New determination of the $N-\Delta(1232)$ axial form factors from weak pion production and coherent pion production off nuclei at T2K and MiniBooNE energies revisited

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- " $N-\Delta(1232)$ axial form factors from weak pion production", Phys. Rev. D 81, 085046 (2010),
E.Hernández, J. Nieves, M. Valverde, and M.J. Vicente Vacas
- "Coherent pion production off nuclei at T2K and MiniBooNE energies revisited", Phys. Rev. D 82,

077303 (2010), E.Hernández, J. Nieves, M. Valverde

## Plan of the talk

- Part I: New determination of $N-\Delta$ axial form factors from old bubble chamber data.
As in K.M. Graczyk et al., Phys. Rev. D 80, 093001 (2009) we include
- Deuteron effects
- Neutrino flux uncertainties

Besides

- Background terms
- Part II:
- Effects on our earlier results on coherent pion production off nuclei at low (T2K or MiniBooNE) energies.
- The $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)}$ SciBooNE ratio.


## Delta Pole Term for weak pion production off the nucleon

The dominant contribution for weak pion production at intermediate energies is given by the $\Delta$ pole mechanism


## $N \rightarrow \Delta$ weak current I

$$
\left\langle\Delta^{+} ; p_{\Delta}=p+q\right| j_{c c+}^{\mu}(0)|n ; p\rangle=\cos \theta_{C} \bar{u}_{\alpha}\left(\vec{p}_{\Delta}\right) \Gamma^{\alpha \mu}(p, q) u(\vec{p})
$$

$$
\begin{aligned}
& \Gamma^{\alpha \mu}(p, q) \\
& =\left[\frac{C_{3}^{V}}{M}\left(g^{\alpha \mu} q-q^{\alpha} \gamma^{\mu}\right)+\frac{C_{4}^{V}}{M^{2}}\left(g^{\alpha \mu} q \cdot p_{\Delta}-q^{\alpha} p_{\Delta}^{\mu}\right)+\frac{C_{5}^{V}}{M^{2}}\left(g^{\alpha \mu} q \cdot p-q^{\alpha} p^{\mu}\right)+C_{6}^{V} g^{\mu \alpha}\right] \gamma_{5} \\
& +\left[\frac{C_{3}^{A}}{M}\left(g^{\alpha \mu} q-q^{\alpha} \gamma^{\mu}\right)+\frac{C_{4}^{A}}{M^{2}}\left(g^{\alpha \mu} q \cdot p_{\Delta}-q^{\alpha} p_{\Delta}^{\mu}\right)+C_{5}^{A} g^{\alpha \mu}+\frac{C_{6}^{A}}{M^{2}} q^{\mu} q^{\alpha}\right]
\end{aligned}
$$

## $N \rightarrow \Delta$ weak current II

- Vector form factors: determined from the analysis of photo and electroproduction
(O. Lalakulich et al., Phys. Rev. D74, 014009 (2006))

$$
\begin{array}{r}
C_{3}^{V}=\frac{2.13}{\left(1-q^{2} / M_{V}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{4 M_{V}^{2}}}, \quad C_{4}^{V}=\frac{-1.51}{\left(1-q^{2} / M_{V}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{4 M_{V}^{2}}}, \\
C_{5}^{V}=\frac{0.48}{\left(1-q^{2} / M_{V}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{0.776 M_{V}^{2}}}, \quad C_{6}^{V}=0(C V C), \quad M_{V}=0.84 \mathrm{GeV}
\end{array}
$$

- Axial form factors:

Use Adler's model $\quad C_{4}^{A}\left(q^{2}\right)=-\frac{C_{5}^{A}\left(q^{2}\right)}{4}, \quad C_{3}^{A}\left(q^{2}\right)=0$
and PCAC $\quad C_{6}^{A}\left(q^{2}\right)=C_{5}^{A}\left(q^{2}\right) \frac{M^{2}}{m_{\pi}^{2}-q^{2}}$
and take (E.A. Paschos et al., Phys. Rev. D69, 014013 (2004))

$$
C_{5}^{A}\left(q^{2}\right)=\frac{C_{5}^{A}(0)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{3 M_{A \Delta}^{2}}}
$$

where with $C_{5}^{A}(0)=1.2$ (as given by the off-diagonal GTR) and $M_{A}=1.05 \mathrm{GeV}$.

## Background Terms

Our model in Phys. Rev. D 76, 033005 (2007) includes background terms required by chiral symmetry. To that purpose we use a $\mathrm{SU}(2)$ non-linear $\sigma$ model Lagrangian.

- No freedom in coupling constants
- We supplement it with well known form factors


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$$
\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+} \text {reaction } \mathbf{I}
$$

Flux averaged $q^{2}$-differential $\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+}$cross section $\int_{M+m_{\pi}}^{1.4 \mathrm{GeV}} d W \frac{d \bar{\sigma}_{\nu_{\mu} \mu^{-}}}{d q^{2} d W}$
$\nu_{\mu} \mathrm{p} \rightarrow \mu^{-} \mathrm{p} \pi^{+}$averaged over the ANL flux, $\mathrm{W}<1.4 \mathrm{GeV}$

$C_{5}^{A}\left(q^{2}\right)=\frac{C_{5}^{A}(0)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{3 M_{A \Delta}^{2}}}$
Results suggested a refit of $C_{5}^{A}$
$C_{5}^{A}(0)=0.867 \pm 0.075$
$M_{A \Delta}=0.985 \pm 0.082 \mathrm{GeV}$
[Phys. Rev. D 76, 033005 (2007)]

ANL data seems to prefer $C_{5}^{A}(0)$ values smaller than the one provided by the off-diagonal GTR

$$
\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+} \text {reaction } \mathbf{I I}
$$



But mind BNL data [T. Kitagaki et al., Phys. Rev. D34, 2554 (1986) ] for which $C_{5}^{A}(0)=1.2$ would be preferred

$$
\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+} \text {reaction III }
$$

## A different $C_{5}^{A}\left(q^{2}\right)$ parameterization is possible



$$
C_{5}^{A}\left(q^{2}\right)=\frac{1.2 \cdot\left(1-\frac{a q^{2}}{b-q^{2}}\right)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}}
$$

Leitner et al. [Phys. Rev. C 79, 034601 (2009)] find $a=-0.25, b=0.04 \mathrm{GeV}^{2}$, $M_{A \Delta}=0.95 G e V$ when only direct $\Delta$ is included

With background terms included one needs $a=-0.361, b=0.0167 G e V^{2}$, $M_{A \Delta}=0.932 G e V$

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## $C_{5}^{A}\left(q^{2}\right)$ comparison



$$
\begin{aligned}
& R_{A}^{2}=-\left.\frac{6}{C_{5}^{A}(0)} \frac{d C_{5}^{A}\left(q^{2}\right)}{d\left(-q^{2}\right)}\right|_{q^{2}=0} \\
& C_{5}^{A}\left(q^{2}\right)=\frac{C_{5}^{A}(0)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}} \cdot \frac{1}{1-\frac{q^{2}}{3 M_{A \Delta}^{2}}} \\
& C_{5}^{A}\left(q^{2}\right)=\frac{1.2 \cdot\left(1-\frac{a q^{2}}{b-q^{2}}\right)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}}
\end{aligned}
$$

In either case BNL data is underestimated

## Can one reconcile ANL \& BNL data and still have $C_{5}^{A}(0) \approx 1.2$ ?

The answer is YES for K.M. Graczyk et al. [Phys. Rev. D 80, 093001 (2009)]

- ANL and BNL data were measured in deuterium
- Deuteron effects were estimated by L. Alvarez-Ruso et al [Phys. Rev. C 59, 3386 (1999)] to reduce the cross section by $5-10 \%$.
- Large uncertainties in the neutrino flux normalization, $10 \%$ for BNL data and $20 \%$ for ANL data.

They made a combined fit to both ANL\&BNL data, assuming that only the $\Delta$ mechanism contributes, including deuteron effects, and treating flux uncertainties as systematic errors. They found

$$
C_{5}^{A}(0)=1.19 \pm 0.08, \quad M_{A \Delta}=0.94 \pm 0.03 \mathrm{GeV}
$$

for a pure dipole parameterization for $C_{5}^{A}\left(q^{2}\right)$.
A very good agreement with the off-diagonal GTR is found!

But no background terms were included!

## Background terms included

In our work in Phys. Rev. D 81, 085046 (2010) we included background terms in a combined fit to ANL \& BNL data that took into account deuteron effects and flux normalization uncertainties.

We used a simpler dipole parameterization for $C_{5}^{A}\left(q^{2}\right)$

$$
C_{5}^{A}\left(q^{2}\right)=\frac{C_{5}^{A}(0)}{\left(1-q^{2} / M_{A \Delta}^{2}\right)^{2}}
$$

In some of the fits we relaxed Adler's constraints allowing

$$
C_{3,4}^{A}\left(q^{2}\right)=C_{3,4}^{A}(0) \frac{C_{5}^{A}\left(q^{2}\right)}{\left.C_{5}^{A}(0)\right)}
$$

exploring the possibility of extracting some direct information on $C_{3,4}^{A}(0)$

## New fit results for the axial form factors

|  | $C_{5}^{A}(0)$ | $M_{A \Delta} / \mathrm{GeV}$ | $C_{3}^{A}(0)$ | $C_{4}^{A}(0)$ | $\chi^{2} / \mathrm{dof}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I* (only $\Delta P$ ) $^{*}$ | $1.08 \pm 0.10$ | $0.92 \pm 0.06$ | Ad | Ad | 0.36 |
| II $^{*}$ | $0.95 \pm 0.11$ | $0.92 \pm 0.08$ | Ad | Ad | 0.49 |
| III (only $\Delta P$ ) | $1.13 \pm 0.10$ | $0.93 \pm 0.06$ | Ad | Ad | 0.32 |
| IV | $1.00 \pm 0.11$ | $0.93 \pm 0.07$ | Ad | Ad | 0.42 |
| V | $1.08 \pm 0.14$ | $0.91 \pm 0.10$ | $-1.0 \pm 1.4$ | Ad | 0.40 |
| VI | $1.08 \pm 0.14$ | $0.86 \pm 0.15$ | Ad | $-1.0 \pm 1.3$ | 0.40 |
| VII | $1.07 \pm 0.15$ | $1.0 \pm 0.3$ | $1 \pm 4$ | $-2 \pm 4$ | 0.44 |

* No deuteron effects included.

Using Adler's constraints we get

$$
C_{5}^{A}(0)=1.00 \pm 0.11, \quad M_{A \Delta}=0.93 \pm 0.07 \mathrm{GeV}
$$

$C_{5}^{A}(0)$ compatible with its GTR value at the $2 \sigma$ level.

## Comparison with ANL \& BNL data



68\% confidence level bands are shown.
Flux uncertainties considered as systematic errors and added in quadratures to the statistical ones.

## Implications for coherent pion production I

Coherent pion production is a low $q^{2}$ process which is dominated by the $\Delta$ mechanism and thus very sensitive to $C_{5}^{A}(0)$


Central values for cross sections increase by 23-30\% compared to our former results [Essentially by a factor $\left.\left(\left.C_{5}^{A}(0)\right|_{\text {new }} /\left.C_{5}^{A}(0)\right|_{\text {former }}\right)^{2}=(1 / 0.867)^{2}=1.33\right]$

## Implications for coherent pion production II




## Implications for coherent pion production III

| Reaction | Experiment | $\bar{\sigma}$ <br> $\left[10^{-40} \mathrm{~cm}^{2}\right]$ | $\sigma_{\exp }$ <br> $\left[10^{-40} \mathrm{~cm}^{2}\right]$ | $E_{\max }$ <br> $[\mathrm{GeV}]$ | $\int d E \phi(E) \sigma(E)$ <br> $\left[10^{-40} \mathrm{~cm}^{2}\right]$ | $\int d E \phi(E)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| CC | $\nu_{\mu}^{12} \mathrm{C}$ | K2K | $6.1 \pm 1.3$ | $<7.7[1]$ | 1.80 | $5.0 \pm 1.0$ |
| CC | $\nu_{\mu}^{12} \mathrm{C}$ | MiniBooNE | $3.8 \pm 0.8$ |  | 1.45 | $3.5 \pm 0.7$ |
| NC | $\nu_{\mu}^{12} \mathrm{C}$ | MiniBooNE | $2.6 \pm 0.5$ | $7.7 \pm 1.6 \pm 3.6[2]$ | 1.34 | $2.2 \pm 0.5$ |
| CC | $\nu_{\mu}^{12} \mathrm{C}$ | T2K | $3.2 \pm 0.6$ |  | 1.45 | $2.9 \pm 0.6$ |

[1] M. Hasegawa et al. [K2K Collaboration], Phys. Rev. Lett. 95, 252301 (2005).
[2] J.L. Raaf, FERMILAB-THESIS-2007-20 (2005) (NOT an official number by the
MiniBooNE Collaboration)

## Coherent NC/CC ratio and the SciBooNE $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NC} \operatorname{coh} \pi^{0}\right)}$ ratio I



We get

$$
\left.\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)}\right|_{0.8 \mathrm{GeV}}=1.45 \pm 0.03
$$

to be compared with the value reported by the SciBooNE Collaboration [Y. Kurimoto et al., Phys. Rev. D 81,111102 (2010)]

$$
\left.\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NC} \operatorname{coh} \pi^{0}\right)}\right|_{\mathrm{SciBooNE}}=0.14_{-0.28}^{+0.30}
$$

A huge factor of 10 discrepancy!

## Coherent NC/CC ratio and the SciBooNE $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)}$ ratio II

The SciBooNE $\sigma\left(\mathrm{NCcoh} \pi^{0}\right)$ cross section is based on the Rein-Sehgal (RS) model

$$
\left.\sigma\left(\mathrm{NCcoh} \pi^{0}\right)\right|_{\mathrm{SciBooNE}}=R_{c o h} \frac{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)_{M C-R S}}{\sigma(C C)_{M C-R S}}=R_{c o h} 1.21 \times 10^{-2}
$$

where $R_{\text {coh }}=0.96 \pm 0.20$ from a fit to data
On the other hand the experimental determination of $\sigma\left(\mathrm{CCcoh} \pi^{+}\right)$by the SciBooNE Collaboration is such that

$$
\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma(C C)}=\left(0.16 \pm 0.17_{-0.27}^{+0.30}\right) \times 10^{-2}
$$

when the Rein- Sehgal model predicts

$$
\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)_{M C-R S}}{\sigma(\text { Total })_{M C-R S}} \approx 1 \times 10^{-2}
$$

Should one trust $\frac{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)_{M C-R S}}{\sigma(C C)_{M C-R S}}$ better than $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)_{M C-R S}}{\sigma(\text { Total })_{M C-R S}}$ ? We think not!

## Coherent NC/CC ratio and the SciBooNE $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)}$ ratio III

Our own conclusion [Phys. Rev. D 80, 013003 (2009)] is that the RS model IS NOT appropriate for low energy neutrinos!

- The $t$ (nuclear momentum transfer square) dependence of the coherent production is fully ascribed to the nuclear form factor, while further and significant $t$-dependences induced by the pion-nucleon interaction are ignored
- The eikonal treatment of the outgoing pion distortion is quite poor for low energy pions.
- Far from the $q^{2}=0$ kinematical point, any PCAC based model, and in particular the RS one, cannot be used to determine the angular distribution of the outgoing pion with respect to the direction of the incoming neutrino. Terms that vanish at $q^{2}=0$, and that are not considered in PCAC based models, provide much more forward peaked outgoing pion-incoming neutrino angular distributions



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## Conclusions

- We have performed a new fit of axial $N \Delta$ transition for factors
- We use ANL \& BNL data
- We include deuteron effects and neutrino flux uncertainties
- We include background terms.

$$
C_{5}^{A}(0)=1.00 \pm 0.11, \quad M_{A \Delta}=0.93 \pm 0.07 \mathrm{GeV}
$$

- As a result we obtain 20-30\% larger coherent cross sections than before
- We believe the large $\frac{\sigma\left(\mathrm{CCcoh} \pi^{+}\right)}{\sigma\left(\mathrm{NCcoh} \pi^{0}\right)}$ ratio by the SciBooNE Collaboration is tied to the use of the RS model which we think is not appropriate for coherent production by low energy neutrinos.

