Scaling FFAG accelerator for muon acceleration

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Abstract. Recent developments in scaling fixed field alternating gradient (FFAG) accelerators have opened new ways for lattice design, with straight sections, and insertions like dispersion suppressors. Such principles and matching issues are detailed in this paper. An application of these new concepts is presented to overcome problems in the PRISM project.

Keywords: Scaling FFAG; transport line; insertion; dispersion suppressor; PRISM.
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INTRODUCTION

Due to their large emittance, muon beams need to be manipulated by systems with large longitudinal and transverse acceptances. The scaling type of Fixed Field Alternating Gradient (FFAG) accelerator has several good features to deal with these large 6D emittances beams. Since the chromaticity is null, particles can be kept far away from harmful resonances. Moreover, since the first order dependence of the time of flight on the transverse amplitude is proportional to the chromaticity [1], scaling FFAGs show little longitudinal amplitude growth for large transverse amplitudes. The use of this type of ring has already been proposed for muon acceleration [2, 3].

To go further, recent developments in scaling FFAGs have opened new ways for lattice design [4, 5]. Indeed, it is possible to guide particles with no overall bend in scaling FFAGs. Different types of cells can be combined in the same lattice. It offers possibilities in terms of shape for rings and transport lines, and also leads to the creation of insertions, such as dispersion suppressors. These new concepts can be applied to overcome problems like in the PRISM project.

ADVANCED SCALING FFAG

Straight sections and circular sections

The classical case of scaling FFAG is a circular section where the general form of the vertical component of the magnetic field B in the mid-plane is [6, 7]

$$B = B_0 \left(\frac{r}{r_0}\right)^k \mathscr{F}(\theta), \tag{1}$$

with r and θ the polar coordinates, B_0 the magnetic field at the radius r_0, $\mathscr{F}$ an arbitrary periodic function, and k the geometrical field index. The periodic dispersion function D for a given momentum p_0 is then defined as

$$D_{circ.}(p_0) = p_0 \left(\frac{\partial r}{\partial p}\right)_{p_0} = \frac{r}{k+1}. \tag{2}$$

But scaling FFAG sections can also be imagined with no overall bend. The general form of the vertical component of the magnetic field B in the mid-plane becomes [4, 5]

$$B = B_0 e^{m(X-X_0)} \mathscr{F}(s), \tag{3}$$

where X is the cartesian abscissa, s the curvilinear abscissa, B_0 the magnetic field at the abscissa X_0, $\mathscr{F}$ an arbitrary periodic function and m the normalized field gradient. The periodic dispersion function D for a given momentum p_0 is then defined as

$$D_{str.}(p_0) = p_0 \left(\frac{\partial X}{\partial p}\right)_{p_0} = \frac{1}{m}. \tag{4}$$

FFAG straight sections could be used with circular FFAG sections, but since field laws in each section are different, it will occur a discontinuity of reference trajectories at the border of these two sections. In consequence, to combine a straight section and a circular section, after matching a special momentum p_0, dispersion can be matched with

$$m = \frac{k+1}{R_0}, \tag{5}$$

with R_0 the radius of the closed orbit for the momentum p_0 at the border of the cell in the circular section. But this matching is done only to the first order in $\frac{R-R_0}{R_0}$, with

R the radius of the closed orbit for a momentum p at the border of the cell in the circular section. Higher orders effects create a reference trajectory mismatch for momenta other than p_0 [4]. By a proper choice of p_0, the mismatch can be minimized for the considered momentum range.

An example has been computed by simulating the insertion of straight sections in the 150 MeV FFAG ring of KURRI, for kinetic energies between 20 MeV and 150 MeV. The mismatch x as a function of kinetic energy is presented in Fig. 1. This maximum mismatch is around one cm. for this case.

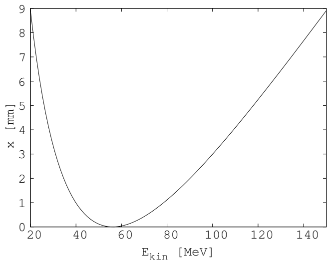

FIGURE 1. Mismatch of reference trajectories between circular cells and straight cells in 150 MeV FFAG ring example.

In the same way, it is possible to match two circular FFAG sections with different radii r_1 and r_2. The dispersion matching condition is kept by adjusting the geometrical field index:

$$\frac{k_1+1}{r_1} = \frac{k_2+1}{r_2}. \tag{6}$$

Once the reference trajectories are matched, the periodic beta-functions of the cells have also to be matched to limit the amplitude of the betatron oscillations. If a correct matching is not achievable, then an insertion with a phase advance multiple of 180 deg. can be done for one of the two different types of cells. This insertion becomes thus transparent, limiting the betatron oscillations.

Dispersion Suppressor

In FFAGs, dispersion suppressors can be useful at the end of a transport line, or to reduce excursion where rf cavities are set in FFAG rings. The effect of a dispersion suppressor would be to decrease the excursion, i.e. to bring closer the reference trajectories around a matched one. This excursion reduction can even be a complete suppression (see Fig. 2).

A principle of a dispersion suppressor in scaling FFAGs is presented in Fig. 2. The components of this scheme are three types of scaling FFAG cells, straight or circular. The area 1 contains FFAG cells with a dispersion D_1 at the border, the area 2, constituting the dispersion suppressor itself, contains FFAG cells with a dispersion D_2 at the borders, and the area 3 contains FFAG cells with a dispersion D_3 at the border. The conditions to have a dispersion suppressor are a phase advance of 180 deg. for the cells of the area 2 and the dispersion D_2 has to verify

$$D_2 = \frac{D_1+D_3}{2}. \tag{7}$$

This principle is based on the linear theory, so is valid as long as the effect of non-linearities is negligible.

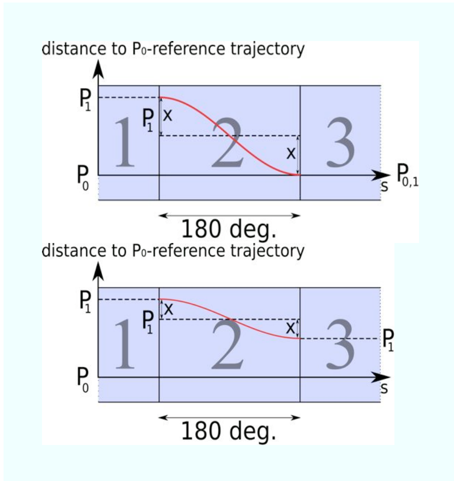

FIGURE 2. Principle of a dispersion suppressor with scaling FFAG cells. The upper scheme shows the case of a complete suppression of the dispersion, the lower one the case of a partial suppression of the dispersion.

Experiment

In order to confirm and study these new concepts, an experiment is planned at Kyoto University Research Reactor Institute. The H- Linac injector of the FFAG complex in the institute will deliver 3.5 MeV and 7 MeV beams in the FFAG beam line. This beam line (see Fig. 3) is a straight FFAG section to confirm the exponential field law, and with a 180 deg. phase advance to study the dispersion suppressor principle.

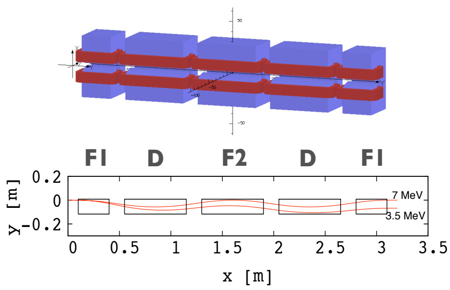

FIGURE 3. Scheme of the FFAG beam line for the experiment at KURRI.

TABLE 1. Parameters of the new PRISM lattice

	Cicular section	Straight section
Type	FDF	FDF
k-value or m-value	2.55	$1.3\,\mathrm{m}^{-1}$
Radius/Length	2.7 m	1.8 m
Horizontal phase advance	60 deg.	27 deg.
Vertical phase advance	90 deg.	97 deg.
Number of cells	2	3

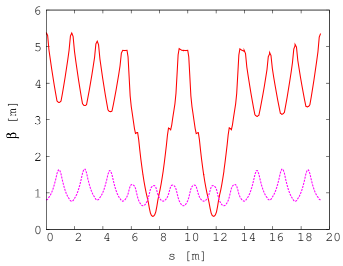

FIGURE 5. Horizontal (plain red) and vertical (dotted purple) beta-functions for half of the ring of the PRISM lattice.

APPLICATION FOR MUONS: PRISM

The PRISM (Phase Rotated Intense Slow Muon beam) project aims to realize a low-energy muon beam with a high-intensity, narrow energy spread and high purity. For this purpose, a scaling FFAG ring has been proposed [8]. Requirements for the FFAG ring include a large transverse and longitudinal acceptance. The original design of the FFAG ring for PRISM is based on 10 identical DFD triplets. If this design fulfills the requirements of acceptance, the excursion is very large and the injection and extraction still remains difficult. To solve this problem, we consider the use of straight cells in the lattice and a new design is proposed (see Fig. 4). Parameters are summarized in Tab. 1.

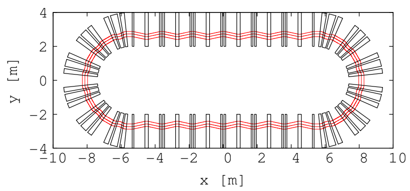

FIGURE 4. Closed orbits of 55 MeV/c, 68 MeV/c and 82 MeV/c muons μ^- in the PRISM lattice with straight sections.

Particle tracking is done using our code in soft edge fields with linear fringe field falloffs. Components of the field off the mid-plane are obtained from a first order Taylor expansion, satisfying the Maxwell equations.

The original PRISM design has a very large dispersion function ($\sim$ 1.2 m) that makes difficult the injection and the extraction. The new proposal starts then from a smaller one ($\sim$ 0.8 m). After minimizing the mismatch of the beta-functions, the bending part of the ring is made transparent to limit the effect of the remaining mismatch on the amplitude of the betatron oscillations. The resulting beta-functions for a momentum of 68 MeV/c are presented in Fig. 5. The working point is chosen in the tune diagram so that it is far from the structural normal resonances. The present working point has a tune of 2.9 in horizontal and 6.3 in vertical.

The transverse acceptance in both planes is studied by tracking over 30 turns a particle with a displacement off the closed orbit and a small deviation in the other transverse direction (1 mm). Collimators ($\pm$ 1 m in horizontal, $\pm$ 30 cm in vertical) are used to identify the lost particles. The regions drawn by the particle with the largest initial stable amplitude in the horizontal and vertical phase spaces are presented in Fig. 6. Horizontal ($\sim 24000\,\pi$.mm.mrad) and vertical ($\sim 6000\,\pi$.mm.mrad) acceptances are then measured by the area of the biggest ellipse included in this region.

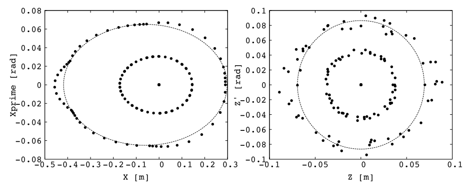

FIGURE 6. Horizontal (left) and vertical (right) phase space. Two particles with an initial displacement of 15 cm and 29 cm (left) and 3.5 cm and 7 cm (right) are tracked in the PRISM lattice over 30 turns. The dotted ellipses are the ones used to measure the acceptance in the middle of the straight section.

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