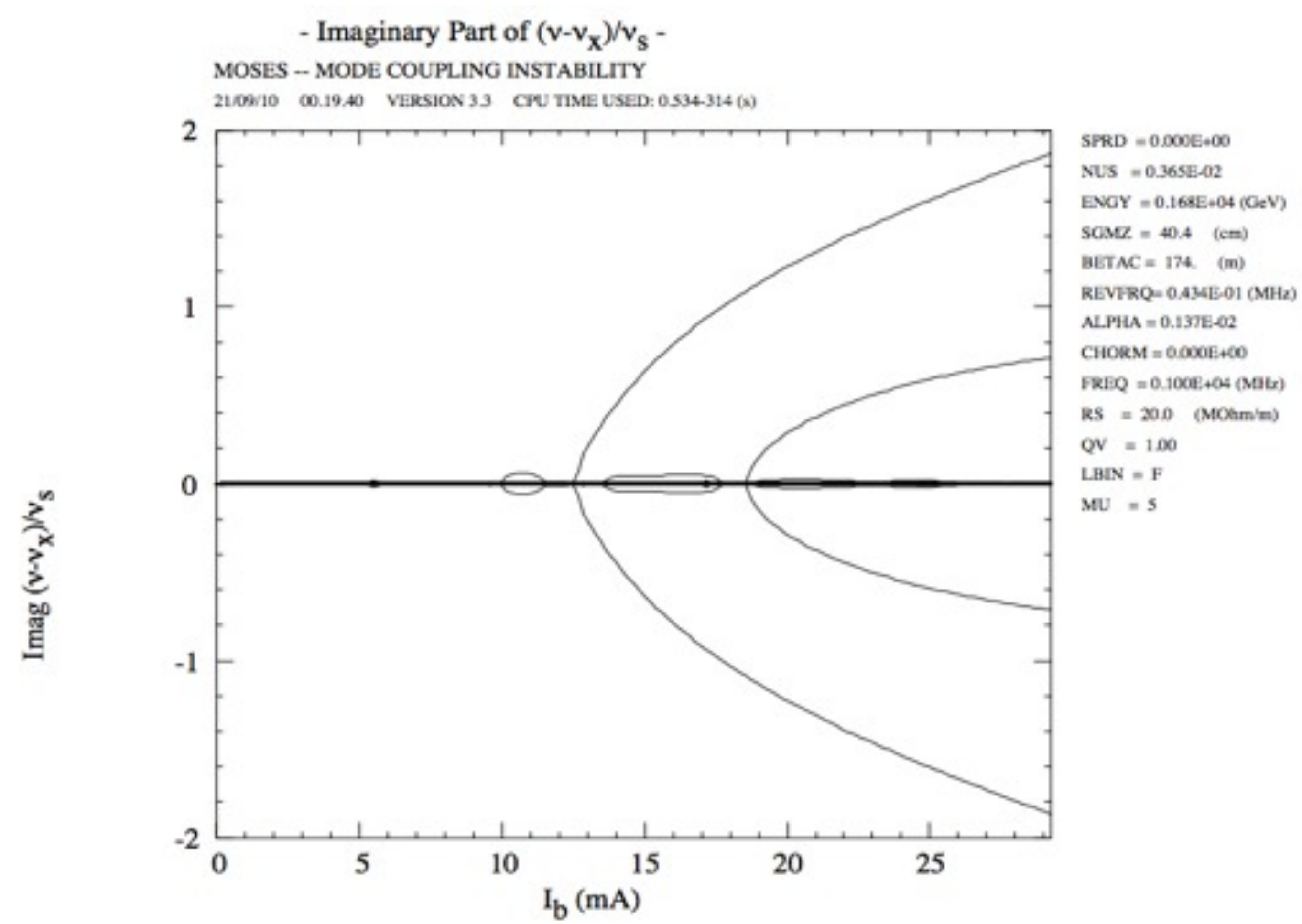


MOSES' Scales

BACKUP

- **To get same scale as MOSES' plots:**
- **X-axis:** $$\frac{1}{\tau} \rightarrow \frac{\Im\left[\Delta Q\right]}{Q_s} = -\frac{1/\tau}{\omega_{rev} Q_s}$$ **since** $$1/\tau = -\Im\left[\Delta Q\right]\omega_{rev}$$
- **Y-axis:** $$N_B \rightarrow \frac{ZeN_B}{T_{rev}} = I_b$$





Back to Ions

THRESHOLD

- Let's apply HEADTAIL and MOSES to ^{6}He and ^{18}Ne
- We will scan over some parameters, ε_l, f_r and ξ, around the working point (shown by grey arrow ↑) to see if we can improve $\boldsymbol{N_b^{th}}$ significantly

- For the Beta Beam Studies the possibility of bunches with ^{18}Ne and ^{6}He was added to HEADTAIL
- Assume that MOSES has same ZeI_b dependency as Sacherer's formula:

$$\left(\frac{1}{\tau}\right)^{m,n}_{\perp_{x,y}} = \frac{-1}{|n|+1} \frac{ZeI_b C\langle\beta_{x,y}\rangle\omega_{rev}}{4\pi E_{tot} L_b} \frac{\sum_{p=-\infty}^{\infty} \Re\left[Z_\perp(\omega_p)\right] h_{|n|}(\omega_p-\omega_\xi)}{\sum_{p=-\infty}^{\infty} h_{|n|}(\omega_p-\omega_\xi)}$$

then we get the ion equivalent threshold by
$\boldsymbol{N_b^{th}} = N_b^{th} / Z$

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Growth Rate From Eq. of Motion

BACKUP

$$\ddot{y} + \left(Q_y \omega_{rev}\right)^2 y = 0$$

$$\ddot{y} + \left(Q_y \omega_{rev}\right)^2 y = Ky$$

$$y(t) = A_1 e^{i\left(Q_y + \Delta Q_y\right)\omega_{rev} t} + A_2 e^{-i\left(Q_y + \Delta Q_y\right)\omega_{rev} t}$$

$$\Delta Q_y = -K / \left(2 Q_y \omega_{rev}^2\right)$$

$$\begin{aligned} \bar{y}(t) &= A e^{i\left(Q_y + \Delta Q_y\right)\omega_{rev} t} \\ &= A e^{i\left(Q_y + \Re\left[\Delta Q_y\right]\right)\omega_{rev} t} e^{-\Im\left[\Delta Q_y\right]\omega_{rev} t} \end{aligned}$$

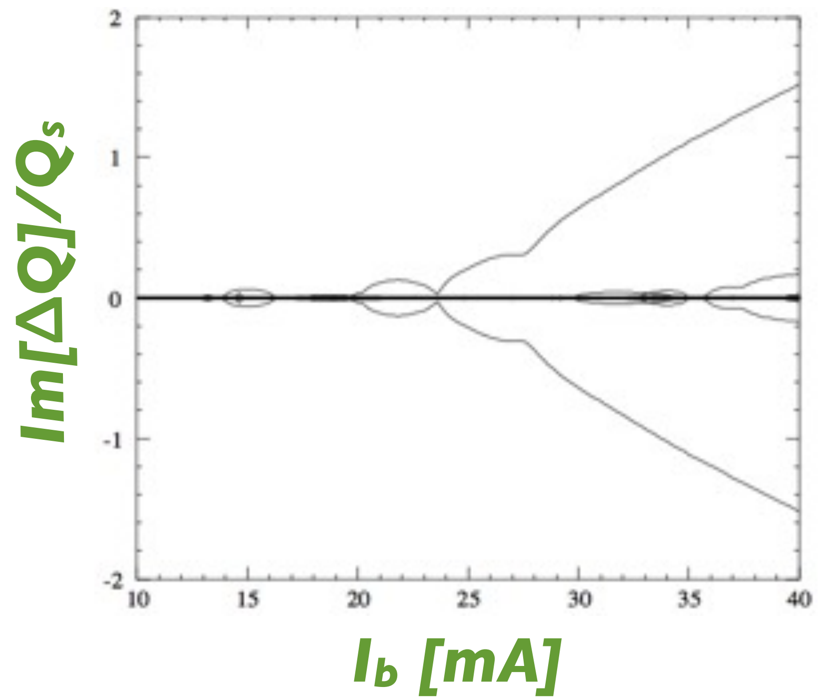
$$1/\tau \equiv -\Im\left[\Delta Q_y\right]\omega_{rev}$$

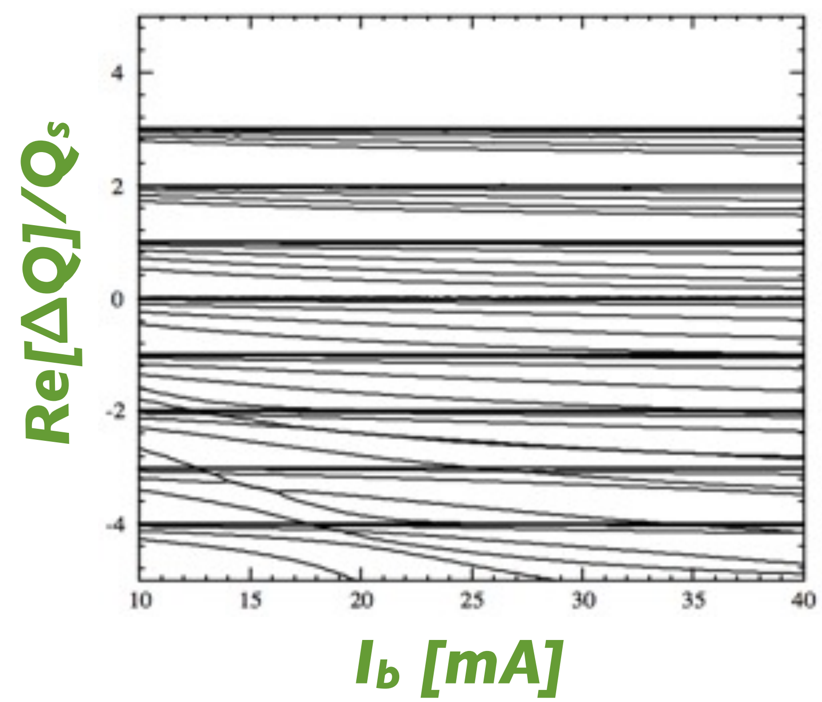
MOSES

- MOSES is a theoretical program
  - ➡ It solves a dispersion integral equation

Y.H.Chin CERN-LEP-TH/88-05

- It gives the *Im[ΔQ]* and *Re[ΔQ]* for different bunch modes and bunch intensities





- *Im[ΔQ]* is connected with the "Rise Time", τ, of the instability with $1/\tau = -Im[\Delta Q]\omega_{rev}$ *(see backup slide)*

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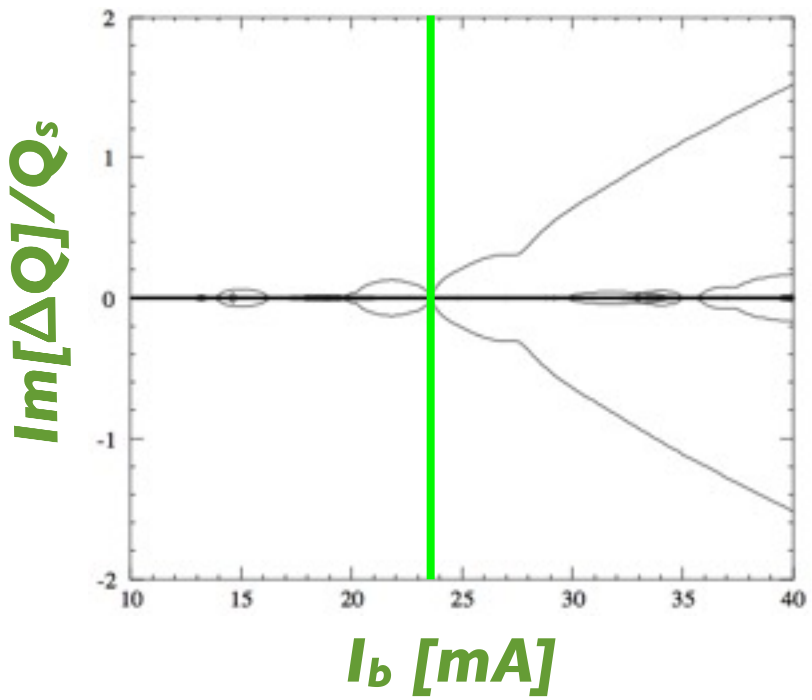
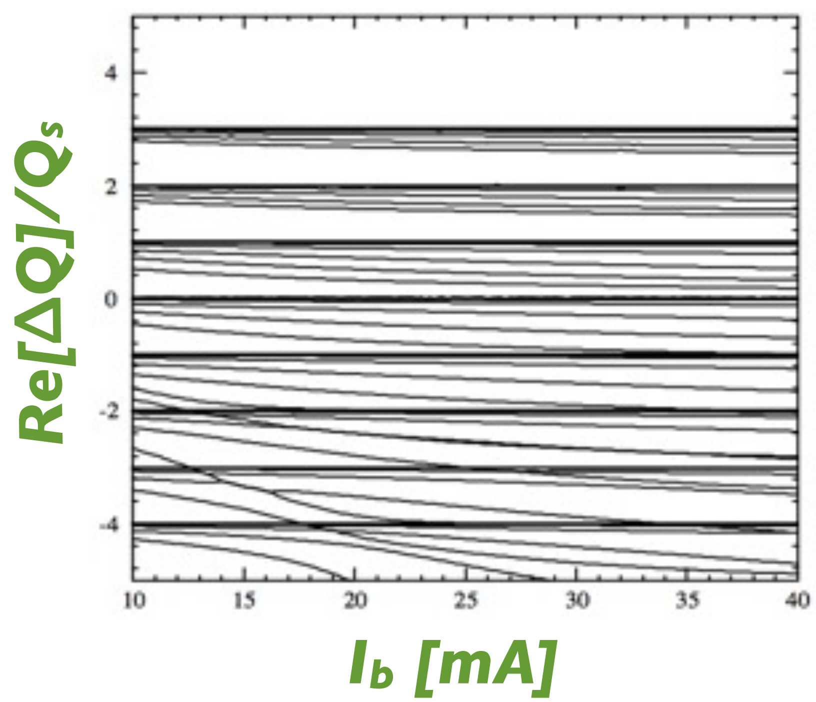
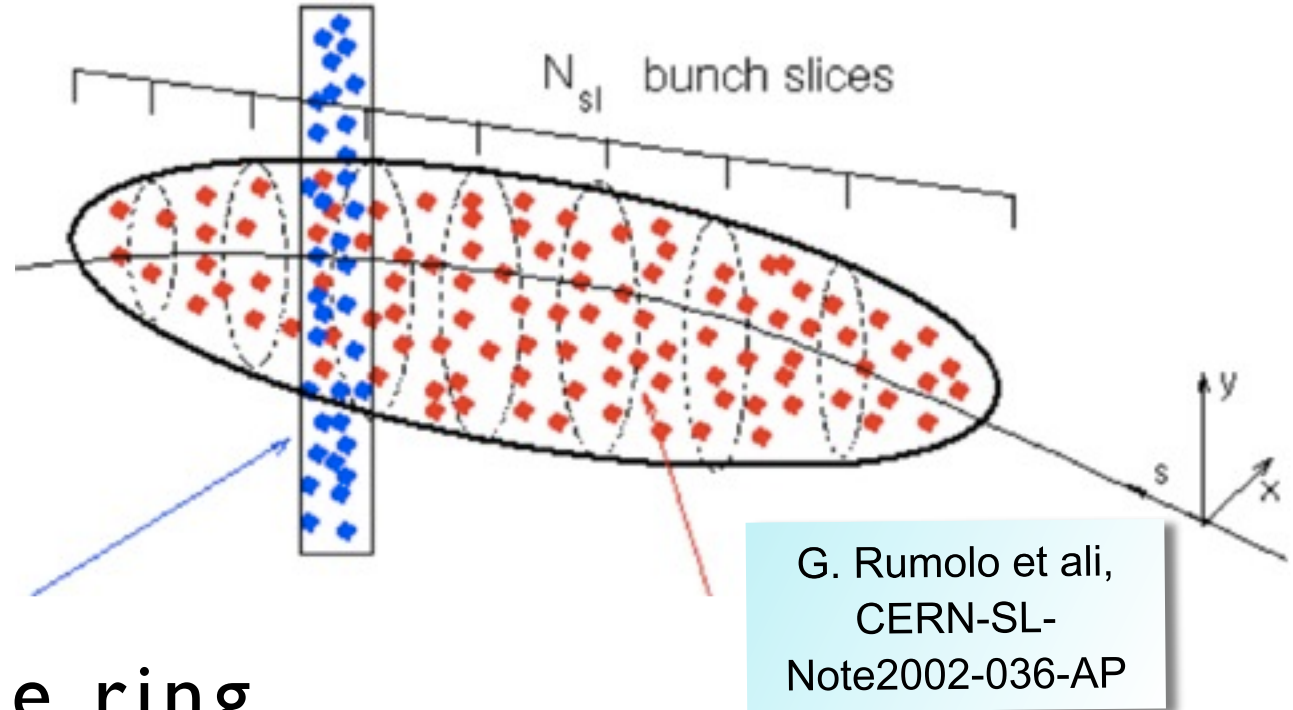




- $Im[\Delta Q]$ is connected with the "Rise Time", τ, of the instability with $1/\tau = -Im[\Delta Q]\omega_{rev}$ *(see backup slide)*
- When the "Growth Rate", $1/\tau$, starts to grow too much gives I_b^{th} and then $\mathbf{N_b^{th}} = T_{rev} I_b^{th} / Ze$

HEADTAIL

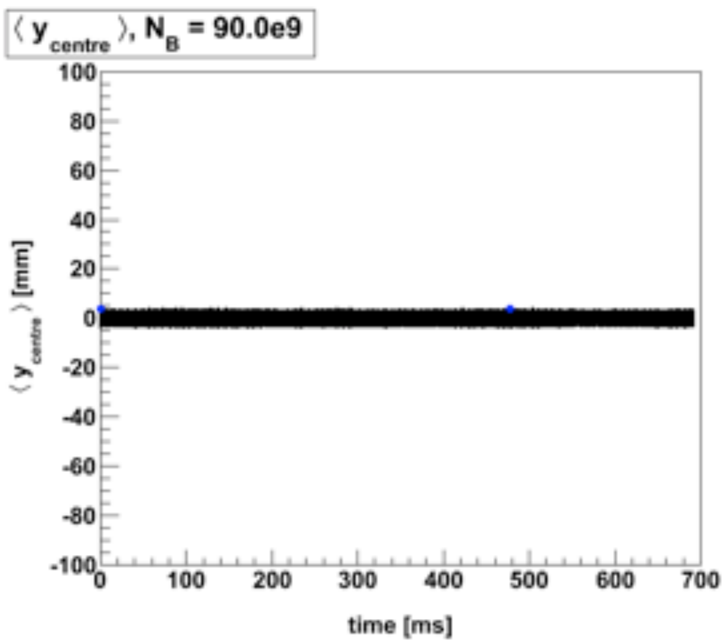
- HEADTAIL is a multi-particle tracking program
  - ➡ A bunch of macro-particles is sliced longitudinally

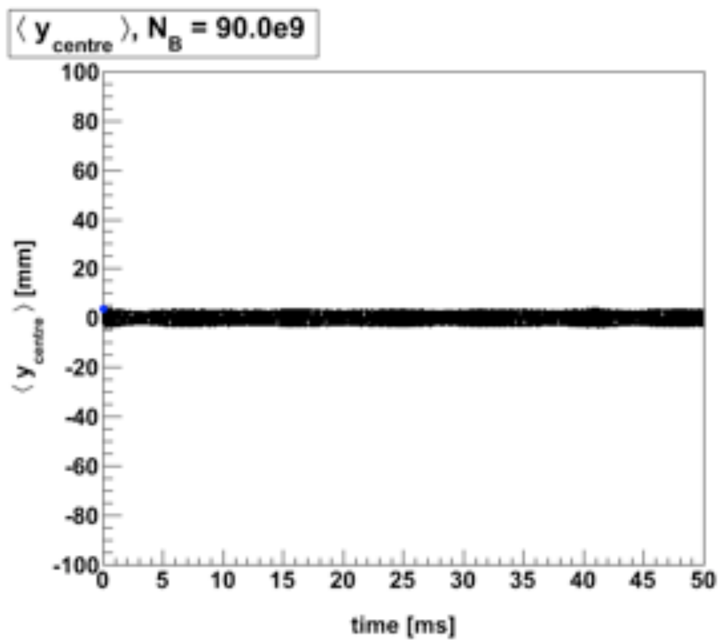


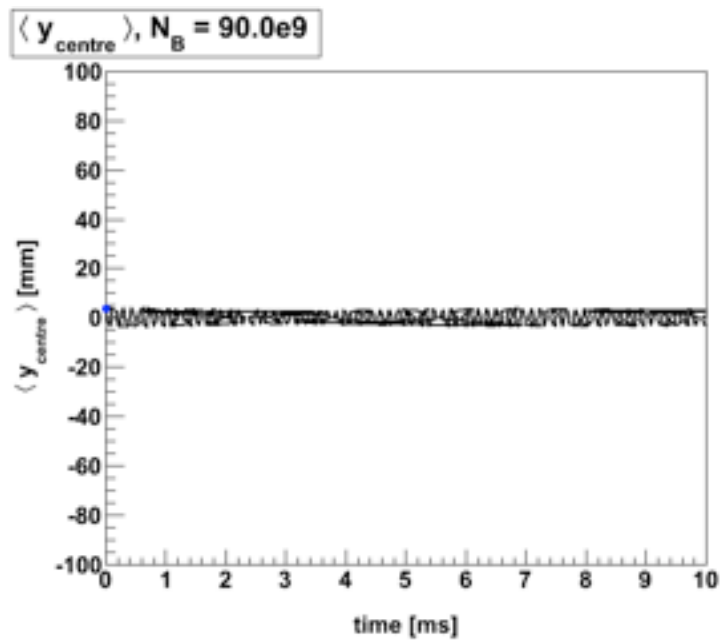
  - ➡ Impedance is assumed to be localized at a few positions around the ring
  - ➡ At each impedance location, each slice leaves a wake field behind and gets a kick by the field generated by the preceding slices
  - ➡ The bunch is then transferred to the next impedance location via a transport matrix

HEADTAIL

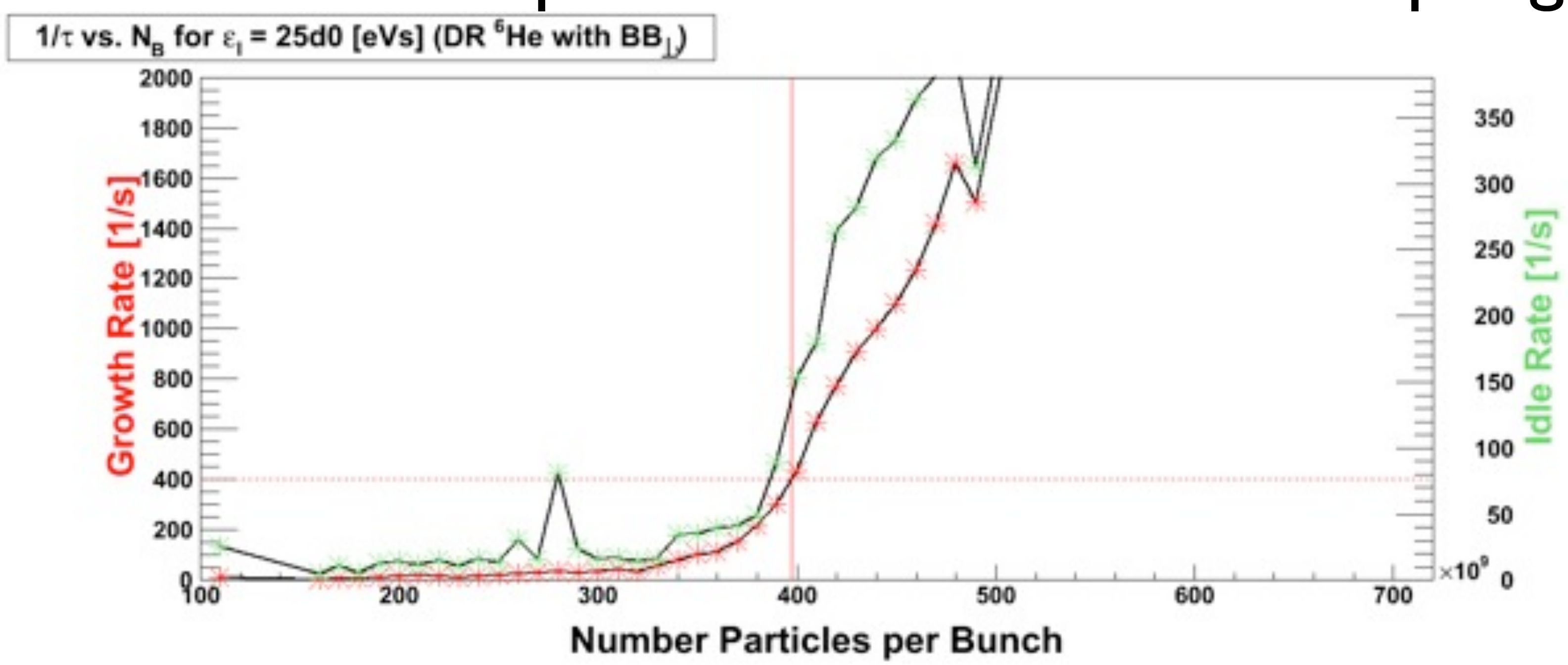
- HEADTAIL gives e.g. the vertical mean beam center shown here for different bunch intensities







- Exponential least square fit to the envelope gives $1/\tau$



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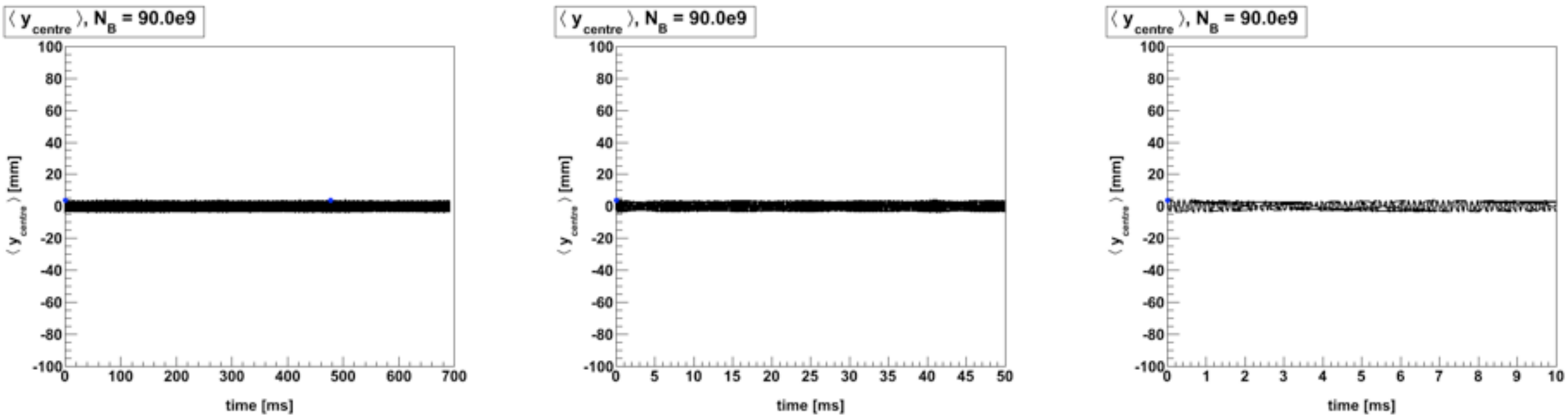

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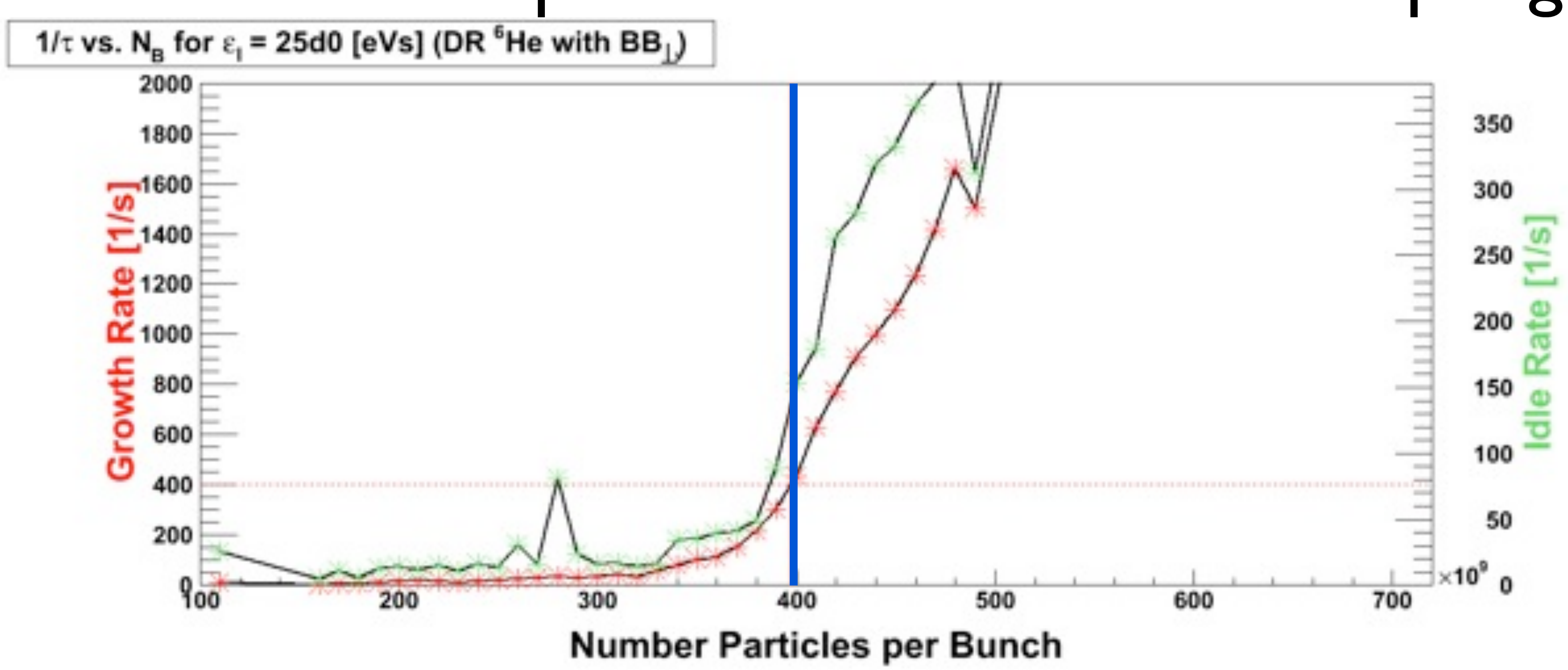

- Threshold Growth Rate defined as $(1/\tau)^{th} = 400$ Hz, which gives intensity limit; $\boldsymbol{N_b}^{th}$