

# NuFact10

12th International Workshop on Neutrino Factories, Superbeams and Beta Beams  
October 20-25, 2010  
Tata Institute of Fundamental Research - Mumbai

## GiBUU and Neutrino-Nucleus scattering

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# NuFact10

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## NEUTRINO-NUCLEUS INTERACTIONS IN A COUPLED-CHANNEL HADRONIC TRANSPORT MODEL

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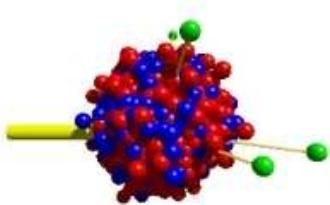
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Dissertation

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Institut für Theoretische Physik

2009

# Gi what?



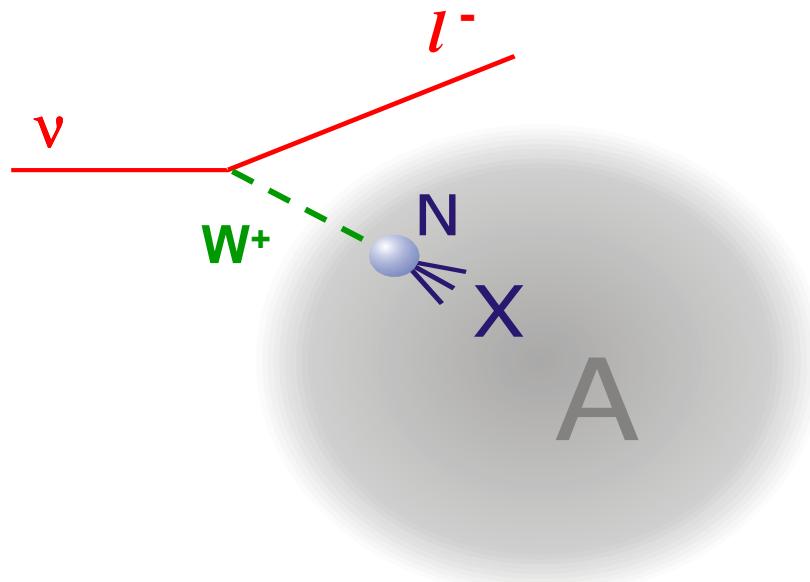
**GiBUU**

The Giessen Boltzmann-Uehling-Uhlenbeck Project

Institut für Theoretische Physik, JLU Giessen

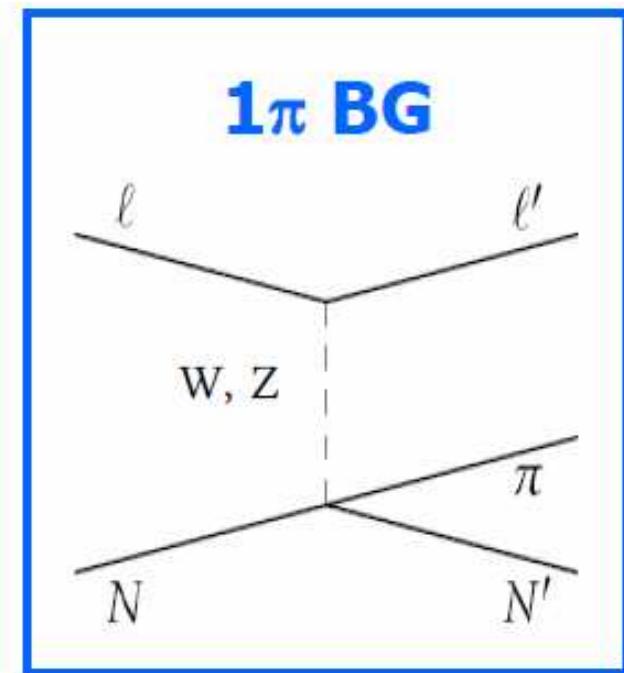
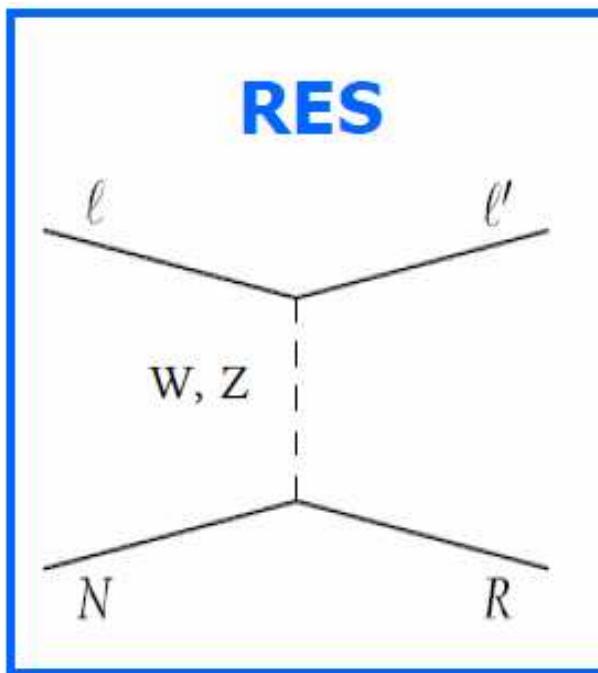
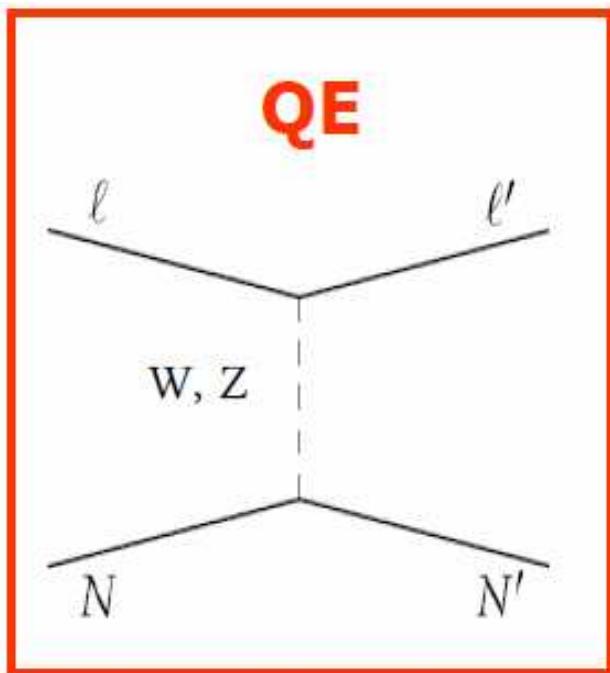
- **GiBUU**: semiclassical transport model in coupled channels
  - Heavy ion collisions
  - pA,  $\pi$ A reactions
  - $\gamma$ A, eA scattering
  - $\nu$ A reactions
    - inclusive and exclusive
    - no new parameters
- Information & code: <http://gibuu.physik.uni-giessen.de/>

# Ingredients



1. Primary interaction:  $\nu_l N \rightarrow l^- (\nu_l) X$
2. Medium modifications
3. Propagation of the final state (**FSI**)

# Elementary $\nu N$ interactions



1. Primary interaction:  $\nu_l N \rightarrow l^- (\nu_l) X$ 
  - on a single nucleon (Impulse Approximation)

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X(p')$

- Cross section:

$$\frac{d\sigma_X}{d\omega d\Omega_{k'}} = \frac{|\vec{k}'|}{32\pi^2} \frac{\mathcal{A}(p'^2)}{[(k \cdot p)^2 - m_l^2 M_N^2]^{1/2}} |\bar{\mathcal{M}}_X|^2$$

- X=R:  $\mathcal{A}(p'^2) = \frac{\sqrt{p'^2}}{\pi} \frac{\Gamma(p')}{(p'^2 - M_R^2)^2 + p'^2 \Gamma^2(p')}$

- X=N:  $\mathcal{A}(p'^2) = \delta(p'^2 - M_N^2)$

- Matrix element:  $|\bar{\mathcal{M}}_X|^2 = C^2 L_{\alpha\beta} H^{\alpha\beta}$

$$C_{EM} = \frac{4\pi\alpha}{q^2}, \quad C_{CC} = \frac{G_F \cos \theta_C}{\sqrt{2}}, \quad C_{NC} = \frac{G_F}{\sqrt{2}}$$

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X(p')$

- Hadronic tensor:  $H^{\alpha\beta} \Rightarrow J_X^\alpha$
- $X = \text{nucleon}$
- $X = \text{resonance with } M_R < 2 \text{ GeV}$ 
  - Spin 1/2:  $P_{11}(1440), S_{11}(1535), S_{31}(1620), S_{11}(1650), P_{31}(1910)$

$$J_{1/2}^\mu = \left[ \frac{(Q^2 \gamma^\mu + q^\mu q^\mu)}{2M_N^2} F_1^V + \frac{i}{2M_N} \sigma^{\mu\alpha} q_\alpha F_2^V + \gamma^\mu \gamma_5 F_A + \frac{q^\mu \gamma_5}{M_N} F_P \right] \begin{Bmatrix} 1 \\ \gamma_5 \end{Bmatrix}$$

- Spin 3/2:  $P_{33}(1232), D_{13}(1520), D_{33}(1700), P_{13}(1720)$

$$J_{3/2}^{\alpha\mu} = \left[ \frac{C_3^V}{M_N} (g^{\alpha\mu} q^\mu - q^\alpha \gamma^\mu) + \frac{C_4^V}{M_N^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) + \frac{C_5^V}{M_N^2} (g_{\alpha\mu} q \cdot p - q^\alpha p^\mu) \right. \\ \left. + \left( \frac{C_3^A}{M_N} (g^{\alpha\mu} q^\mu - q^\alpha \gamma^\mu) + \frac{C_4^A}{M_N^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M_N^2} q^\alpha q^\mu \right) \gamma_5 \right] \begin{Bmatrix} \gamma_5 \\ 1 \end{Bmatrix}$$

- Spin > 3/2:  $D_{15}(1675), F_{15}(1680), F_{35}(1905), F_{37}(1950)$ 
  - Treated as spin 3/2
  - negligible contributions to c.s.

# Elementary $\nu N$ interactions

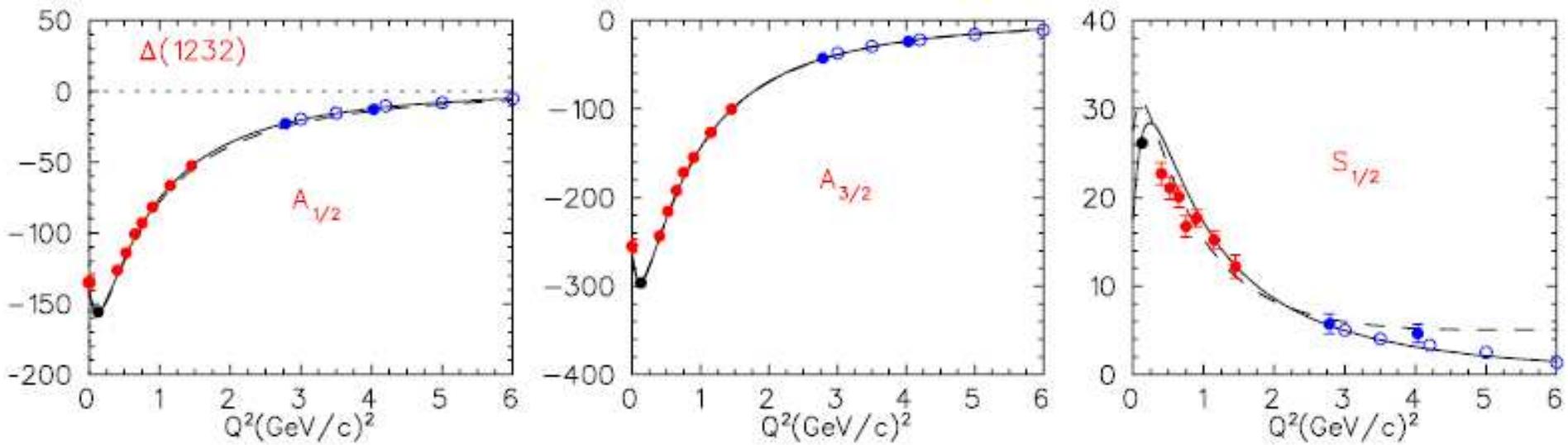
- $l(k) + N(p) \rightarrow l'(k') + X(p')$ 
  - N-R **vector** form factors:
  - **MAID** Drechsel, Kamalov, Tiator, EPJA 34 (2007) 69

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 1/2 | \epsilon_\mu^+ J_{\text{EM}}^\mu | N, J_z = -1/2 \rangle \zeta$$
$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 3/2 | \epsilon_\mu^+ J_{\text{EM}}^\mu | N, J_z = 1/2 \rangle \zeta$$
$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \langle R, J_z = 1/2 | \epsilon_\mu^0 J_{\text{EM}}^\mu | N, J_z = 1/2 \rangle \zeta$$

- Helicity amplitudes  $\Rightarrow$  Vector form factors

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X(p')$ 
  - N-R vector form factors:
  - MAID Drechsel, Kamalov, Tiator, EPJA 34 (2007) 69
    - Example: N- $\Delta(1232)$



- Helicity amplitudes  $\Rightarrow$  Vector form factors

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X(p')$ 
  - N-R axial form factors:
    - PCAC
    - $\pi$ -pole dominance of the pseudoscalar form factor
  - Spin 1/2:

$$F_P(Q^2) = \frac{(M_R \pm M_N)M_N}{Q^2 + m_\pi^2} F_A(Q^2)$$

$$F_A(0) = -C_I \sqrt{2} f_\pi \frac{f}{m_\pi} \quad C_I = \begin{cases} \sqrt{2}, & I = 1/2 \\ -\sqrt{1/3}, & I = 3/2 \end{cases}$$

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 + \frac{Q^2}{M_{AR}^2}\right)} \quad M_{AR} = 1 \text{ GeV}$$

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X(p')$ 
  - N-R axial form factors:
    - PCAC
    - $\pi$ -pole dominance of the pseudoscalar form factor

## ■ Spin 3/2:

$$C_6^A(Q^2) = \frac{M_N^2}{Q^2 + m_\pi^2} C_5^A(Q^2)$$

$$C_5^A(0) = -C_I \sqrt{2} f_\pi \frac{f}{m_\pi} \quad \begin{aligned} C_I &= \sqrt{2}, \quad I = 1/2 \\ C_I &= -\sqrt{1/3}, \quad I = 3/2 \end{aligned}$$

$$C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_{AR}^2}\right)} \quad f/m_\pi \leftarrow \text{RN}\pi \text{ coupling}$$

$$M_{AR} = 1 \text{ GeV}$$

Except for the  $\Delta(1232)$   
Fit to ANL

$$\blacksquare \text{ Adler: } C_4^A = -\frac{1}{4} C_5^A \quad C_3^A = 0$$

# Elementary $\nu N$ interactions

- $l(k) + N(p) \rightarrow l'(k') + X$

  - N-R axial form factors:

    - PCAC

    - $\pi$ -pole dominance of the

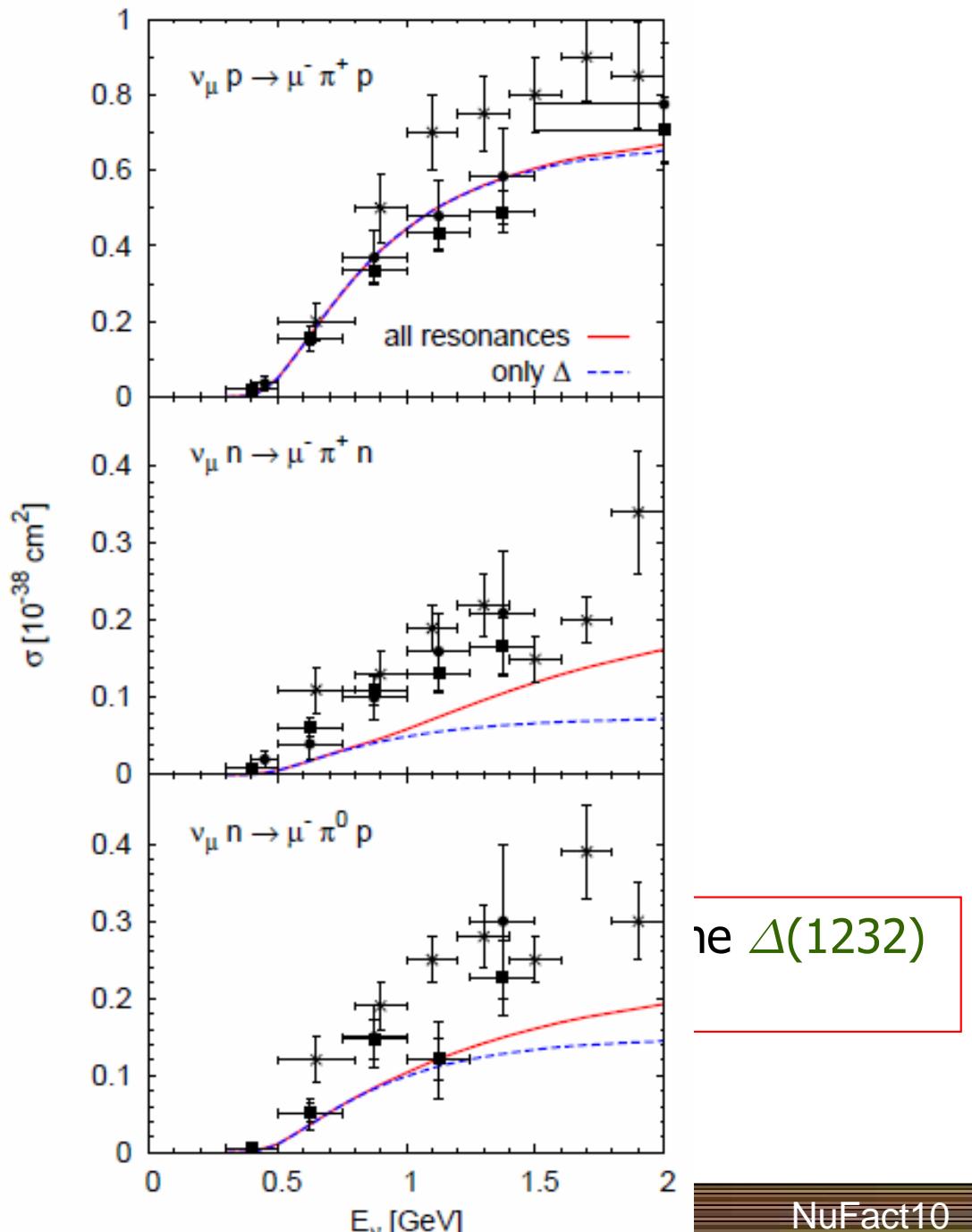
  - Spin 3/2:

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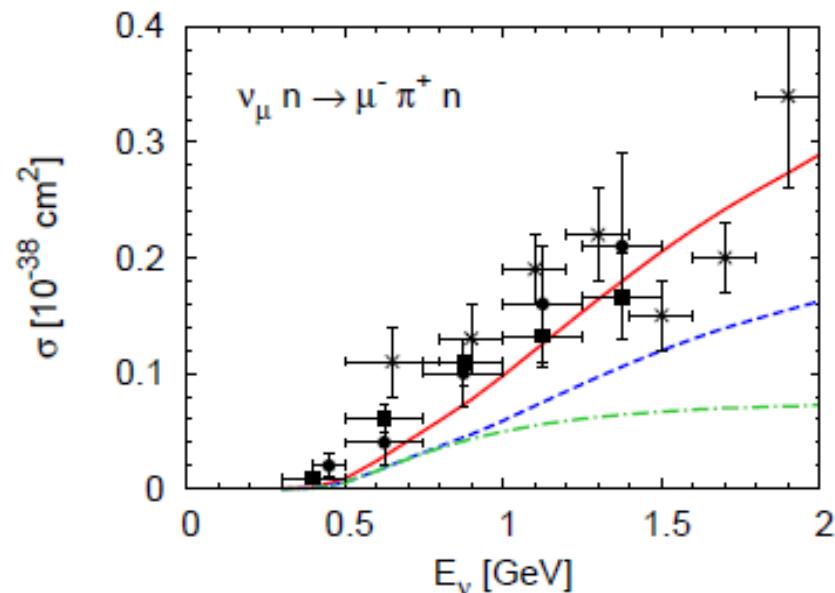
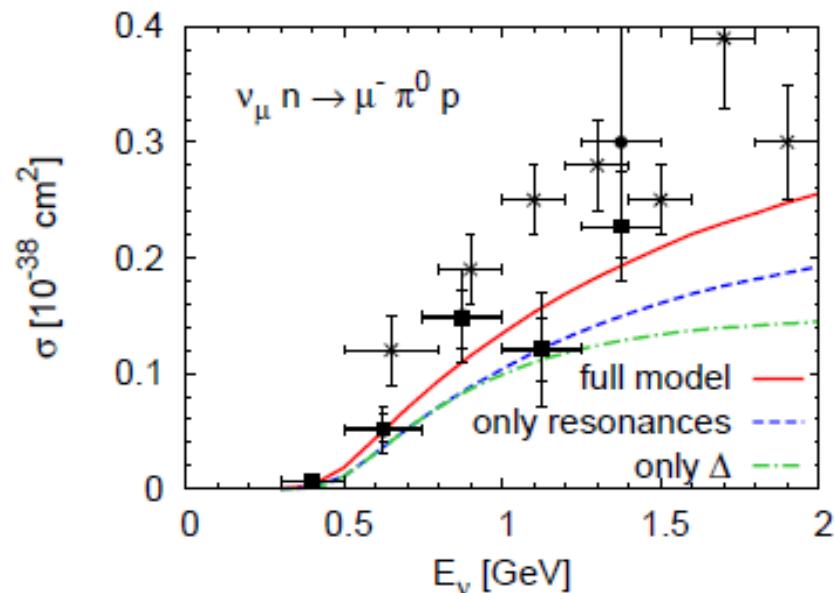
# Elementary $\nu N$ interactions

- $\ell(k) + N(p) \rightarrow \ell'(k') + X(p')$

- Non-resonant  $\pi$  background:

$$d\sigma_{BG} = (1 + b^{\pi N}) d\sigma_{BG}^V$$

- Vector part: obtained from  $e N \rightarrow e' N' \pi$  amplitudes (**MAID**) subtracting resonance contribution
- Axial part:
  - same structure as in the vector part assumed
  - constant  $b^{\pi N}$  fitted to **ANL** data



# $\nu$ N in the nuclear medium

## ■ Local Relativistic Fermi Gas

$$p_F(r) = [\frac{3}{2}\pi^2 \rho(r)]^{1/3}$$

### ■ Fermi Motion of initial nucleons:

$$f(\vec{r}, \vec{p}) = \Theta(p_F(r) - |\vec{p}|)$$

### ■ Pauli blocking of final nucleons:

$$P_{\text{Pauli}} = 1 - \Theta(p_F(r) - |\vec{p}|)$$

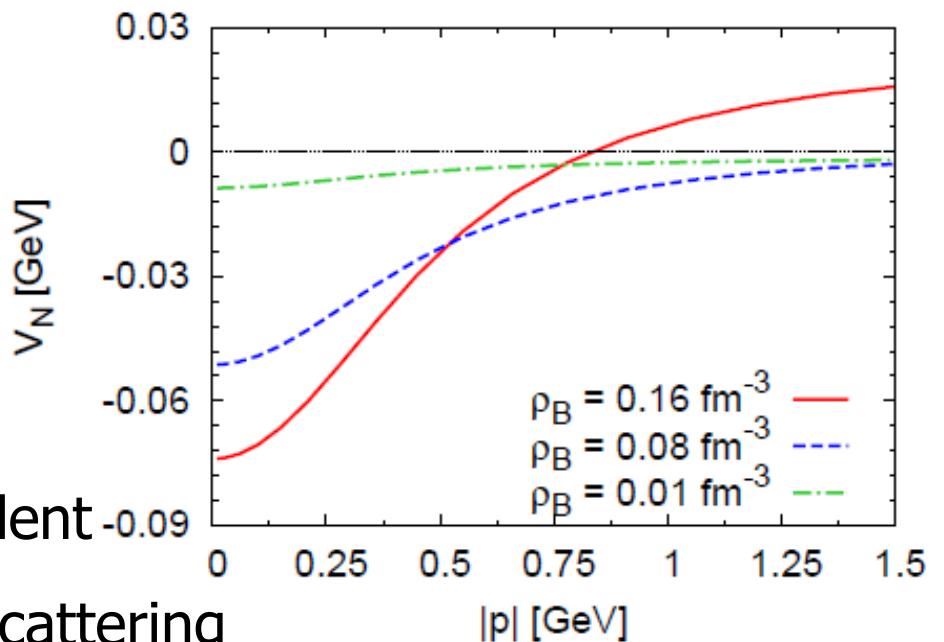
### ■ Mean field potential

■ Density and momentum dependent

■ Parameters fixed in p-Nucleus scattering

■ Nucleons acquire **effective masses**

$$M_{\text{eff}} = M + U(\vec{r}, \vec{p})$$



# $\nu N$ in the nuclear medium

## ■ Spectral functions

$$\blacksquare S(p) = -\frac{1}{\pi} \frac{\text{Im}\Sigma(p)}{[p^2 - M^2 - \text{Re}\Sigma(p)]^2 + [\text{Im}\Sigma(p)]^2}$$

$\Sigma \leftarrow$  selfenergy

## ■ Hole spectral function:

- The correlated part of  $S_h$  is neglected

$$\text{Im}\Sigma \approx 0 \quad S_h(p) \rightarrow \delta(p^2 - M_{\text{eff}}^2)$$

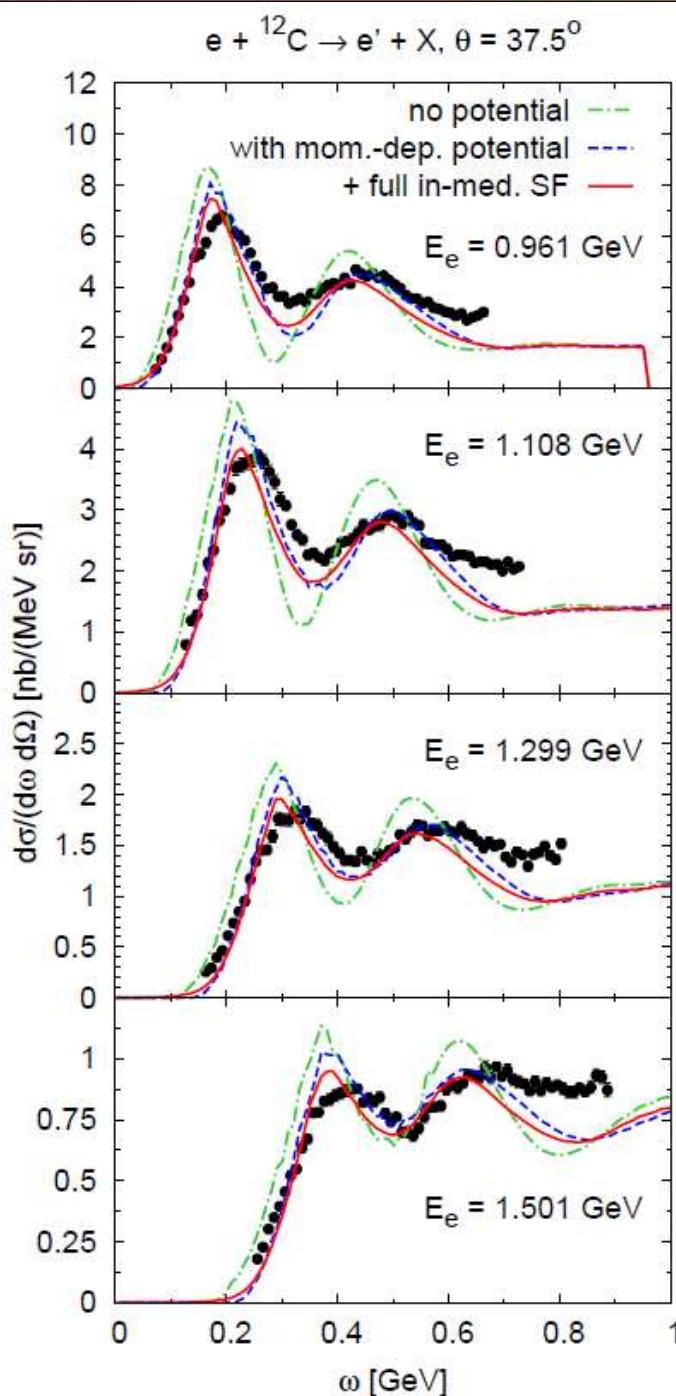
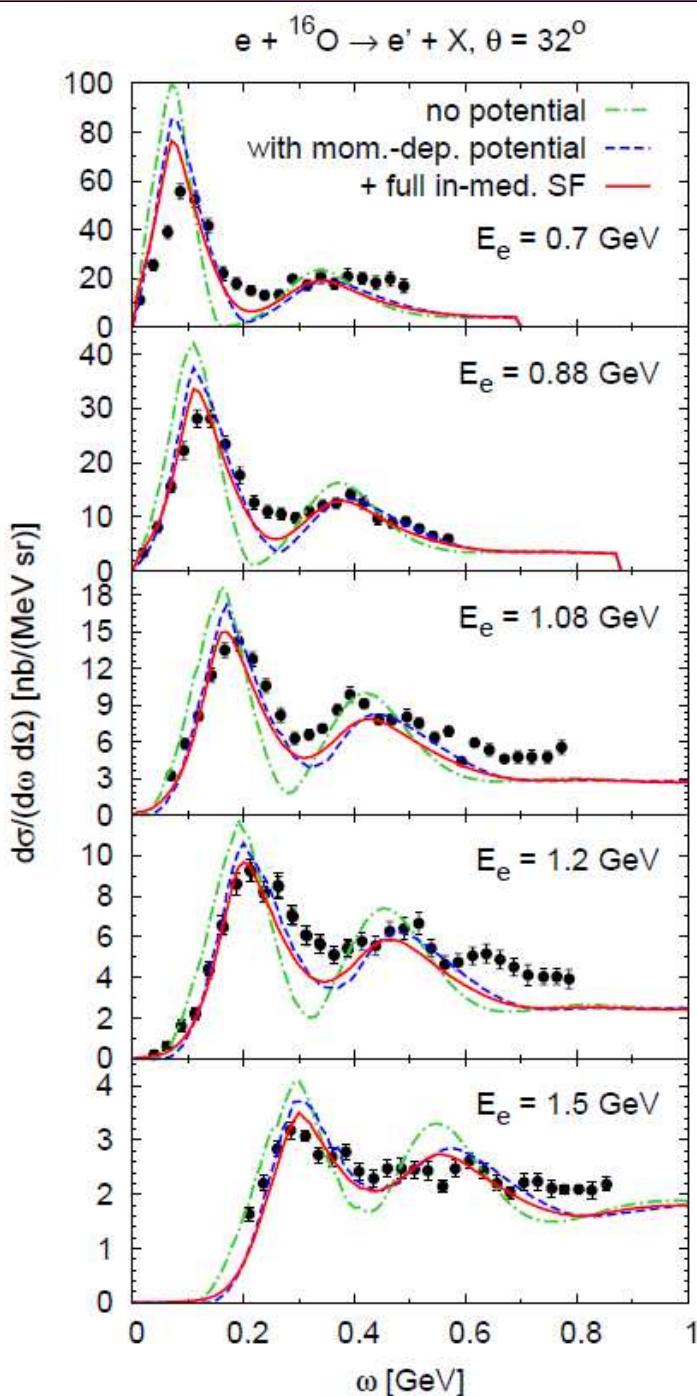
## ■ Particle spectral function:

- $\text{Im}\Sigma = -\sqrt{(p^2)}\Gamma_{\text{coll}}(p, r)$ ,  $\Gamma_{\text{coll}} = \langle \sigma_{XN} v_{\text{rel}} \rangle$  ← collisional broadening

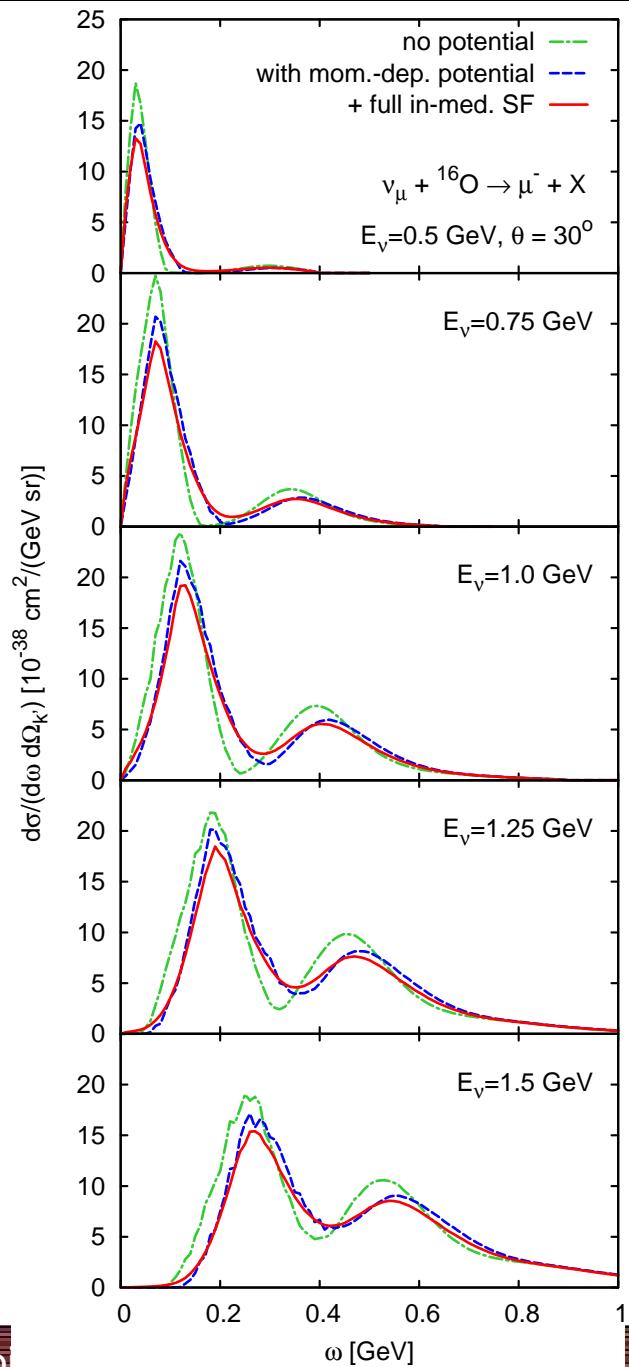
- $\text{Re}\Sigma$  is obtained from  $\text{Im}\Sigma$  with a dispersion relation fixing the pole position:

$$p_0^{(\text{pole})} = \sqrt{\vec{p}^2 + M_{\text{eff}}^2}$$

# Inclusive $(e, e')$ cross section



# Inclusive $(\nu, \mu^-)$ cross section



# FSI: transport model

- The time evolution of phase space density  $f_i(\vec{r}, \vec{p}, t)$  ( $i=N, \Delta, \pi, \rho, \dots$ ) is determined by the **Boltzmann-Uehling-Uhlenbeck** (BUU) equation.

$$\frac{df_i}{dt} = \left( \partial_t + (\nabla_{\vec{p}} H) \nabla_{\vec{r}} - (\nabla_{\vec{r}} H) \nabla_{\vec{p}} \right) f_i(\vec{r}, \vec{p}, t) = I_{coll} [f_i, f_N, f_\pi, f_\Delta, \dots]$$

- **Hamiltonian:**  $H = \sqrt{(m_i + U)^2 + \vec{p}^2}$
- Equations coupled mainly through the **collision integral**

- Accounts for changes in  $f_i(\vec{r}, \vec{p}, t)$
- Elastic & inelastic processes
- Decay of unstable particles
- Pauli blocking

- Most important processes:



# FSI: transport model

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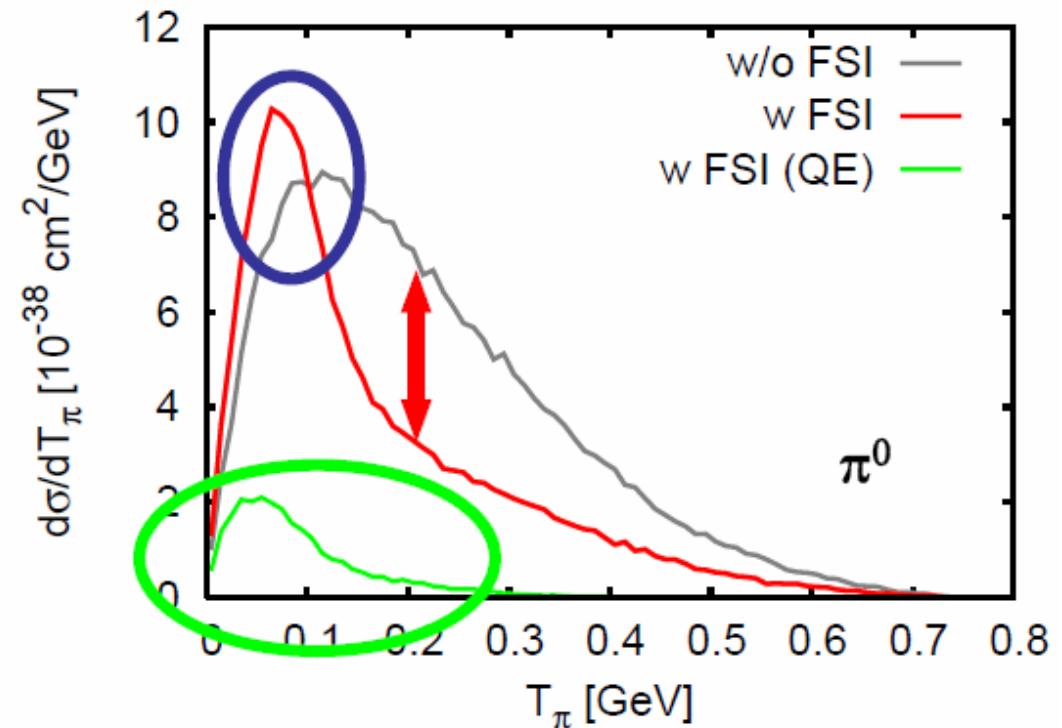
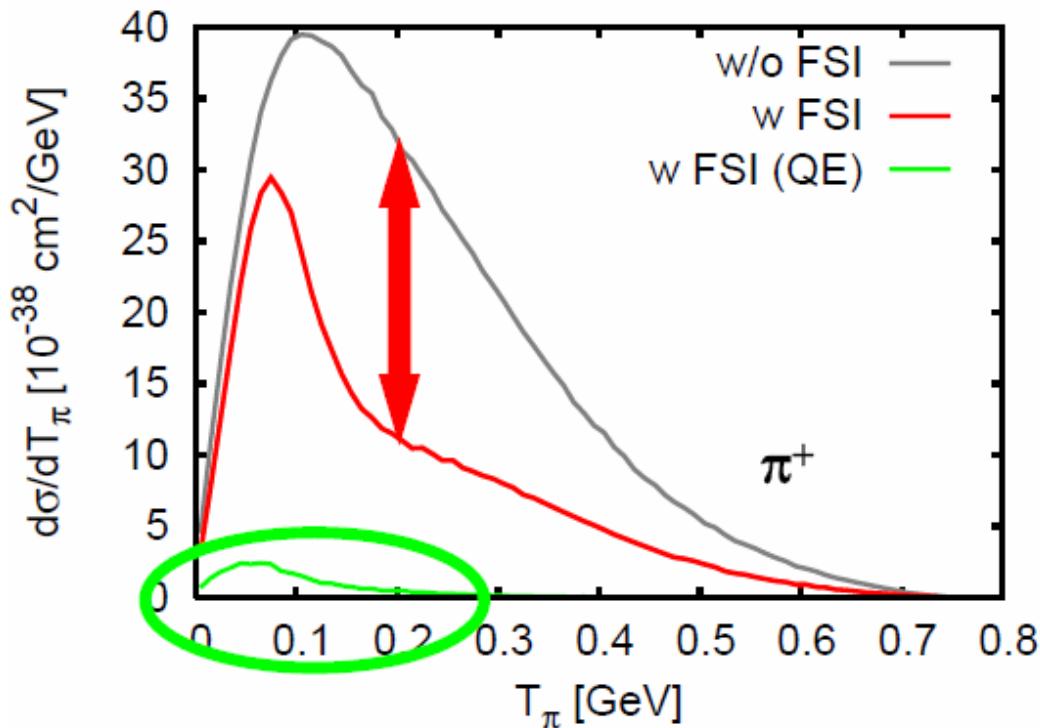
FSI →

- **absorption**
- **charge exchange**
- **redistribution of energy**
- **production of new particles**

# $1\pi$ production

## ■ Effects of FSI on pion kinetic energy spectra

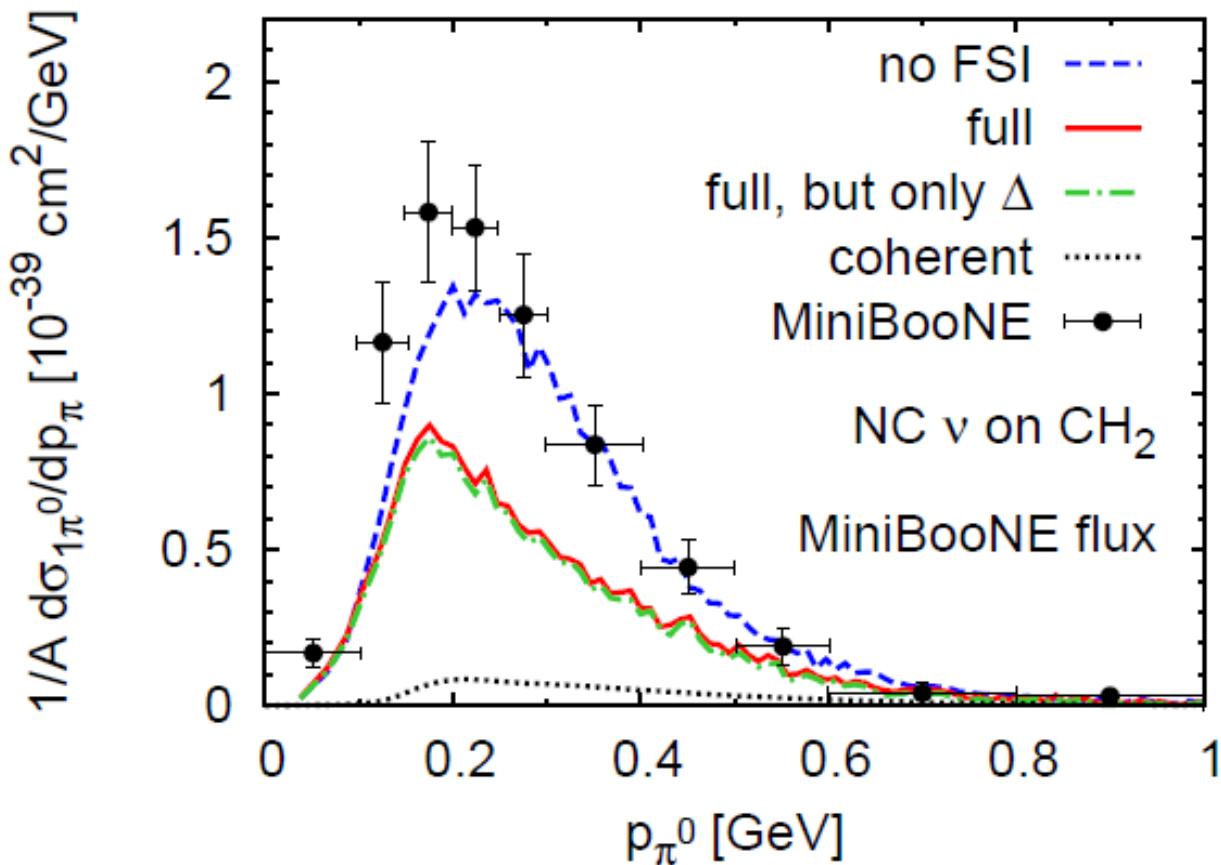
- strong absorption in  $\Delta$  region
- side-feeding from dominant  $\pi^+$  into  $\pi^0$  channel
- secondary pions through FSI of initial QE protons



$$\nu_\mu + {}^{56}\text{Fe} \rightarrow \mu^- \pi X \quad E_\nu = 1 \text{ GeV}$$

# $1\pi$ production

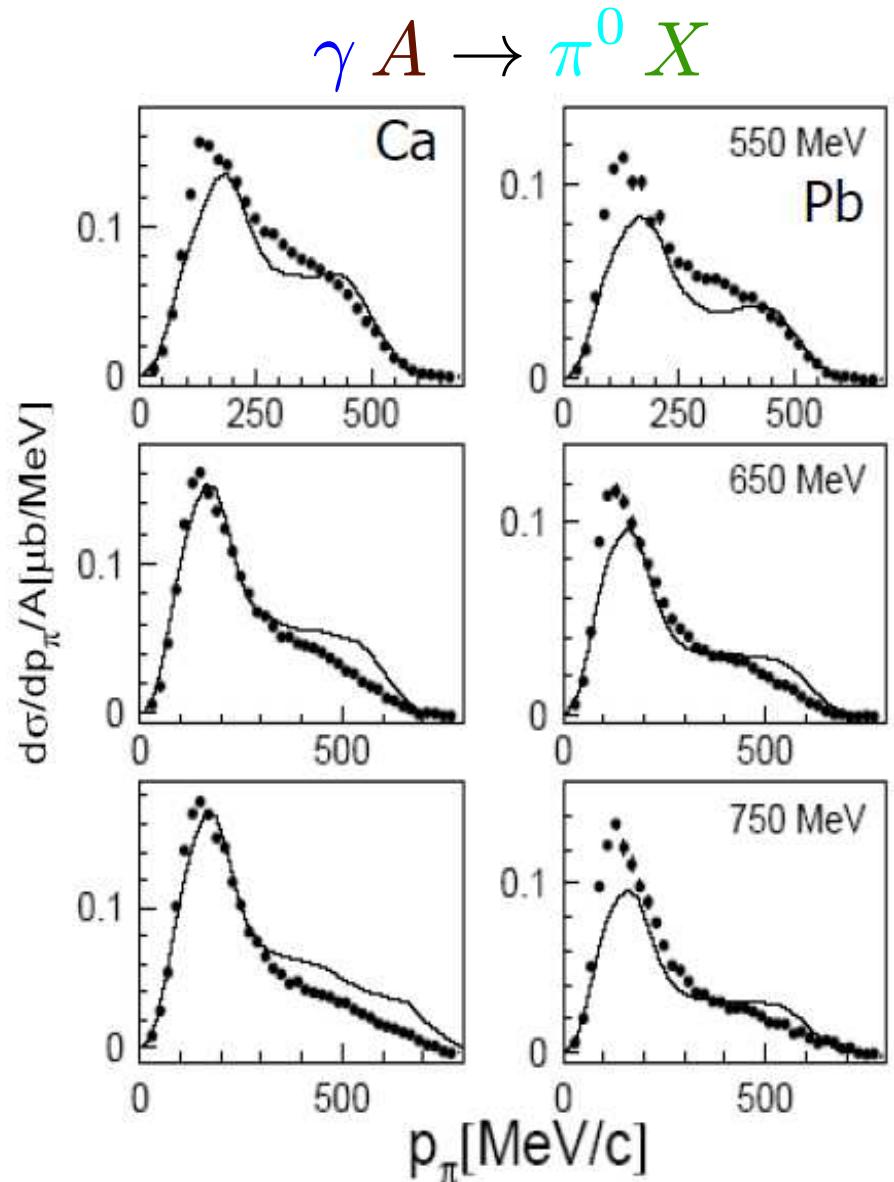
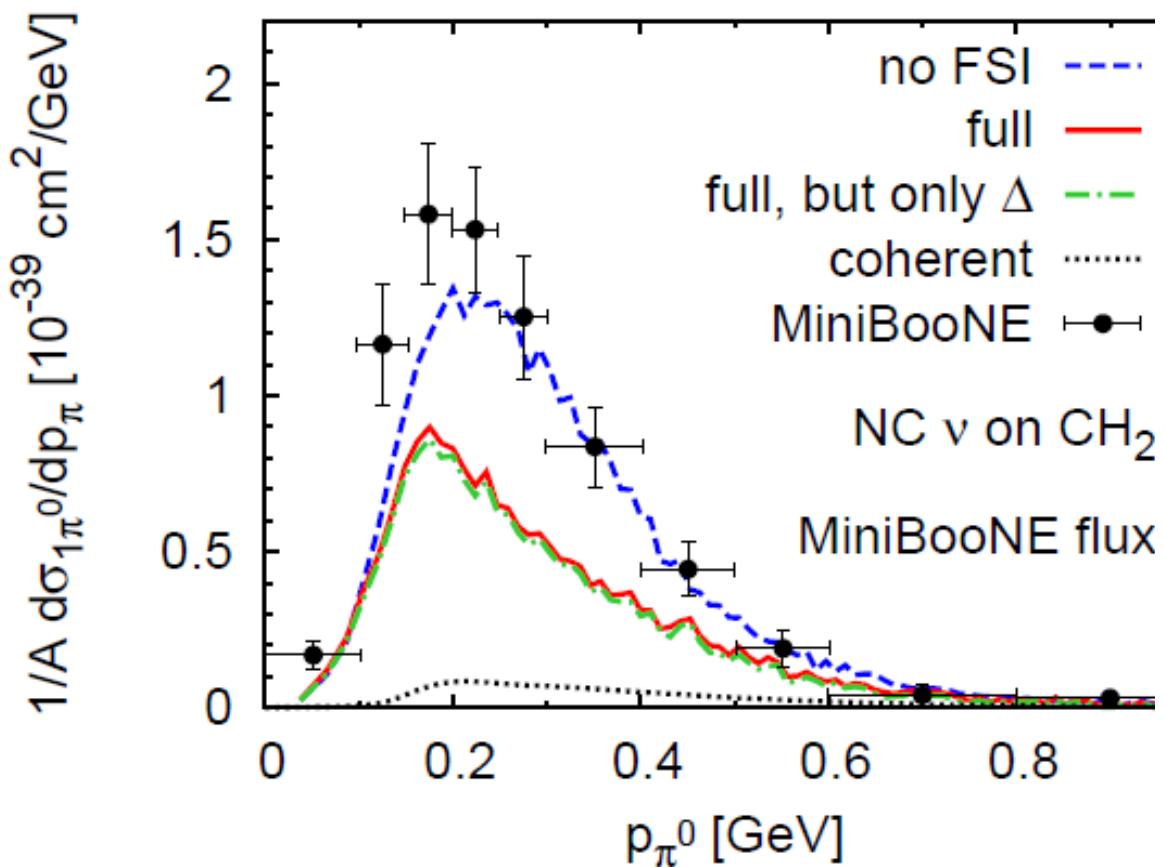
- NC  $\pi^0$  production at MiniBooNE



- Good description of **shape**
- Integrated  $\sigma$  : factor 2 **smaller**

# $1\pi$ production

- NC  $\pi^0$  production at MiniBooNE

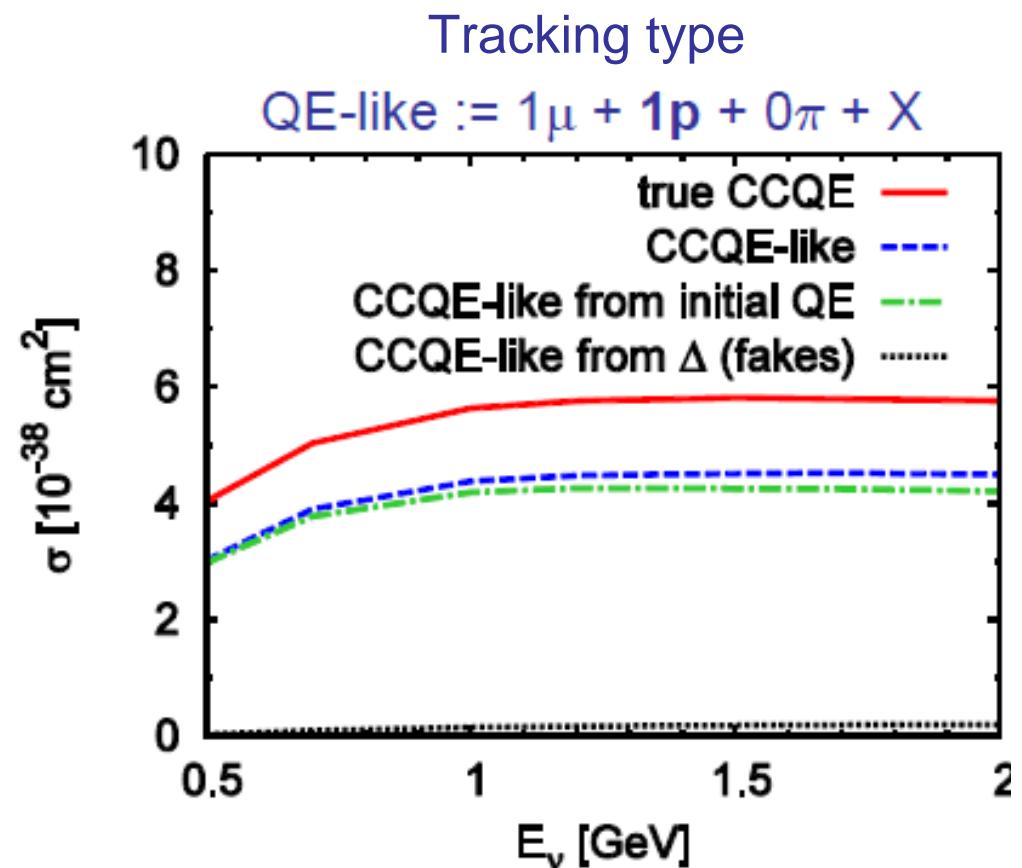
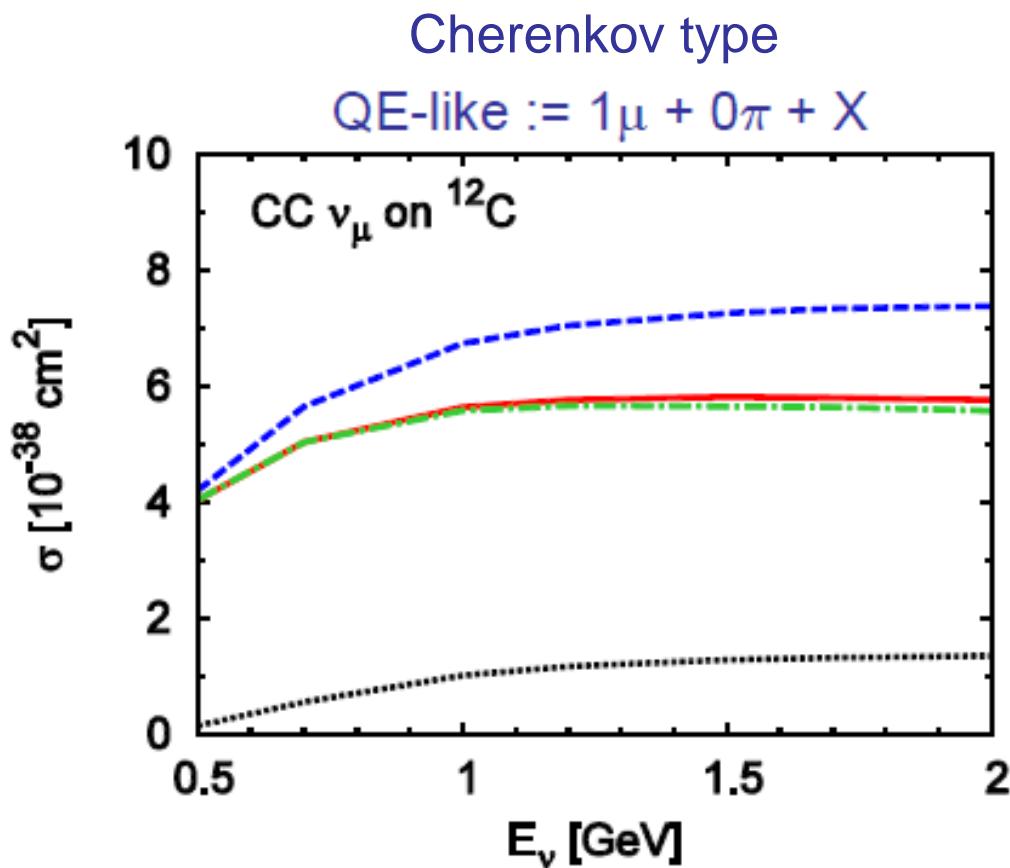


- Good description of **shape**
- Integrated  $\sigma$  : factor 2 **smaller**
- **Better** agreement in  $\pi^0$  photoproduction

TAPS, EPJA 22 (2004)

# QE & $1\pi$ entanglement

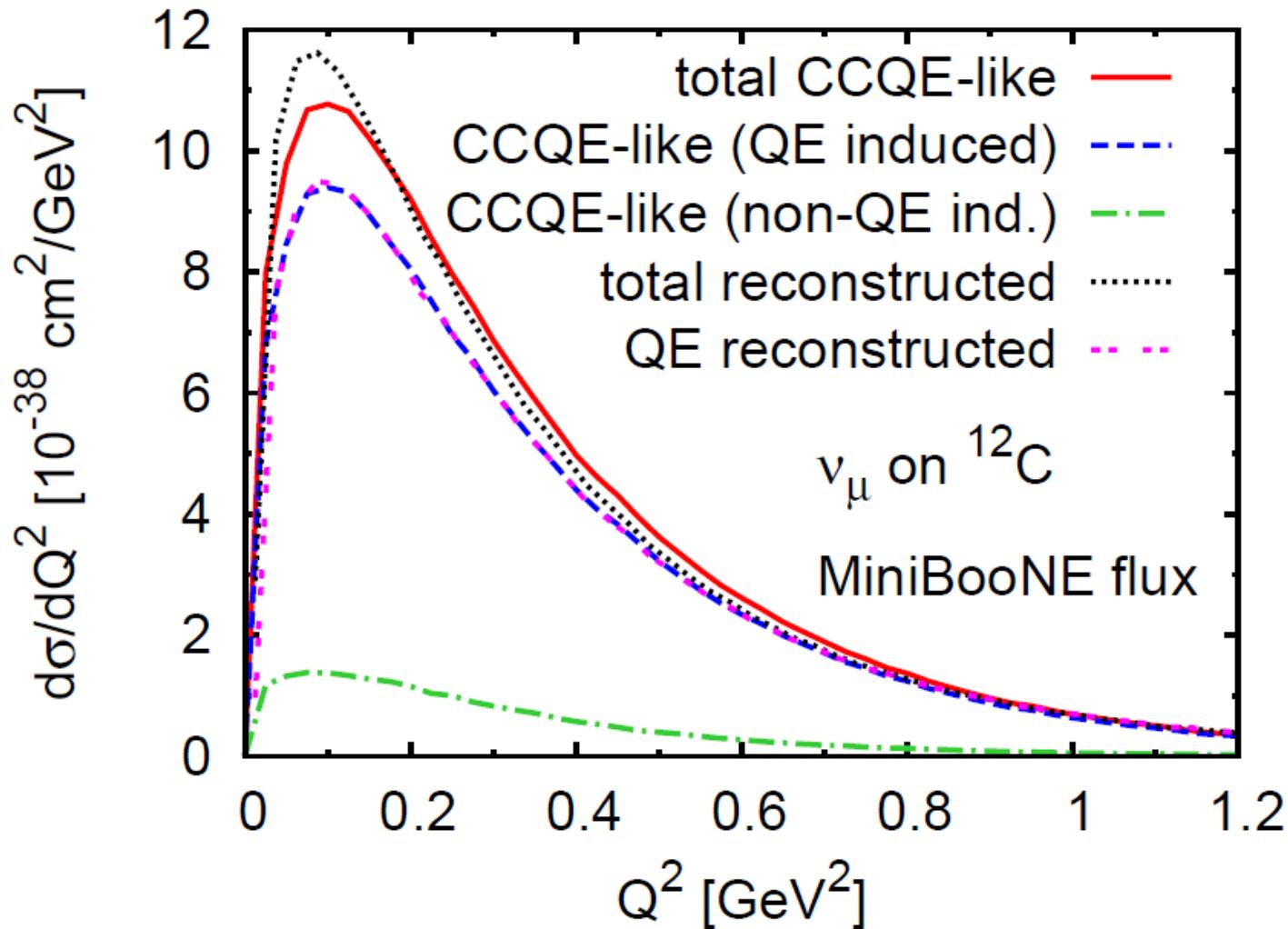
- Clean separation between QE and  $1\pi$  in nuclei is hard to achieve
- Depends on the detection technique
- For example:



- $\sim 20\%$  of CCQE events are misidentified: has to be corrected with MC

# $E_\nu$ reconstruction

- Influenced by QE &  $1\pi$  entanglement



# Conclusions

- **GiBUU** provides a framework to study different processes:
  - Heavy ion collisions,  $pA$ ,  $\pi A$ ,  $\gamma A$ ,  $eA$
  - $\nu A$  (inclusive and exclusive) **without new parameters**
- **FSI** modifies considerably the distributions through rescattering, charge-exchange and absorption
- A realistic description of **FSI** is important to **oscillation** experiments