Gauged $L_\mu - L_\tau$ and different muon neutrino and anti-neutrino oscillations: MINOS and beyond

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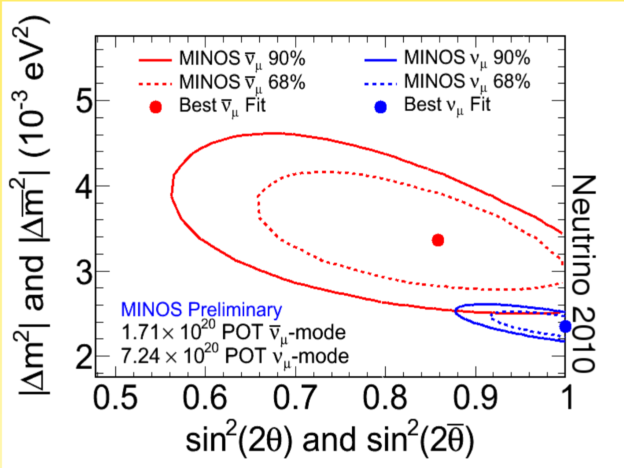
NuFact10, 24/10/10



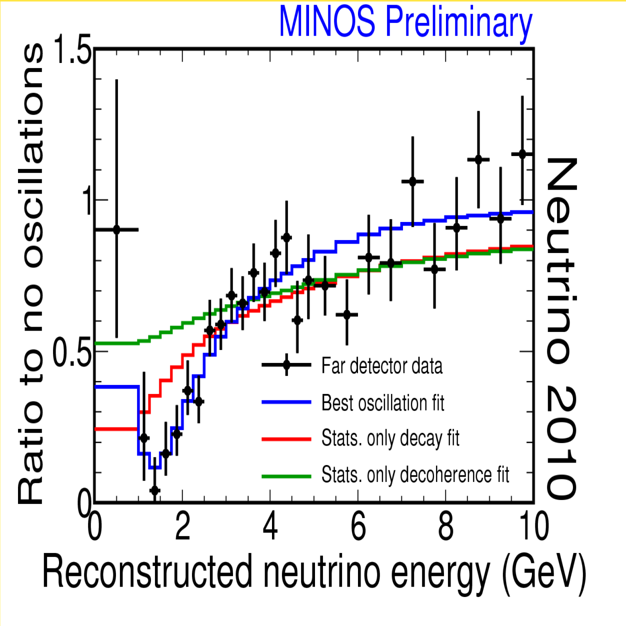
based on Heeck, W.R., arXiv:1007.2655 [hep-ph]

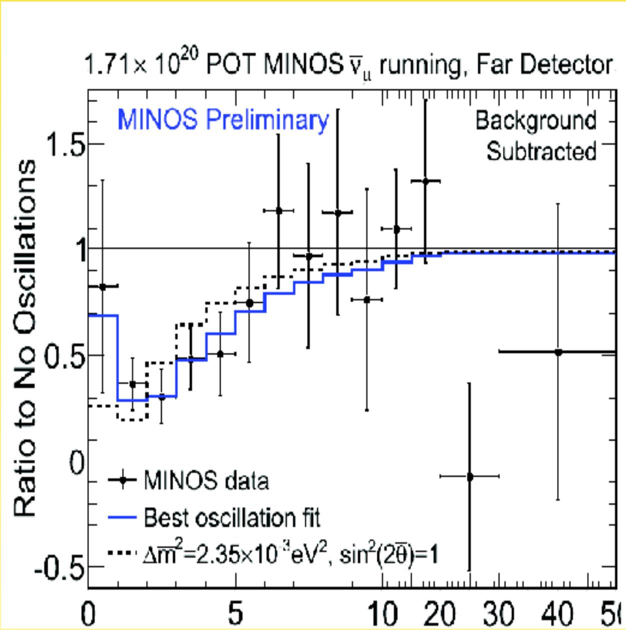
Outline

- MINOS data
- Gauged $L_\alpha - L_\beta$, Z' and neutrinos
- Gauged $L_\mu - L_\tau$, MINOS and neutrinos
- On NSIs
- Other aspects of $L_\mu - L_\tau$
- Comments on CPT violation

MINOS data

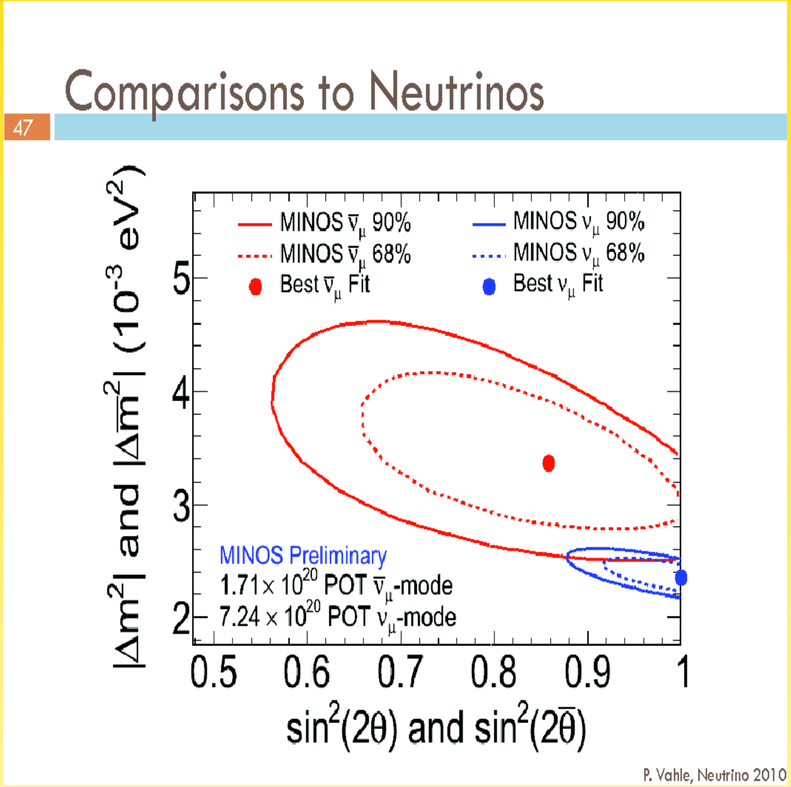
- 7.24×10^{20} POT for ν_μ, 1.71×10^{20} POT for $\bar{\nu}_\mu$
- ν_μ: 2451 if no oscillations, 1986 observed
- $\bar{\nu}_\mu$: 155 if no oscillations, 97 observed

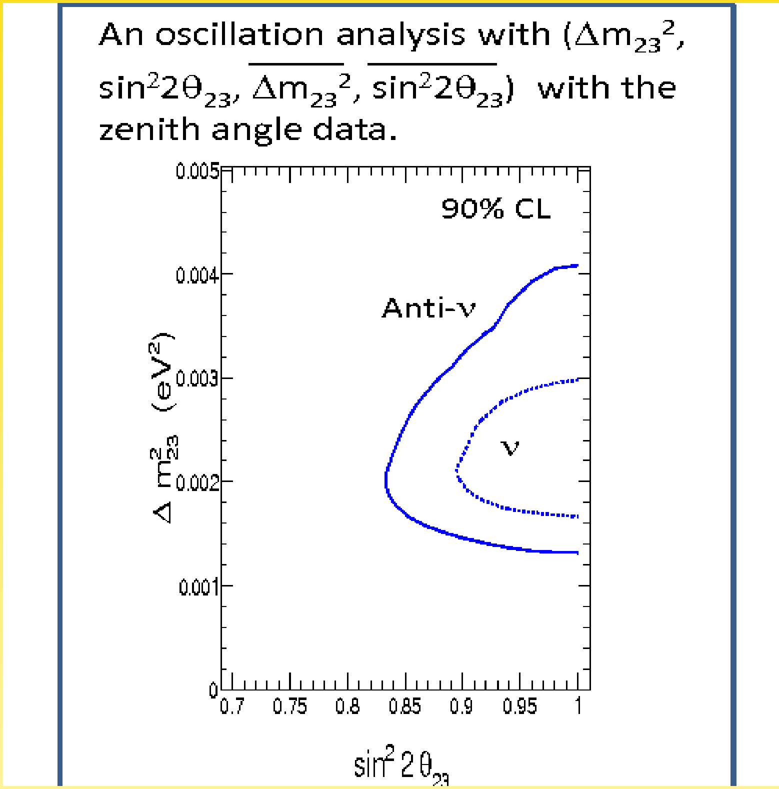




$$\Delta m^2 = \left(2.35^{+0.11}_{-0.08}\right) \times 10^{-3}\,\mathrm{eV}^2\ , \quad \sin^2 2\theta > 0.91$$

$$\overline{\Delta m^2} = \left(3.36^{+0.45}_{-0.40}\right) \times 10^{-3}\,\mathrm{eV}^2\ , \quad \sin^2 2\bar{\theta} = 0.86 \pm 0.11$$





SK cannot rule this out yet

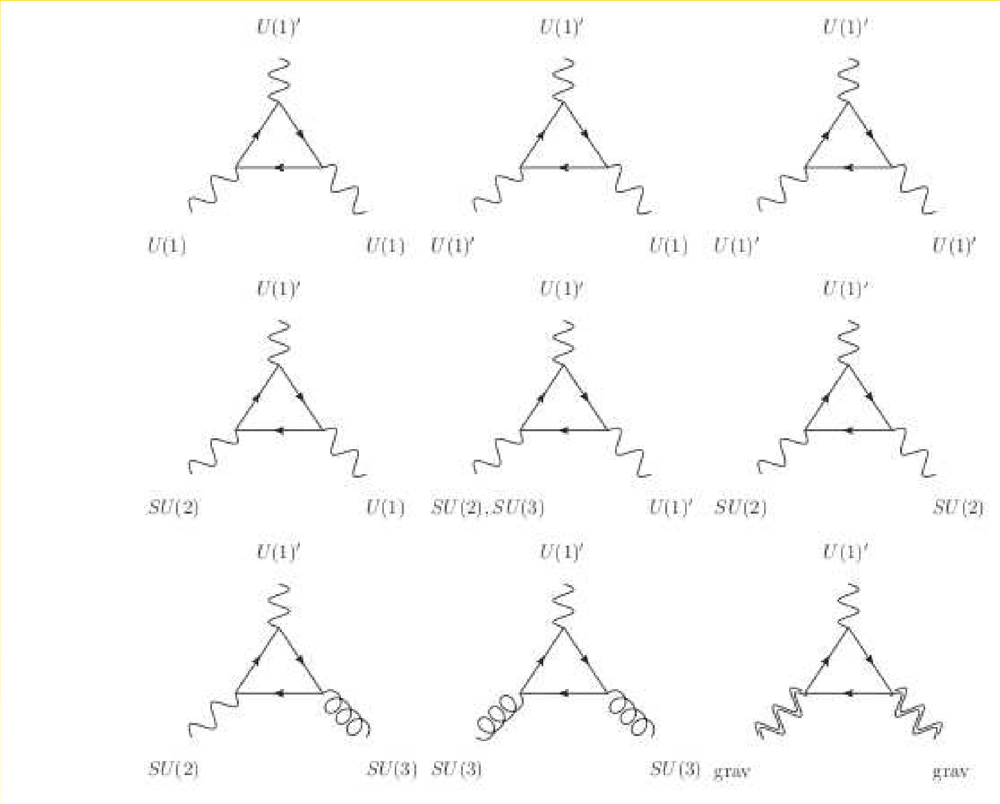
Is this...

- ...Non-Standard Interaction (`Mann, Cherdack, Musial, Kafka, 1006.5720; Kopp, Machado, Parke, 1009.0014`)?
- ...sterile neutrino (plus gauged Z' from $U(1)$ according to $B - L$) (`Engelhardt, Nelson, Walsh, 1002.4452`)?
- ...gauged ultra-light Z' from $U(1)$ according to $L_\mu - L_\tau$ (`Heeck, W.R., 1007.2655`)?
- ...CPT violation? (`Barenboim, Lykken, 0908.2993; Choudhury, Datta, Kundu, 1007.2923`)?
- ...nothing and will go away (`common sense`)?

Gauged $L_\alpha - L_\beta$

$L_e - L_\mu$ or $L_e - L_\tau$ or $L_\mu - L_\tau$ can be gauged **without anomaly** in SM

(Foot, 1991)



- SM: $\sum Y_i = 0$ in each family
- extra $U(1)$: anomalies cancel among different (lepton) generations
- example $L_e - L_\mu$: ν_e, L_e have $Q = 1$, ν_μ, L_μ have $Q = -1$
- there is an extra Z' which couples to ν_e, L_e and ν_μ, L_μ with coupling g'
- no expectation for mass scale...

- if Z' from $L_e - L_\alpha$ is ultra-light: particles in Sun (or Earth) create potential for terrestrial neutrinos (Joshipura, Mohanty, PLB **584**, 103 (2004))

$$V = \frac{g'^2}{4\pi} \frac{N_e}{R} \equiv \alpha \frac{N_e}{R}$$

Scale: $m'_Z \leq 1/\mathrm{A.U.} \simeq 10^{-18}$ eV...

V must be added to Hamiltonian:

$$\Rightarrow \mathcal{H}_{e\mu} = \frac{\Delta m^2}{4\,E} \left(\begin{array}{cc} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{array} \right) + \left(\begin{array}{cc} V & 0 \\ 0 & -V \end{array} \right)$$

$\leftrightarrow$ looks like NSI, but **does not depend on matter density!**

$\leftrightarrow$ also works for vacuum oscillations!

- V changes sign for anti-neutrinos!

$\Rightarrow P(\nu_\alpha \to \nu_\alpha) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha)$ without CPT violation

$$V = \alpha \frac{N_e}{R_{\mathrm{A.U.}}} = \alpha \frac{6 \times 10^{56}}{8 \times 10^{17}\,\mathrm{eV}^{-1}} \simeq 8 \times 10^{38}\,\alpha\,\mathrm{eV}$$

- atmospheric neutrinos

$$\frac{\Delta m^2_{\mathrm{A}}}{4E} \simeq 6 \times 10^{-13} \left(\frac{\mathrm{GeV}}{E}\right) \mathrm{eV}$$

Limits $\alpha_{e\mu} \leq 5.5 \times 10^{-52}$ and $\alpha_{e\tau} \leq 6.4 \times 10^{-52}$

(Joshipura, Mohanty, PLB **584**, 103 (2004))

- solar neutrinos

$$\frac{\Delta m^2_{\odot}}{4E} \simeq 2 \times 10^{-11} \left(\frac{\mathrm{MeV}}{E}\right) \mathrm{eV}$$

Limits $(\theta_{13} = 0)$ $\alpha_{e\mu} \leq 3.4 \times 10^{-53}$ and $\alpha_{e\tau} \leq 2.5 \times 10^{-53}$

(Bandyopadhyay, Dighe, Joshipura, PRD **75**, 093005 (2007))

stronger than limits from equivalence principle!

Gauged $L_\mu - L_\tau$

never considered for oscillation physics, but very interesting because

$$L_e - L_\tau : m_\nu = \begin{pmatrix} 0 & 0 & a \\ \cdot & b & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \text{ not successful}$$

but $L_\mu - L_\tau$ has zeroth order mass matrix

$$L_\mu - L_\tau : m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$$

masses $a, \pm b$ and $U_{e3} = 0$, $\theta_{23} = \pi/4$

automatically μ–τ symmetric!

flavor $\leftrightarrow$ gauge

Gauged $L_\mu - L_\tau$

$$\mathcal{L} = -\frac{1}{4} Z'_{\mu\nu}\, Z'^{\mu\nu} + \frac{1}{2} M'^2_Z\, Z'_\mu\, Z'^\mu - g'\, j'^\mu\, Z'_\mu - \frac{\sin\chi}{2}\, Z'^{\mu\nu}\, B_{\mu\nu} + \delta M^2\, Z'_\mu\, Z^\mu$$

with new current

$$j'^\mu = \bar{\mu}\,\gamma^\mu\,\mu + \bar{\nu}_\mu\,\gamma^\mu\, P_L\,\nu_\mu - \bar{\tau}\,\gamma^\mu\,\tau - \bar{\nu}_\tau\,\gamma^\mu\, P_L\,\nu_\tau$$

Diagonalizing kinetic and mass terms gives

$$\mathcal{L}_A = -e\,(j_{\mathsf{EM}})_\mu\, A^\mu$$

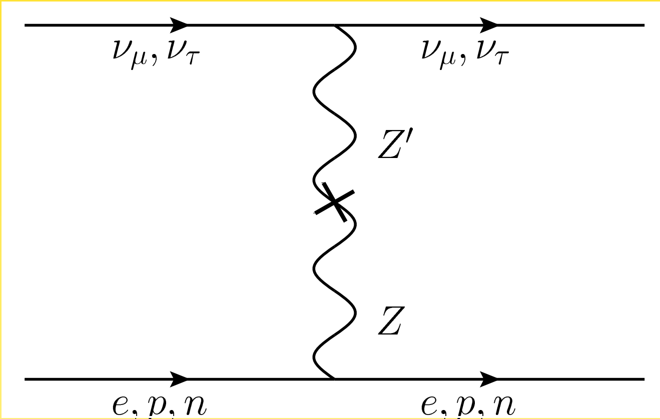
$$\mathcal{L}_{Z_1} = -\left(\frac{e}{s_W\, c_W}\left((j_3)_\mu - s_W^2\,(j_{\mathsf{EM}})_\mu\right) + g'\,\xi\,(j')_\mu\right) Z_1^\mu$$

$$\mathcal{L}_{Z_2} = -\left(g'\,(j')_\mu - \frac{e}{s_W\, c_W}\,(\xi - s_W\,\chi)\left((j_3)_\mu - s_W^2\,(j_{\mathsf{EM}})_\mu\right) - e\, c_W\,\chi\,(j_{\mathsf{EM}})_\mu\right) Z_2^\mu$$

$\Rightarrow$ Z-Z' mixing

Potential through Z-Z' mixing:

$$V = g'\,(\xi - s_W\,\chi)\,\frac{e}{4\,s_W\,c_W}\,\frac{N_n}{4\pi R_{A.U.}} \equiv \alpha\,\frac{e}{4\,s_W\,c_W}\,\frac{N_n}{4\pi R_{A.U.}}$$



With $\eta = 2\,E\,V/\Delta m^2$:

$$\sin^2 2\theta_V = \frac{\sin^2 2\theta}{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 = \Delta m^2\sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

Recall: V changes sign for anti-neutrinos!!

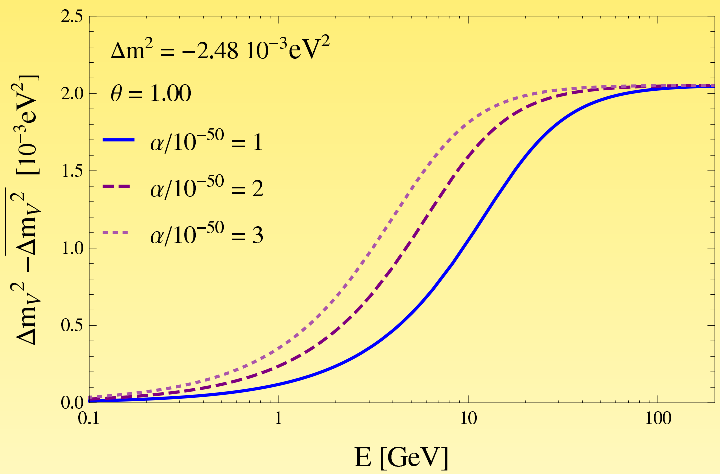
$$\sin^2 2\theta_V = \frac{\sin^2 2\theta}{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 = \Delta m^2 \sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\Delta m_V^2 - \overline{\Delta m_V^2} = \Delta m^2 \sqrt{1 - 4\,\eta\,\cos 2\theta + 4\,\eta^2} - \Delta m^2 \sqrt{1 + 4\,\eta\,\cos 2\theta + 4\,\eta^2}$$

$$\simeq -4\,\eta\,\Delta m^2\,\cos 2\theta$$

$\Rightarrow$ works only with non-maximal θ



Gauged $L_\mu - L_\tau$ and MINOS

First, fit MINOS data with $\alpha = 0$:

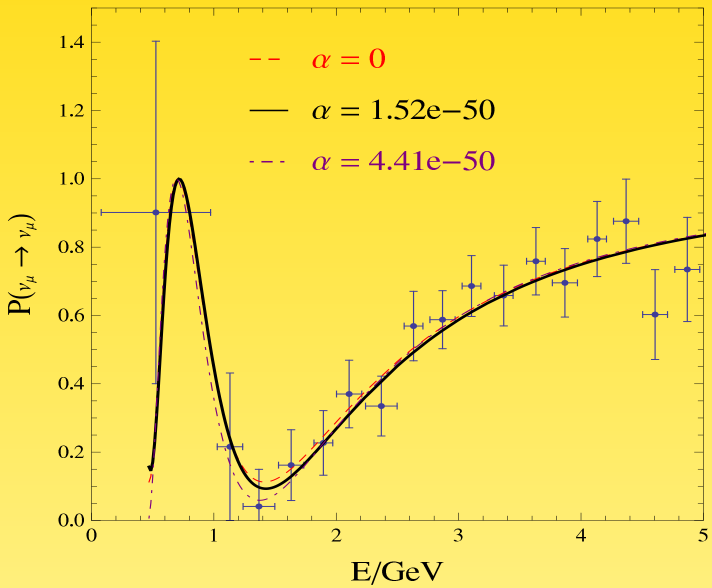
$\Delta m^2 = 2.28 \times 10^{-3}$ eV^2, $\sin^2 2\theta = 0.94$ for neutrinos

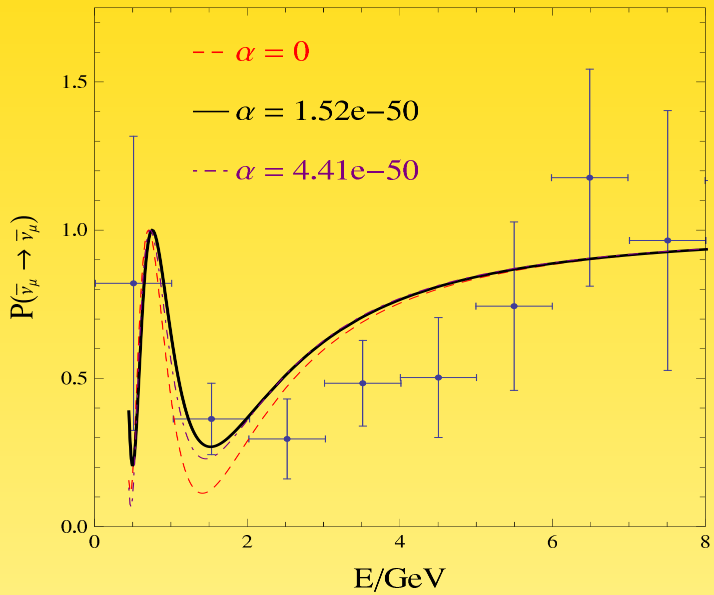
$\overline{\Delta m^2} = 3.38 \times 10^{-3}$ eV^2, $\sin^2 2\overline{\theta} = 0.81$ for anti-neutrinos

to be compared with official MINOS result:

$\Delta m^2 = 2.35 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2 2\theta > 0.91$ for neutrinos

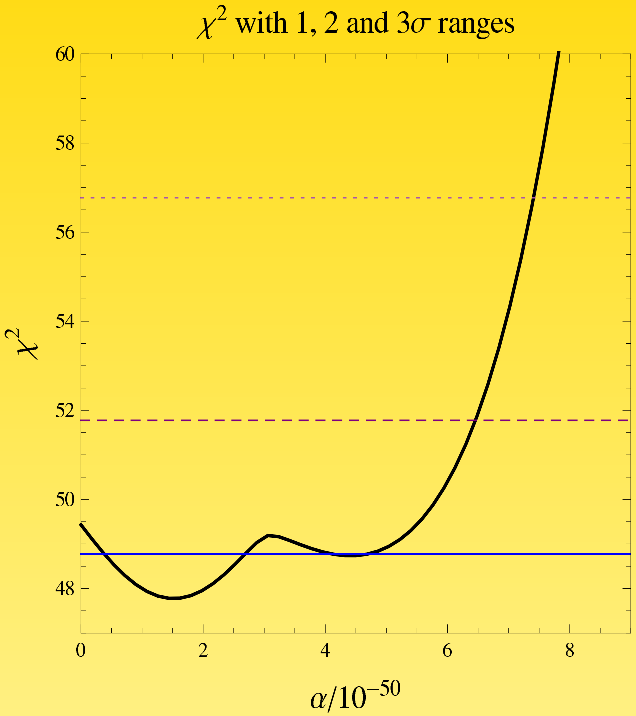
$\overline{\Delta m^2} = 3.36 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2 2\overline{\theta} = 0.86$ for anti-neutrinos

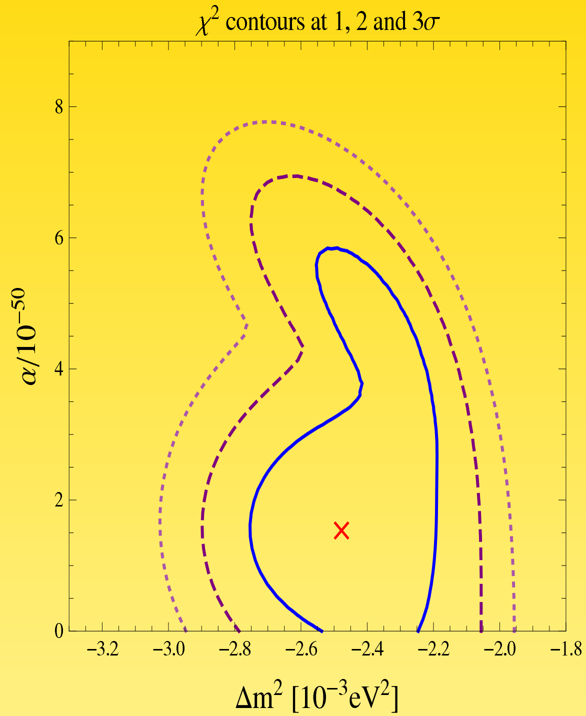




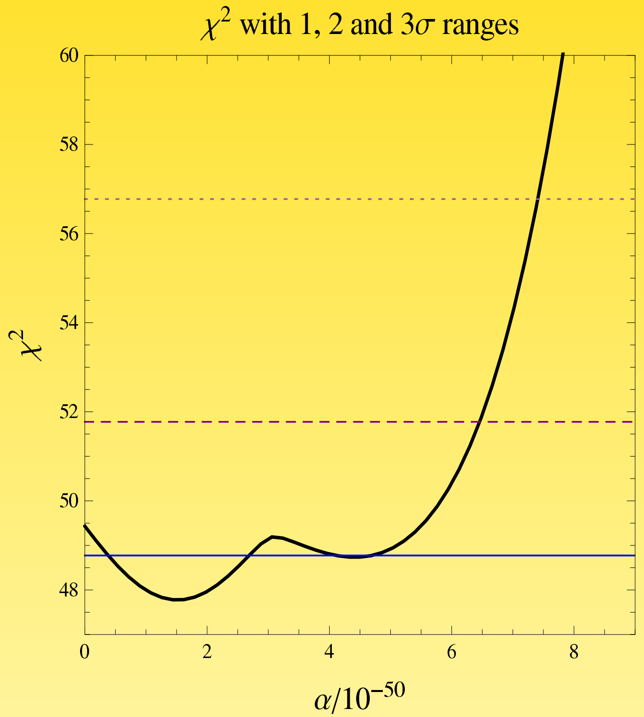
$$\sin^2 2\theta = 0.83\pm0.08\,,\;\; \Delta m^2 = (-2.48\pm0.19)\times10^{-3}\,\mathrm{eV}^2\,,\;\; \alpha = \left(1.52^{+1.17}_{-1.14}\right)\times10^{-50}$$

with $\chi^2_{\min}/N_{\rm dof} = 47.77/50$, (without α: $\chi^2_{\min}/N_{\rm dof} = 49.43/51$)

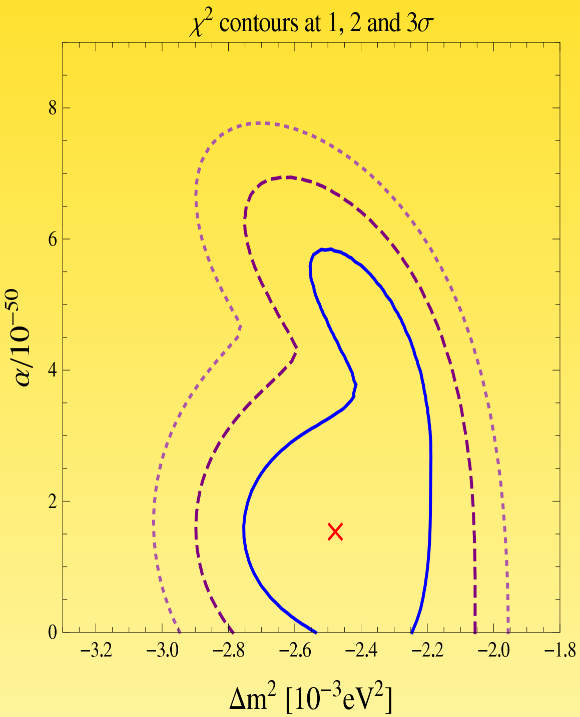




$$P(\theta, \Delta m^2, \alpha) = P(\theta, -\Delta m^2, -\alpha) = P(\theta{+}\pi/2, \Delta m^2, -\alpha) = P(\theta{+}\pi/2, -\Delta m^2, \alpha)$$



χ² with 1, 2 and 3σ ranges
χ²
α/10⁻⁵⁰



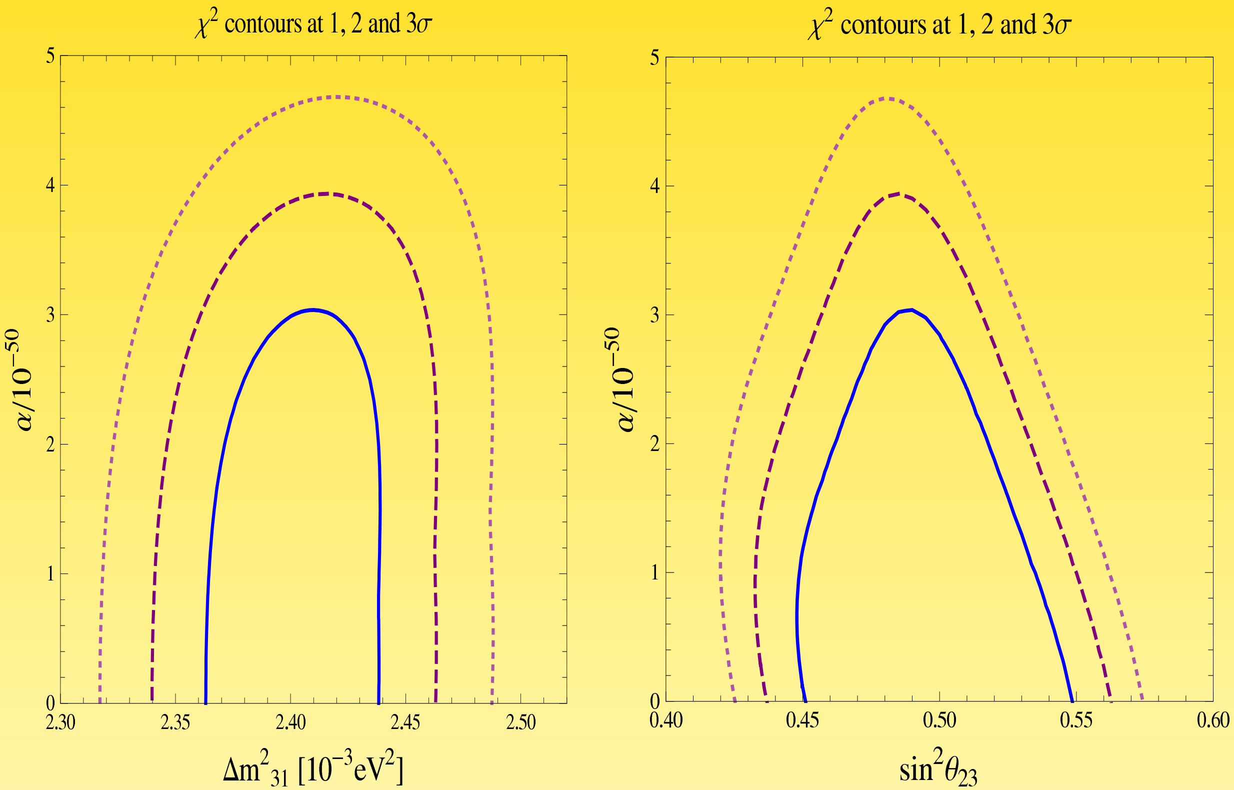
χ² contours at 1, 2 and 3σ
α/10⁻⁵⁰
Δm² [10⁻³eV²]

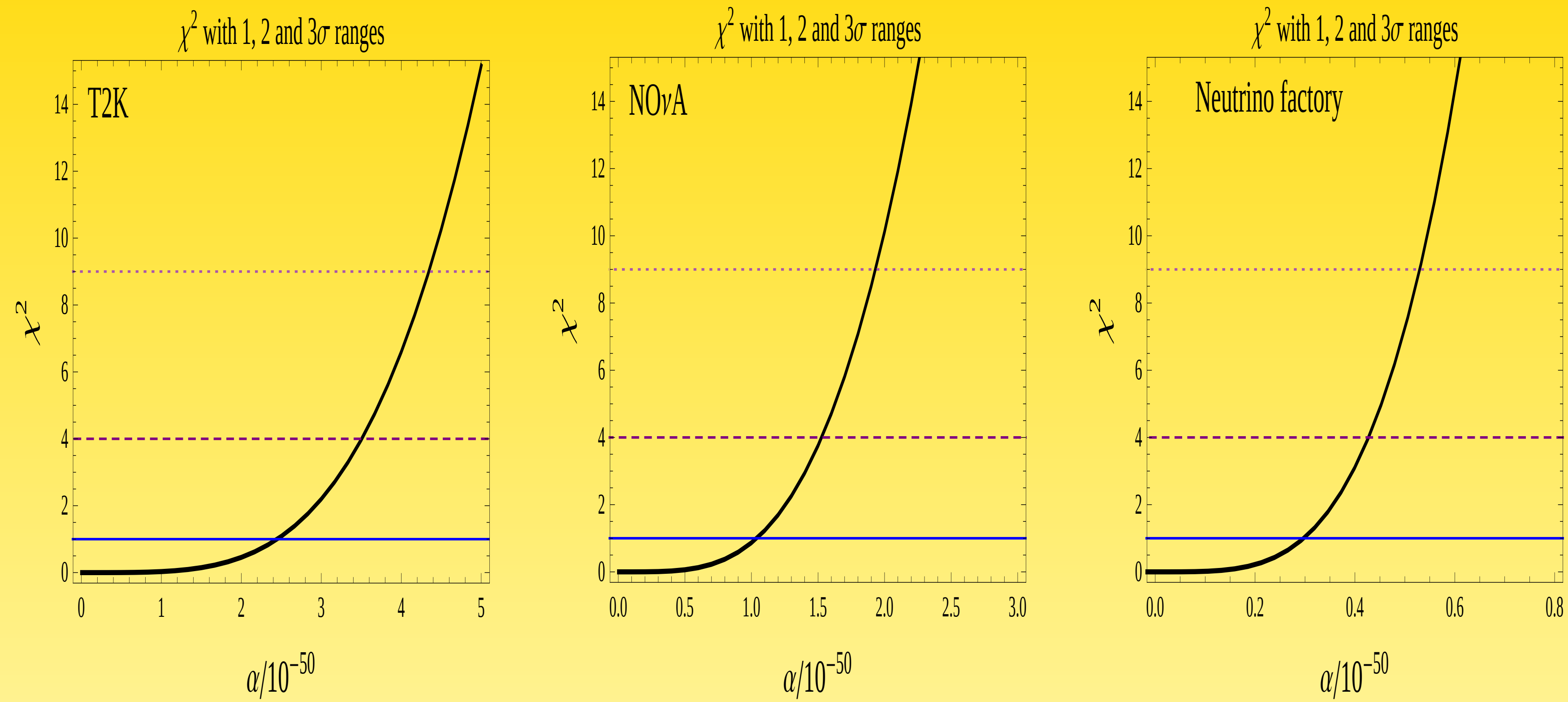
GLoBES:

Experiment	Baseline	Running-time [years]	Beam-energy [GeV]	mass
T2K	295 km	$5\ \nu + 5\ \overline{\nu}$	$0.2 - 2$	22.5 kt
T2HK	$295\,\mathrm{km}$	$4\ \nu + 4\ \overline{\nu}$	$0.4 - 1.2$	$500\,\mathrm{kt}$
SPL	$130\,\mathrm{km}$	$2\ \nu + 8\ \overline{\nu}$	$0.01 - 1.01$	$500\,\mathrm{kt}$
NOνA	812 km	$3\ \nu + 3\ \overline{\nu}$	$0.5 - 3.5$	15 kt
Nufact	3000 km	$4\ \nu + 4\ \overline{\nu}$	$4 - 50$	50 kt

θ_{12}	$\arcsin\sqrt{0.318} \pm 0.02\,(3\%)$
θ_{13}	0 ± 0.2
θ_{23}	$\arcsin\sqrt{0.500} \pm 0.07\,(9\%)$
δ_{CP}	$\in [0, 2\pi]$
$\Delta m^2_{21}\ [10^{-5}\,\mathrm{eV}^2]$	$7.59 \pm 0.23\,(3\%)$
$\Delta m^2_{31}\ [10^{-3}\,\mathrm{eV}^2]$	$2.40 \pm 0.12\,(5\%)$

Correlations (e.g., T2K)





$$\alpha \leq 4.30$$
$$(\alpha = 1.52 \pm 0.46)$$

$$\alpha \leq 1.93$$
$$(\alpha = 1.52 \pm 0.27)$$

$$\alpha \leq 0.53$$
$$(\alpha = 1.52^{+0.11}_{-0.21})$$

Experiment	Sensitivity to $\alpha/10^{-50}$ at 99.73% CL
T2K (ν-run)	11.8
T2K	4.3
T2HK	1.7
SPL	7.5
NOνA	1.9
Combined Superbeams	1.4
Nufact	0.53

Comparison of MINOS result

$$\mathcal{H}_{e\mu} = \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V_{e\mu} & 0 \\ 0 & -V_{e\mu} \end{pmatrix}$$
$$= \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & V_{\mu\tau} \end{pmatrix}$$

and

$$\mathcal{H}_{\mu\tau} = \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -V_{e\tau} \end{pmatrix}$$
$$= \frac{\Delta m^2}{4\,E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} V_{\mu\tau} & 0 \\ 0 & -V_{\mu\tau} \end{pmatrix}$$

okay with atm. limits, in conflict with solar ($\alpha \leq 4 \times 10^{-51}$)

(in Sun n_e not $\propto n_n \ldots$)

- looks in $\mathcal{H}$ like NSI, hence apply NSI limits

$$\alpha = 10^{-50} \Rightarrow |\epsilon^{\oplus}_{\mu\mu}| \simeq 0.25$$

current limit

$$|\epsilon^{\oplus}_{\mu\mu}| \lesssim 0.068 \Rightarrow \alpha \simeq 10^{-51}$$

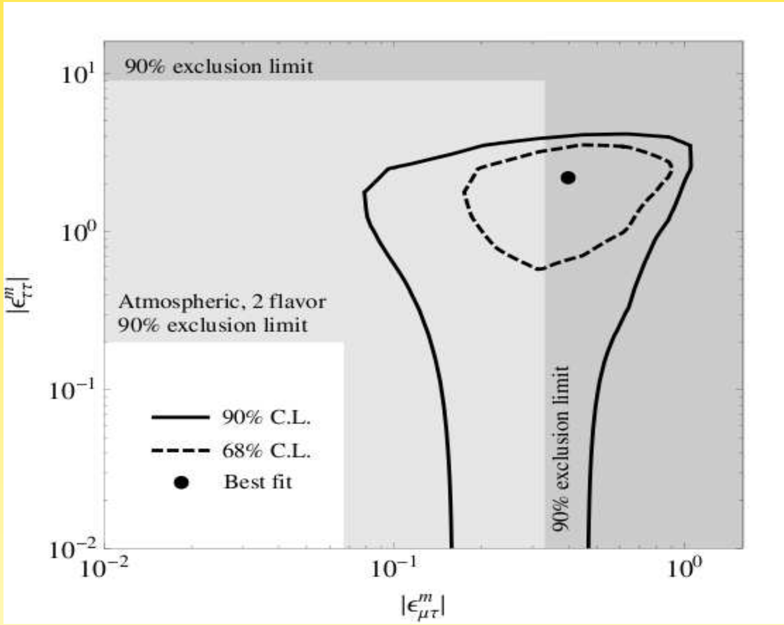
- 3-flavor effects...?

Non-Standard Interactions

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}}\, \epsilon^f_{\alpha\beta} \left(\overline{\nu}_\alpha \gamma_\mu \nu_\beta\right) \left(\overline{f}\gamma^\mu f\right)$$

and $\epsilon_{\alpha\beta} \longrightarrow \epsilon^*_{\alpha\beta}$ for anti-neutrinos

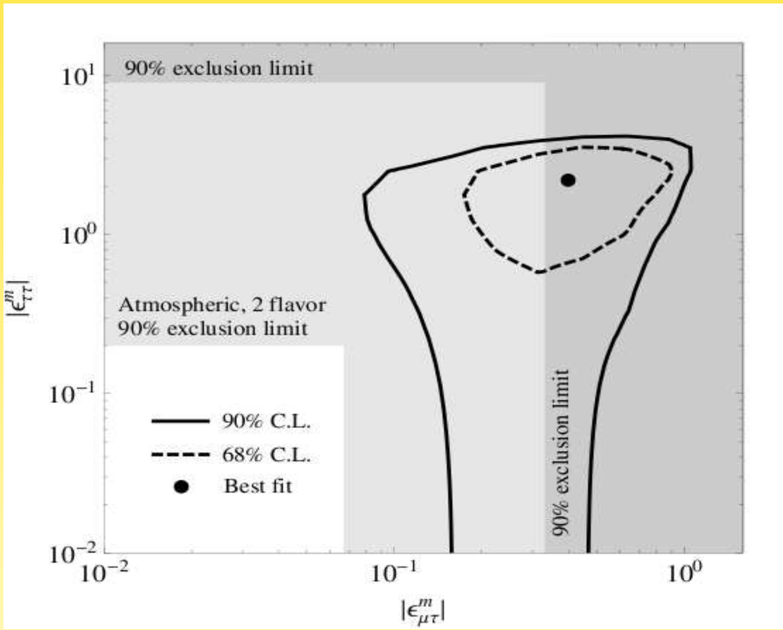
$$\mathcal{H} = \frac{1}{2E}\left[U \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2_{32} \end{pmatrix} U^\dagger + A \begin{pmatrix} \epsilon^\oplus_{\mu\mu} & \epsilon^\oplus_{\mu\tau} \\ \epsilon^{\oplus *}_{\mu\tau} & \epsilon^\oplus_{\tau\tau} \end{pmatrix}\right]$$



Kopp, Machado, Parke, 1009.0014 (only $\epsilon^\oplus_{\mu\tau}$: Mann *et al.*, 1006.5720)

NSIs

$$\mathcal{H} = \frac{1}{2E}\left[U\begin{pmatrix}0 & 0\\ 0 & \Delta m^2_{32}\end{pmatrix}U^\dagger + A\begin{pmatrix}\epsilon^{\oplus}_{\mu\mu} & \epsilon^{\oplus}_{\mu\tau}\\ \epsilon^{\oplus *}_{\mu\tau} & \epsilon^{\oplus}_{\tau\tau}\end{pmatrix}\right]$$

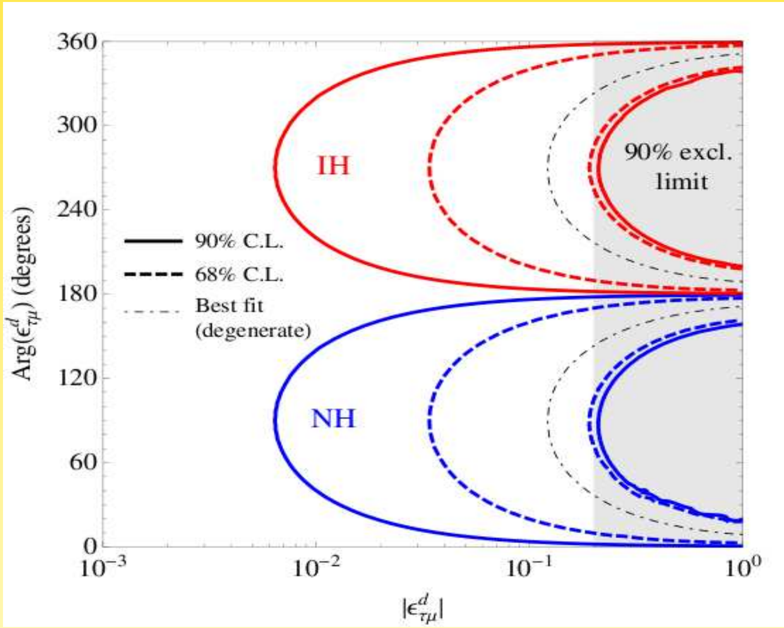


Charged Current NSIs

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{u}\gamma^\mu d]\, [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

leads to interference of

$\nu_\mu \rightsquigarrow \nu_\tau + N \to X + \mu$ and $\nu_\mu + N \to X + \mu$



Kopp, Machado, Parke, 1009.0014

Gauge Invariance strikes back!

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{u}\gamma^\mu d]\, [\bar{\mu}\gamma_\mu P_L \nu_\tau]$$

gives 1-loop diagram for $\tau \to \mu\, \pi^0$: $|\epsilon^d_{\tau\mu}| \leq 0.2$

`Kopp, Machado, Parke, 1009.0014`

BUT: gauge invariant term

$$\mathcal{L}_{\rm NSI} \supset -2\sqrt{2}\, G_F\, \epsilon^d_{\tau\mu}\, V_{ud}\, [\bar{U}\gamma^\mu D]\, [\bar{L}_\mu \gamma_\mu L_\tau]$$

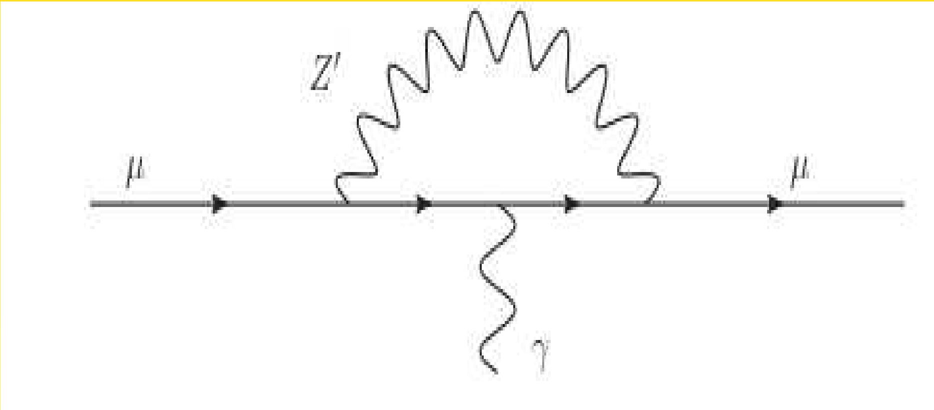
gives tree-level diagram for $\tau \to \mu\, \pi^0$: $|\epsilon^d_{\tau\mu}| \leq 10^{-4}$

`Gavela, Talk@NOW2010`

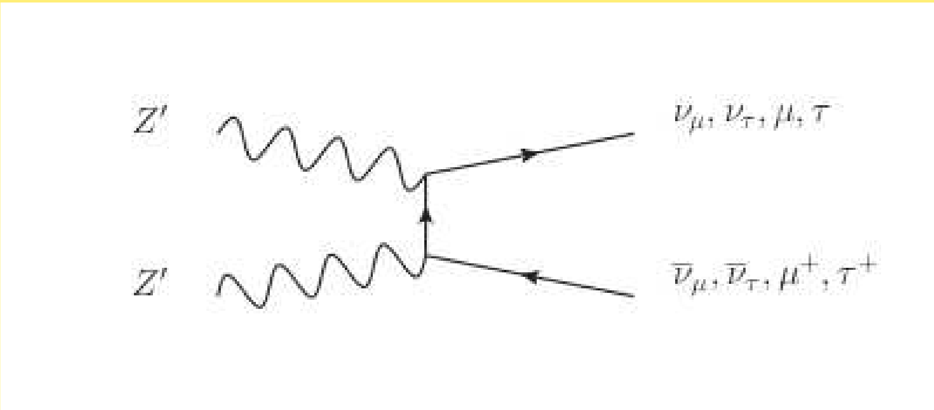
$\Leftrightarrow$ **this argument does <u>not</u> apply to gauged $U(1)$!**

Other aspects/limits of $L_\mu - L_\tau$

- $\Delta a_\mu = {g'}^2/(8\pi^2)$



- BBN: $\Gamma(Z'\,Z' \to \nu_{\mu,\tau}\,\nu_{\mu,\tau}) \propto g'^2\,T \Rightarrow g' \lesssim 10^{-5}$



- other EW precision: there are only $\sim 10^8\ Z\ldots$

Other aspects/limits of $L_\mu - L_\tau$

- coupling of Z' with electromagnetic current gives modified charge

$$\frac{Q(\mu^+)}{Q(e^+)} \simeq 1 + \frac{g'}{e}\left((\xi - s_W\,\chi)(\frac{1}{4} - s_W^2)/(s_W\,c_W) + c_W\chi\right)$$

measured to be 1 ± 10^{-9}

Equivalence principle is violated:

$$V(r) = \frac{e\,(\xi - s_W\,\chi)}{4\,s_W\,c_W}\,N_n\,\frac{e^{-rM_2}}{4\pi\,r}$$

gravitational potential between 2 bodies with neutron content N_{n_1} and N_{n_2}:

$$V_{\rm grav}(r) = -G_N\frac{m_1\,m_2}{r}\left(1 - \left(\frac{e\,(\xi - s_W\,\chi)}{4\,s_W\,c_W}\right)^2\frac{N_{n_1}}{m_1}\frac{N_{n_2}}{m_2}\frac{1}{4\pi\,G_N}e^{-rM_2}\right)$$

Use the limits from `Adelberger` *et al.*, `PPNP` **62**, `102 (2009)` who analyze

$$V_{\rm grav}(r) = -G_N\frac{m_1\,m_2}{r}\left(1 + \tilde{\alpha}\frac{N_{n_1}}{\mu_1}\frac{N_{n_1}}{\mu_2}e^{-r/\lambda}\right)$$

this gives limits depending on range:

$$\alpha/g' \leq 5\times10^{-24} \quad \text{Sun-Earth}$$
$$\alpha/g' \leq 1\times10^{-22} \quad \text{Earth}$$

$\Rightarrow$ neutrinos give best limits on leptonic fifth forces :-)

Other aspects/limits of $L_\mu - L_\tau$

- Neutrino masses tend to be quasi-degenerate

$$m_\nu = \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$$

breaking generates both $m_{Z'}$ and non-zero entries in m_ν

$m_{Z'} \sim g' \langle \Phi' \rangle$ and e.g., $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle^2/\Lambda$ or $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle \langle \Phi \rangle/\Lambda$ or $(m_\nu)_{\alpha\beta} \lesssim \langle \Phi' \rangle$

- neutral scalar χ is present, $m_\chi \simeq \lambda \langle \Phi \rangle$, with dangerous $Z \to Z' \chi$

- if heavy Z':
  - integrate out to get $\epsilon_{\mu\mu} = -\epsilon_{\tau\tau} \propto (1/M_Z')^2$
  - cf. with $\tau \to 3\mu$, a_μ,...
  - $\mu^+\mu^-$ collider

$L_\mu - L_\tau$ and Dark Matter

Baek, Ko, JCAP **0910**, 011 (2009)

add Dirac fermion ψ charged under $L_\mu - L_\tau$

relic density from $\psi\bar{\psi} \to Z' \to (\mu^+\mu^-, \tau^+\tau^-, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau)$

annihilation as well

annihilation is automatically leptophilic $\leftrightarrow$ PAMELA frenzy

CPT Violation

Obvious (and most drastic!!) "explanation" of MINOS (MiniBooNE, solar/KamLAND. . .)

Question: if CPT is violated, can particles be their own anti-particles, i.e., is lepton number violated???

What about Dirac/Majorana??

Barenboim, Beacom, Borissov, Kayser, PLB **537** (2002) 227

Define $\xi = \text{CPT}$

CPT properties are $\xi|\nu\rangle = e^{i\zeta}\,|\overline{\nu}\rangle$ and $\xi|\overline{\nu}\rangle = e^{i\zeta}\,|\nu\rangle$

A Majorana particle is defined as

$$\xi|\nu\rangle = e^{i\zeta}\,|\nu\rangle$$

Now we introduce CPT-violation in a one-family example

$$M_\nu = \begin{pmatrix} \mu + \Delta & y^* \\ y & \mu - \Delta \end{pmatrix} \text{ for basis } \nu, \overline{\nu}$$

y mixes ν and $\overline{\nu}$ and hence L is violated

diagonalize M_ν to get eigenstates m_+, m_- and mixing $\tan 2\theta = \frac{|y|}{\Delta}$

$$|\nu_+\rangle = \cos\theta\,|\nu\rangle + e^{i\phi}\,\sin\theta\,|\overline{\nu}\rangle\ ,\quad m_+ = \mu + \sqrt{|y^2| + \Delta^2}$$

$$|\nu_-\rangle = -\sin\theta\,|\nu\rangle + e^{i\phi}\,\cos\theta\,|\overline{\nu}\rangle\ ,\quad m_- = \mu - \sqrt{|y^2| + \Delta^2}$$

Mixing angle $\tan 2\theta = \frac{|y|}{\Delta}$

$$\xi|\nu_+\rangle = e^{i(\zeta-\phi)}\left(\sin\theta\,|\nu\rangle + e^{i\phi}\,\cos\theta\,|\overline{\nu}\rangle\right)$$

$$\xi|\nu_-\rangle = -e^{i(\zeta-\phi)}\left(-\cos\theta\,|\nu\rangle + e^{i\phi}\,\sin\theta\,|\overline{\nu}\rangle\right)$$

$\Rightarrow$ $\nu_\pm$ are Majorana particles if and only if $\theta = \pi/4$

but $\theta = \pi/4$ means $\Delta = 0$ and thus CPT conservation:-)

if $\Delta \neq 0$: CPT is violated: neutrinos are no longer Majorana fermions;

if $y \neq 0$: L is violated and $0\nu\beta\beta$ can occur

$\Rightarrow$ observation of $0\nu\beta\beta$ implies non-zero y but not that neutrinos are CPT self-conjugate

Neutrino-less double beta decay?

amplitude is

$$\mathcal{A} \propto \sum m_i \, U_{ei} \, \overline{U}_{ei}$$

$$= m_+ \, U_{+\nu} \, U_{+\overline{\nu}} + m_- \, U_{-\nu} \, U_{-\overline{\nu}}$$

$$= m_+ \, \cos\theta \, e^{i\phi} \, \sin\theta + m_- \, (-\sin\theta) \, e^{i\phi} \, \cos\theta$$

$$\propto (m_+ - m_-) \, \cos\theta \, \sin\theta = y \neq U_{ei}^2 \, m_i$$

in general 3-flavor case, also Δ will appear

CPT counter part of $0\nu\beta\beta$ gives same result :-)

phenomenology of 0$\nu\beta\beta$ different from usual picture

Summary

It's very hard to explain MINOS data[a]

[a] Also see Osamu's talk why all solutions presented so far probably don't work. . .

Derivation of Potential

Consider the time-like components, note that $j^0_{\mathsf{EM}} = 0$ and use

$$j_3^0 = -\frac{1}{2}\,\bar{e}_L\,\gamma^0\,e_L + \frac{1}{2}\,\bar{p}_L\,\gamma^0\,p_L - \frac{1}{2}\,\bar{n}_L\,\gamma^0\,n_L = -\frac{1}{4}\,(n_e - n_p + n_n) = -\frac{n_n}{4}$$

since the axial-part will result in a spin-operator in the non-relativistic limit and we assume the Sun is not polarized. The equation of motion for Z_2^0, following from the Euler-Lagrange equation

$$\partial_\nu \frac{\delta}{\delta(\partial_\nu\,Z_{2\mu})}\left(-\frac{1}{4}\,Z_{2\alpha\beta}\,Z_2^{\alpha\beta}\right) - \frac{\delta}{\delta\,Z_{2\mu}}\left(\frac{1}{2}M_2^2\,Z_{2\alpha}\,Z_2^\alpha + \mathcal{L}_{Z_2}\right) = 0$$

is therefore

$$(\partial^2 + M_2^2)\,Z_2^0 = (\xi - s_W\,\chi)\frac{e}{s_W\,c_W}\frac{n_n}{4}$$

In the static case outside of the Sun this is $(n_n(\vec{x}) = N_n\,\delta^{(3)}(\vec{x}))$:

$$(\Delta - M_2^2)\,Z_2^0 = -(\xi - s_W\,\chi)\frac{e}{s_W\,c_W}\frac{1}{4}N_n\,\delta^{(3)}(\vec{x})$$

with the well-known solution

$$V(r) = Z_2^0 = (\xi - s_W\,\chi)\frac{e}{s_W\,c_W}\frac{1}{4}N_n \times \frac{e^{-rM_2}}{4\pi\,r}$$

In the limit $M_2 \to 0$ the potential, for ν_μ and ν_τ respectively, on Earth is:

$$V_{\mu,\tau} = \pm\, g'\,(\xi - s_W\,\chi)\frac{e}{4\,s_W\,c_W}\frac{N_n}{4\pi R_{\mathrm{A.U.}}}$$