

NPDF FOR NEUTRINO & CHARGED LEPTON DATA

Karol Kovařík

ITP KIT Karlsruhe, Germany

in collaboration with I.Schienbein, J.Y.Yu, F.Olness, J.Owens,
J.Morfin, C.Keppel, T. Stavreva

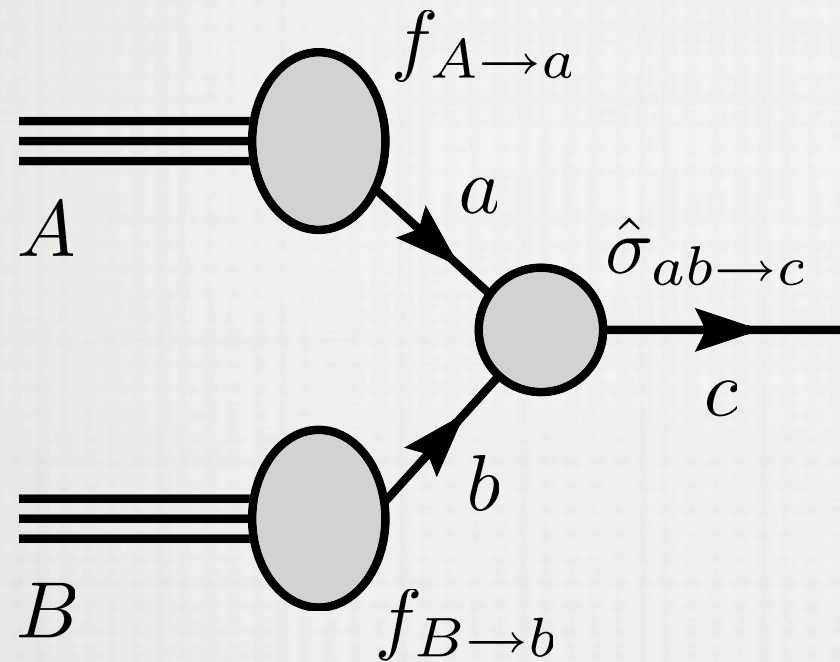
[arXiv:0710.4897 \[hep-ph\]](https://arxiv.org/abs/0710.4897) [arXiv:0907.2357 \[hep-ph\]](https://arxiv.org/abs/0907.2357)

OUTLINE

1. Introduction to parton distribution functions
2. Nuclear effects in PDFs
3. Global nPDF analysis & new CTEQ framework
4. Neutrino DIS vs. nPDF
5. Conclusions and Outlook

INTRO TO PDFs

Factorization & PDFs



Parton distribution functions (PDFs)

$$f_{A \rightarrow a}(x, \mu_F)$$

- universal, non-perturbative objects
- describe the structure of hadrons (in terms of partons - quarks & gluons)
- obey DGLAP evolution equations

The hard cross-section $\hat{\sigma}_{ab \rightarrow c}$

- free of long distance effects
- calculable in pQCD
- process dependant

$$\sigma = f_{A \rightarrow a} \otimes f_{B \rightarrow b} \otimes \hat{\sigma}_{ab \rightarrow c}$$

\uparrow \uparrow \uparrow
 from experiment from pQCD

INTRO TO PDFS

- How to determine PDFs $f_{A \rightarrow a}(x, \mu_F)$ from experimental data ?
- Scale dependence of PDFs obeys the DGLAP equations

$$\frac{df_i(x, \mu_F)}{d \ln \mu_F^2} = \frac{\alpha_S(\mu_F)}{2\pi} \int_x^1 \frac{dy}{y} P_{ij}(y) f_j(x/y, \mu_F)$$

- The set of equations can be solved exactly in momentum space
 - **BUT** one needs the knowledge of initial PDFs at scale μ_0 for all values of $x \in (0, 1)$
- Current global analysis - CTEQ, MRST etc...
 - solve set of DGLAP equations numerically
 - input needed only for as low x as needed
 - at the input scale one chooses a parameterization in x to fit data

INTRO TO PDFS

● CTEQ framework to fit PDFs from experimental data

CTEQ6M hep-ph/0201195

- the input scale set to $\mu_0 = Q_0 = 1.3 \text{ GeV}$
- parameterization of the PDFs in x

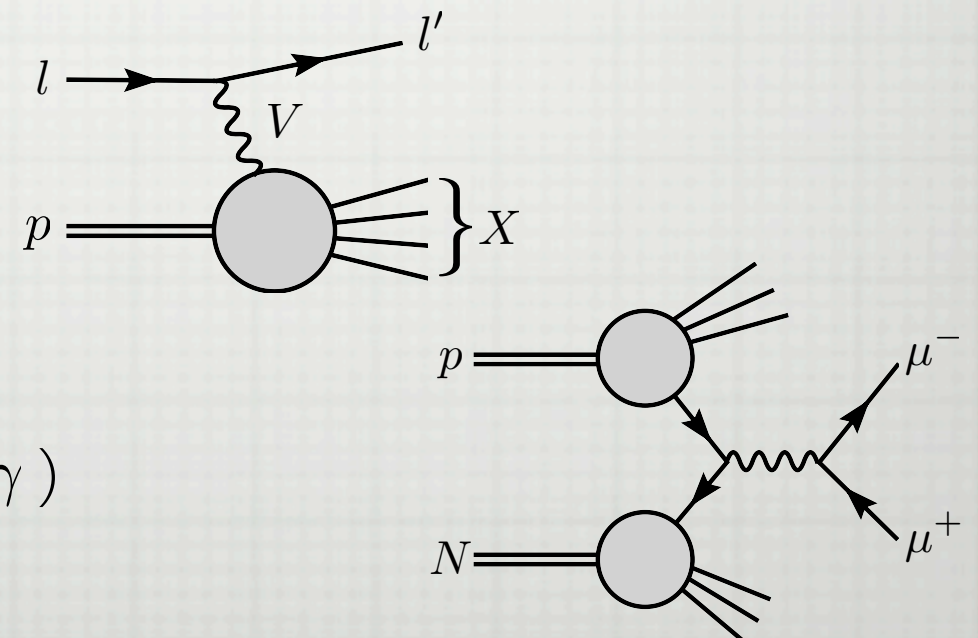
$$x f_k(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

- make sure # of free parameters not too high - CTEQ approx. 20 free params
- carefully choose data sets & kinematic cuts to constrain free parameters
- perform χ^2 fit to data

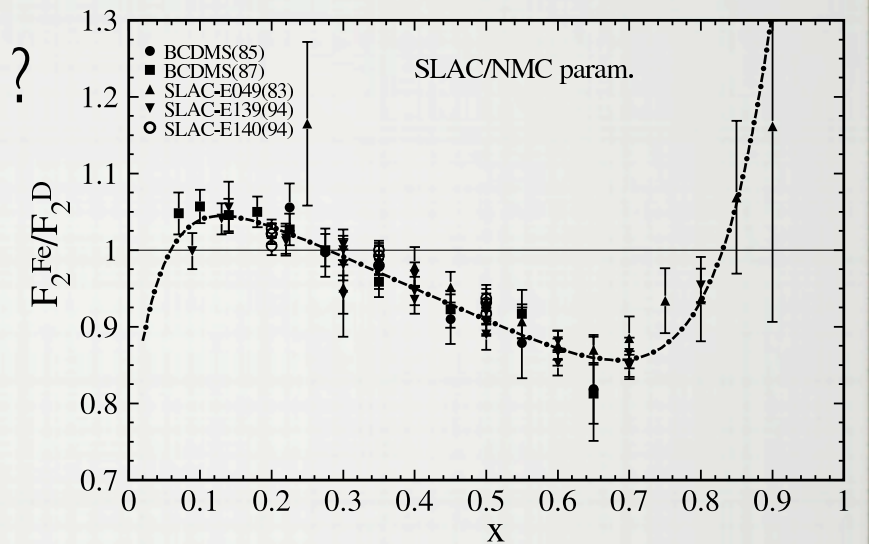
● Which data sets are included ?

- Deep Inelastic Scattering ($l^\pm p, l^- d, \nu N, \bar{\nu} N$)
- Neutrino DIS di-muon production
- Drell-Yan & vector boson production (W^\pm, Z^0, γ)
- hadronic jet data



NUCLEAR PDFS

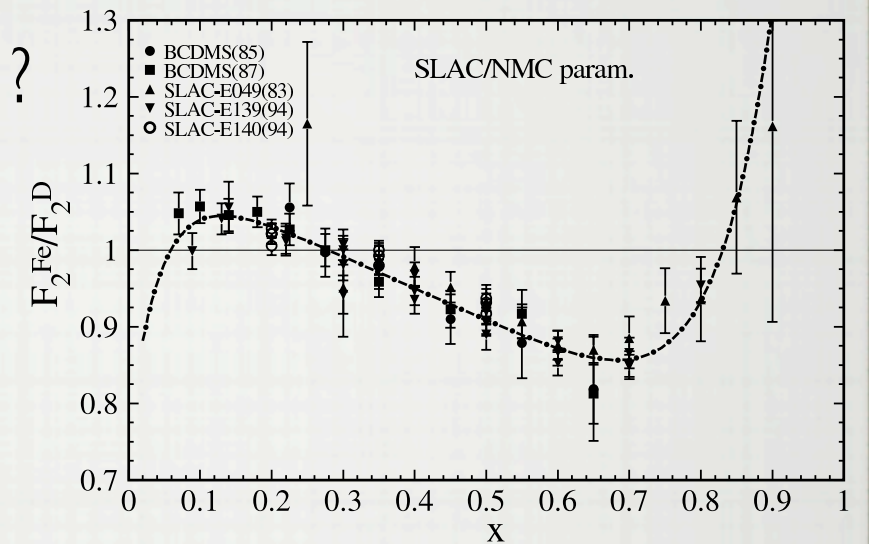
- What are nuclear parton density functions (nPDF) ?
 - parton densities for partons in bound proton & neutron



NUCLEAR PDFS

● What are nuclear parton density functions (nPDF) ?

- parton densities for partons in bound proton & neutron

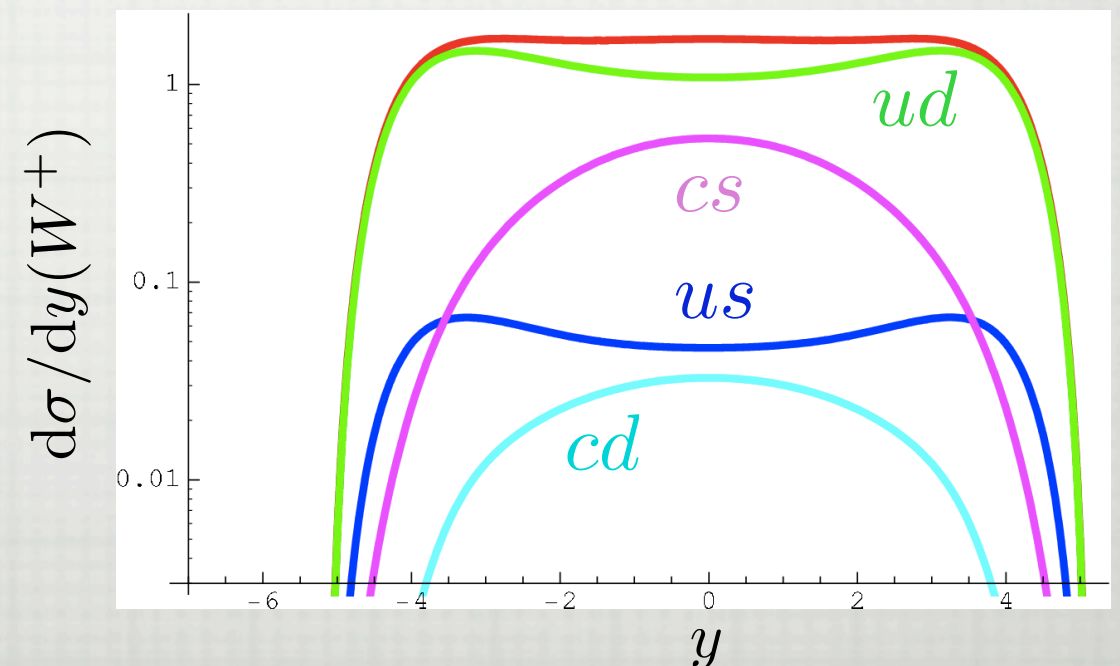


● Where are nuclear parton density functions useful ?

I. Strange quark content of the proton

strange PDF from neutrino DIS with heavy nuclei - nuclear effects very important

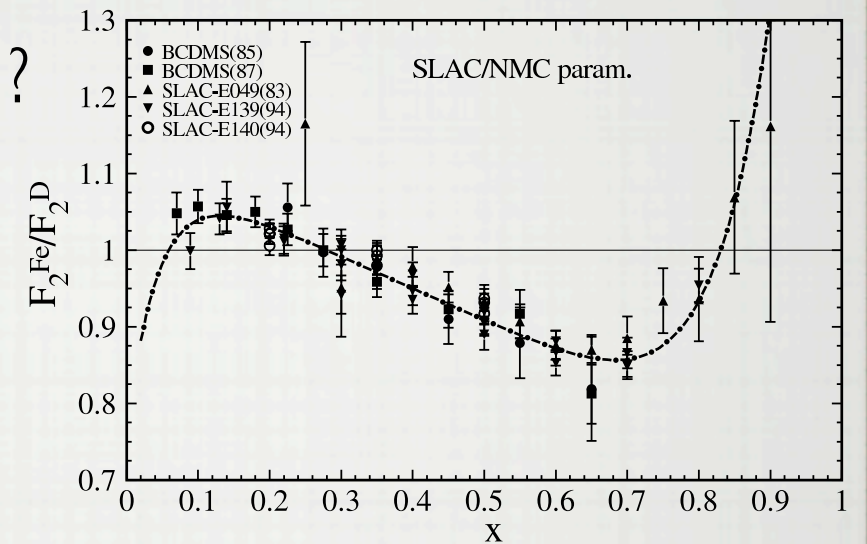
crucial for: W-boson production at the LHC
(standard candle process)



NUCLEAR PDFS

What are nuclear parton density functions (nPDF) ?

- parton densities for partons in bound proton & neutron

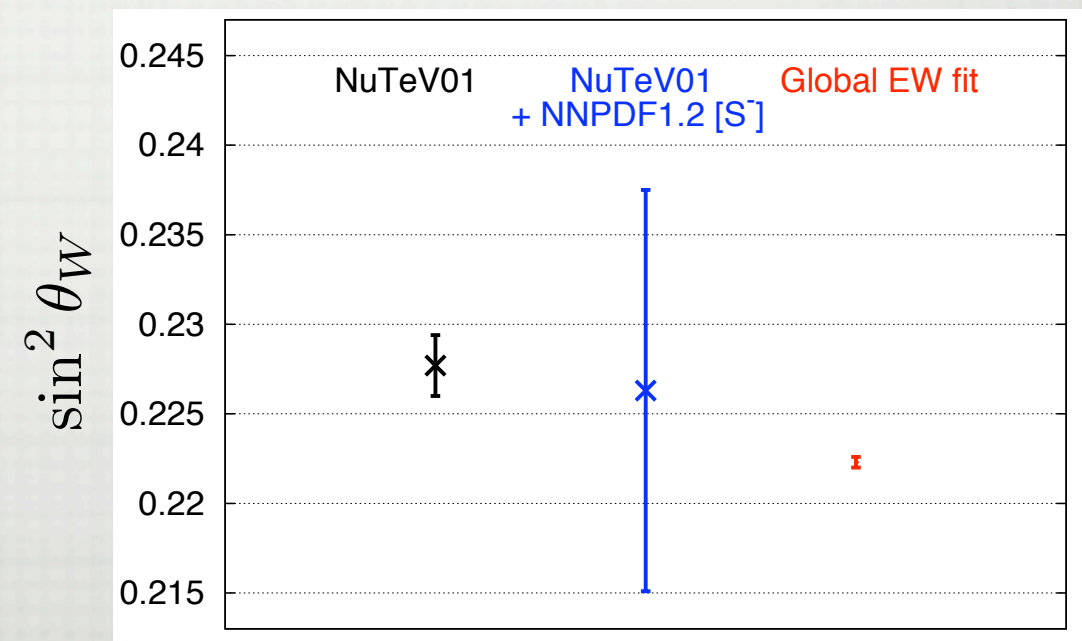


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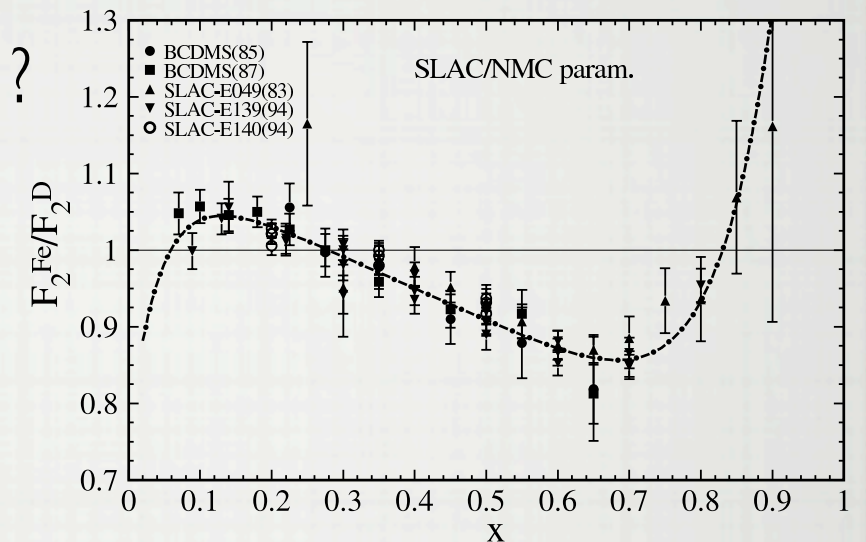
crucial for: determining weak mixing angle
from NuTeV experiment



NUCLEAR PDFS

● What are nuclear parton density functions (nPDF) ?

- parton densities for partons in bound proton & neutron



● Where are nuclear parton density functions useful ?

1. Strange quark content of the proton
2. Collisions of protons and nuclei at RHIC, ALICE & CMS
3. Neutrino scattering experiments e.g. MINERvA
4. Neutrino oscillations experiments e.g. MINOS
5. Even new physics - direct detection of dark matter

NUCLEAR PDFS

- Review of existing global analyses of nuclear PDF

DE FLORIAN, SASSOT'04 [PRD69(2004)074028]
LO, NLO

$\chi^2/\text{dof} = 0.76$

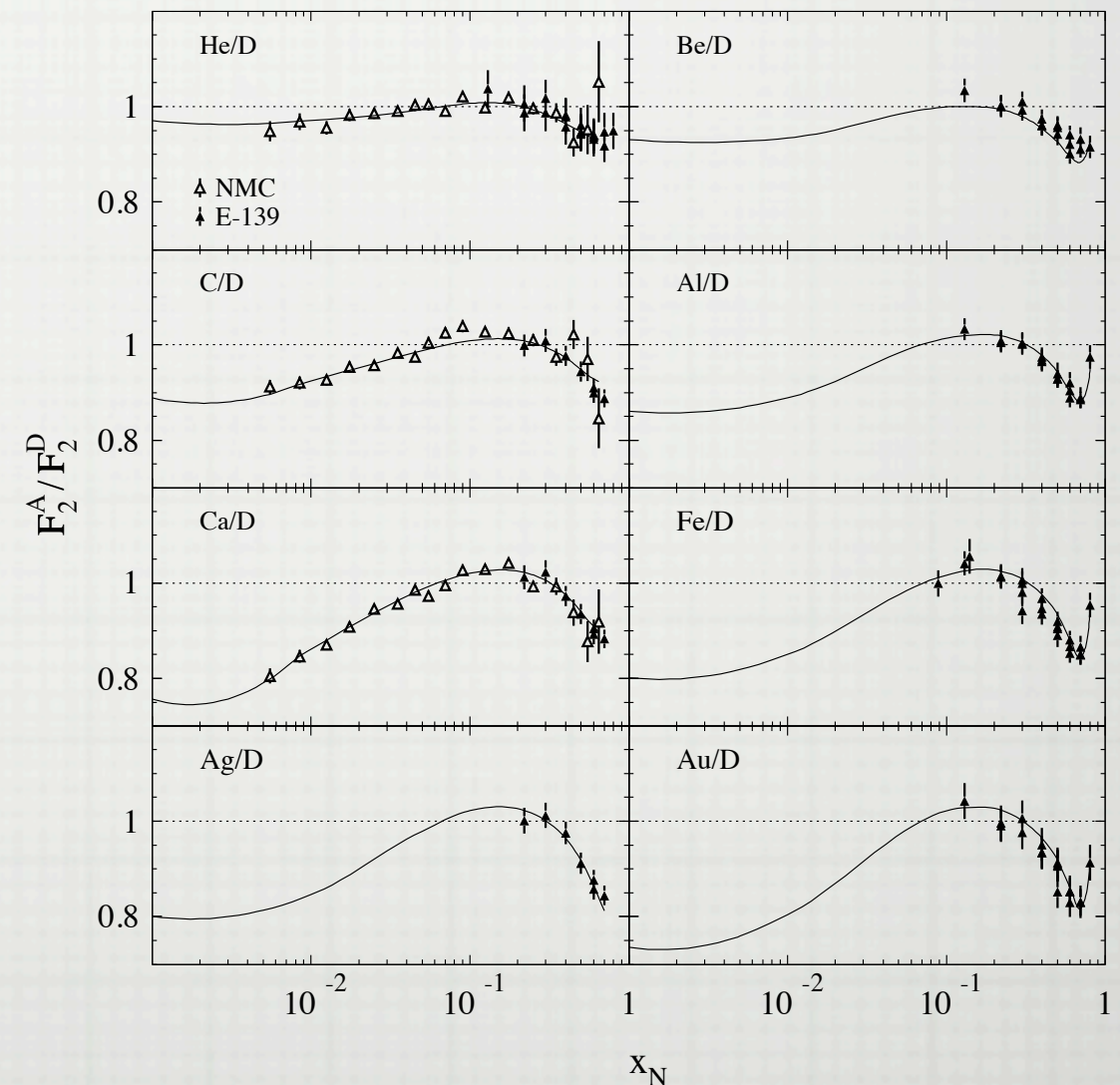
- first NLO analysis of nuclear data
- the only group using convolution relation

$$f_i^A(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i(x_N/y, Q_0^2)$$

- typical nucleon density for valence quarks

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] \\ + n_v \left(\frac{y}{A}\right)^{\alpha_v} \left(1 - \frac{y}{A}\right)^{\beta_v} + n_s \left(\frac{y}{A}\right)^{\alpha_s} \left(1 - \frac{y}{A}\right)^{\beta_s}$$

- the only framework using evolution in Mellin space & have PDFs also for $x_N > 1$
- only standard DIS data sets (semi-global)
- no error analysis



NUCLEAR PDFS

- Review of existing global analyses of nuclear PDF

ESKOLA, PAUKKUNEN, SALGADO'09 [JHEP0904(2009)065]
 LO, NLO, ERROR PDFS

$\chi^2/\text{dof} = 0.8$

- uses multiplicative factor

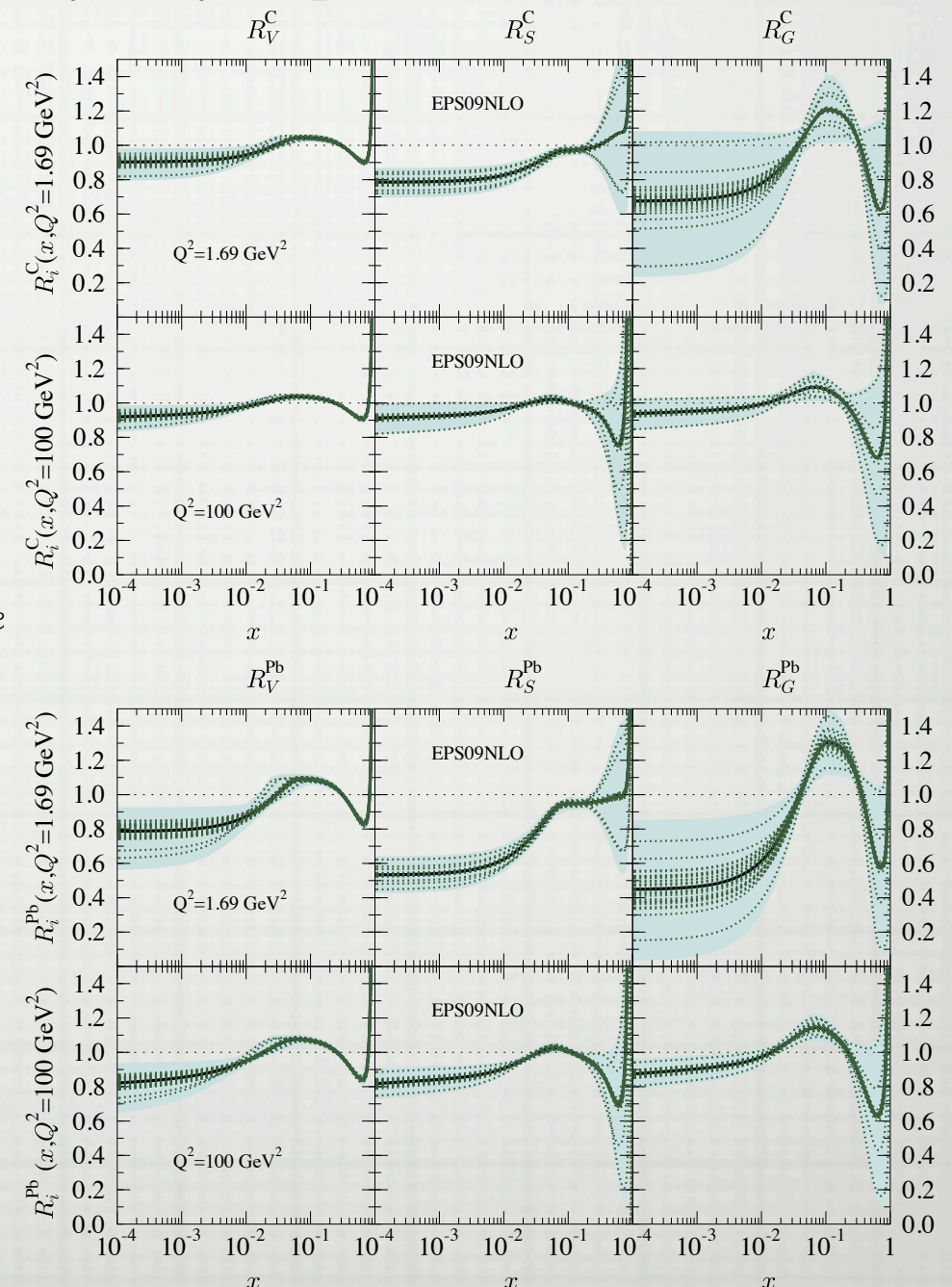
$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

where proton PDF in CTEQ6.1M and factor is a complicated piecewise defined function

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

with A-dependent parameters

- neglects region $x > 1$
- includes all current DIS & DY data set & π^0 RHIC data to constrain gluon
- use Hessian method to produce error PDFs



NUCLEAR PDFS

Why another set of NPDFs ?

1. Better estimate of the nPDF uncertainty

- PDFs underestimate the true error since they do not take into account the variation of the framework (parameterization) & all other physical assumptions & technical details

2. Flexible CTEQ nuclear correction factor

- we would like a NPDF analysis with a close relation to the existing CTEQ proton analysis - allows to calculate nuclear correction factors in a flexible way (Q dependent & based on global analysis)

3. Include neutrino DIS data in the fit

NUCLEAR CTEQ

- CTEQ framework for nuclear PDF - based on CTEQ6M proton fit

- functional form for bound protons same as for free proton PDF (restrict x to $0 < x < 1$)

$$x f_k(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5} \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s}$$

$$\bar{d}(x, Q_0) / \bar{u}(x, Q_0) = c_0 x^{c_1} (1 - x)^{c_2} + (1 + c_3 x)(1 - x)^{c_4}$$

- coefficients with A-dependance (reduces to proton for $A=1$)

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1} (1 - A^{-c_{k,2}}), \quad k = \{1, \dots, 5\}$$

- PDF for a nucleus with A-nucleons out of which Z-protons

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A - Z}{A} f_i^{n/A}(x, Q)$$

Note: PDF of neutron are related to the proton by isospin symmetry

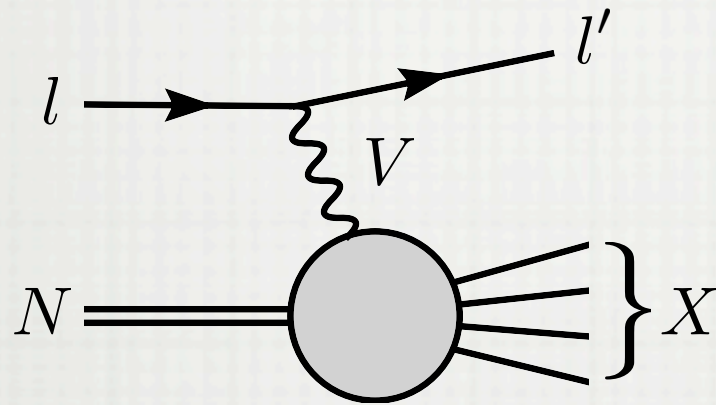
- Input scale and other input parameters as in CTEQ6M proton analysis

$$Q_0 = m_c = 1.3 \text{ GeV} \quad m_b = 4.5 \text{ GeV} \quad \alpha_s(m_Z) = 0.118$$

NUCLEAR CTEQ

- Experiments included in the analysis:

Deep Inelastic Scattering



$$l + N \rightarrow l' + X$$

CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

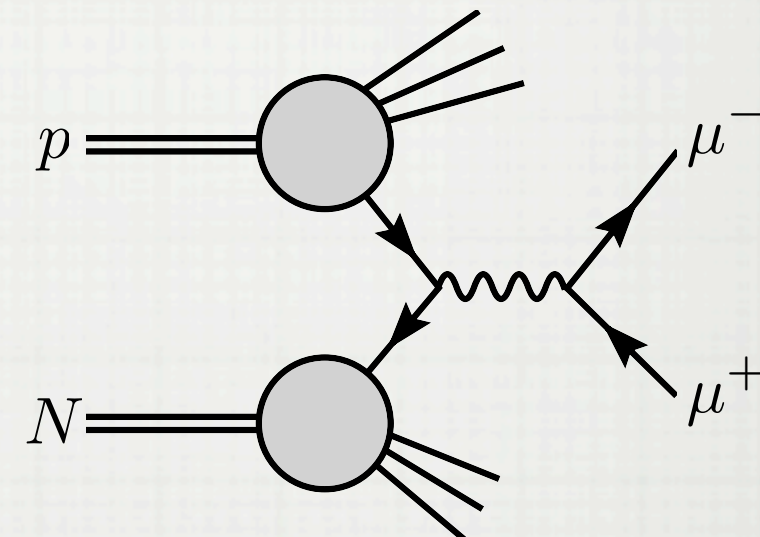
DESY Hermes

$N = (\text{D, He, N, Kr})$

SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

Drell-Yan process



$$p + N \rightarrow \mu^+ \mu^- + X$$

FNAL E-772 & E-886

$N = (\text{D, C, Ca, Fe, W})$

NUCLEAR CTEQ

● NPDF fit properties:

- we fit nuclear data with NLO QCD predictions & include heavy quark effects (ACOT)
- added nuclear observables to CTEQ fitting routines (need to treat 2 nuclei at once)

$$\text{DIS: } F_2^A / F_2^{A'} \quad \text{Drell-Yan: } \sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$$

- applied standard CTEQ kinematical cuts $Q > 2\text{GeV}$ & $W > 3.5\text{GeV}$

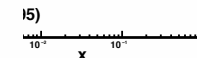
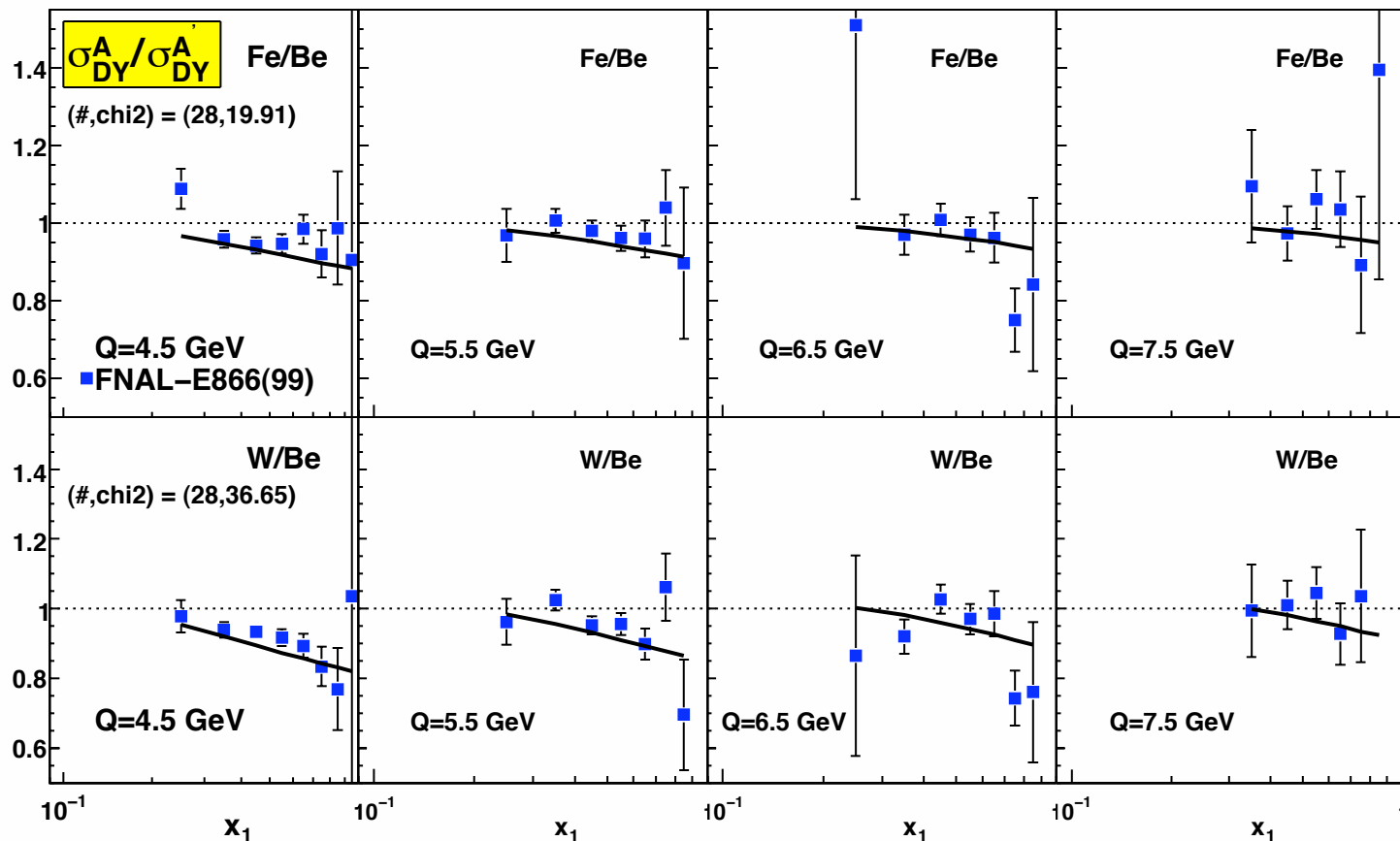
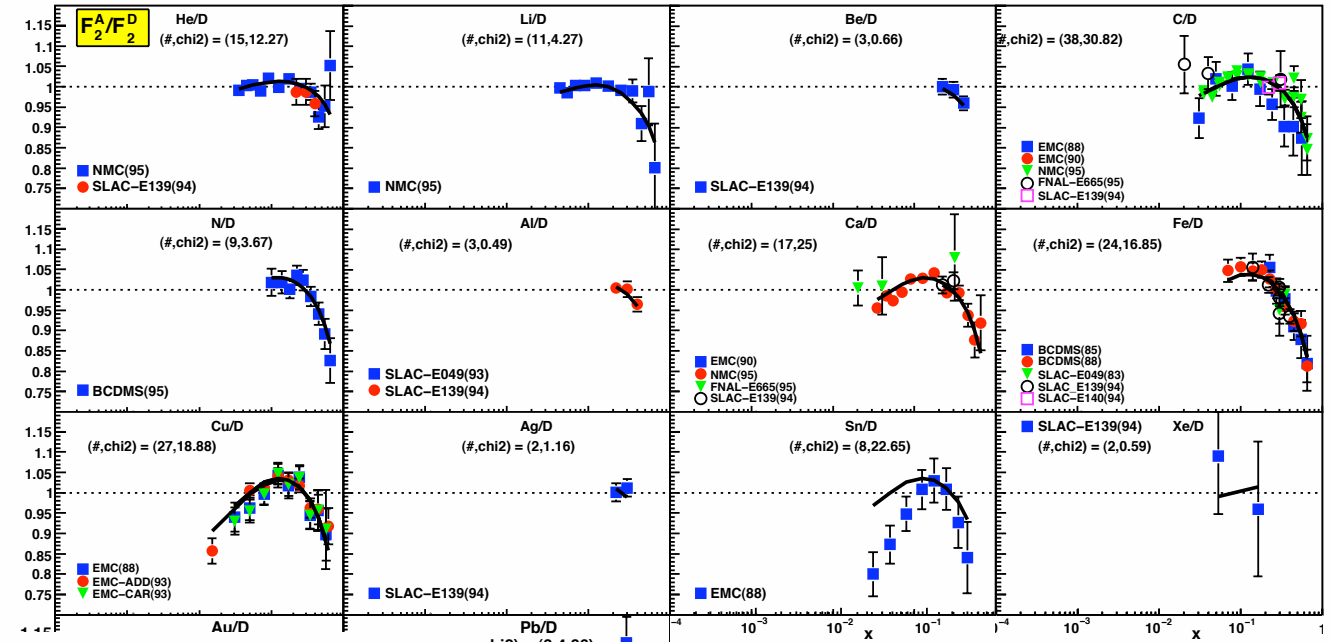
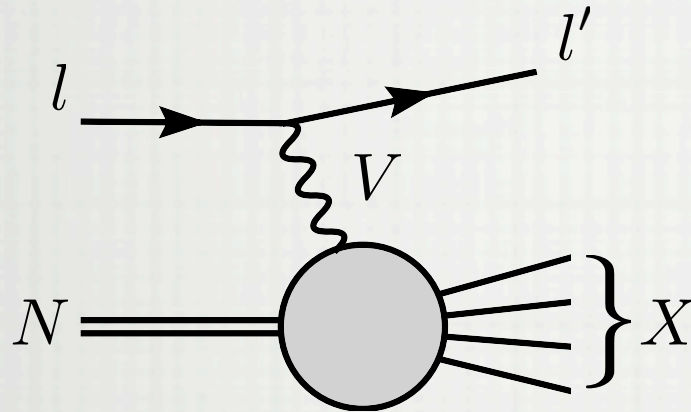
● NPDF fit results:

- 708 (1233) data points after (before) cuts
- 32 free parameters - 675 degrees of freedom
- overall $\chi^2/\text{dof} = 0.95$
- individually for different data sets

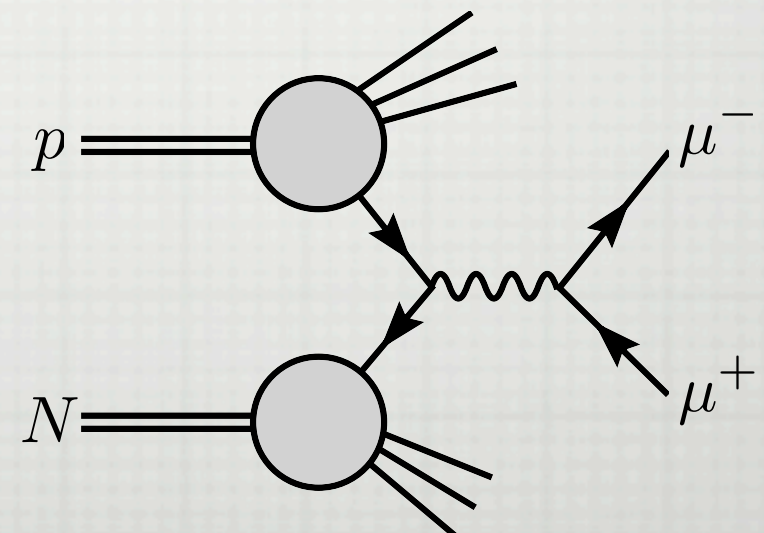
- for F_2^A / F_2^D $\chi^2/\text{pt} = 0.92$
- for $F_2^A / F_2^{A'}$ $\chi^2/\text{pt} = 0.69$
- for $\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$ $\chi^2/\text{pt} = 1.08$

NUCLEAR CTEQ

Deep Inelastic Scattering

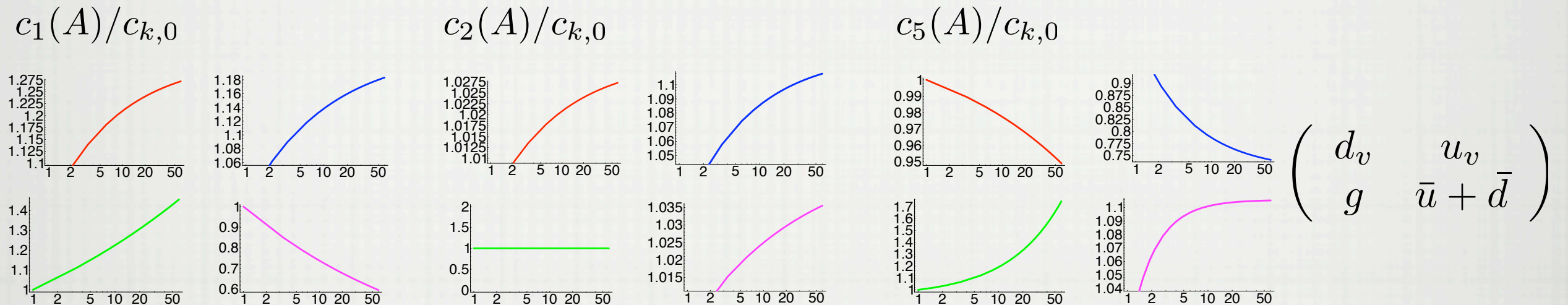


Drell-Yan process



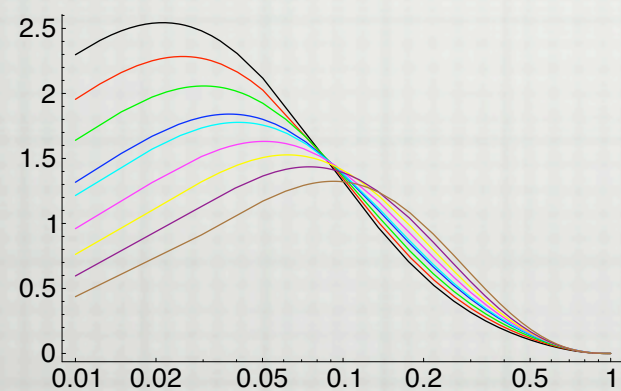
NUCLEAR CTEQ

CTEQ parameters dependent on atomic number A - $c_k(A)/c_{k,0}$

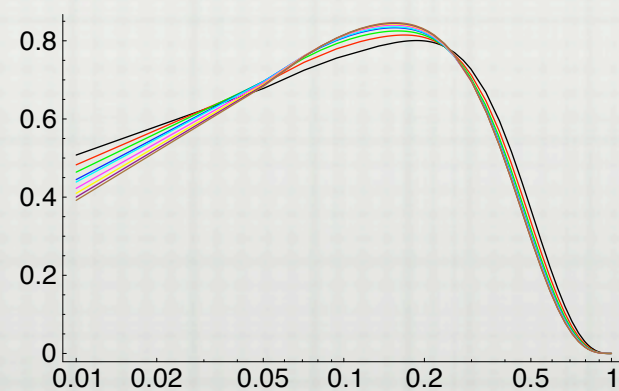


Parton density functions for bound partons as a function of x

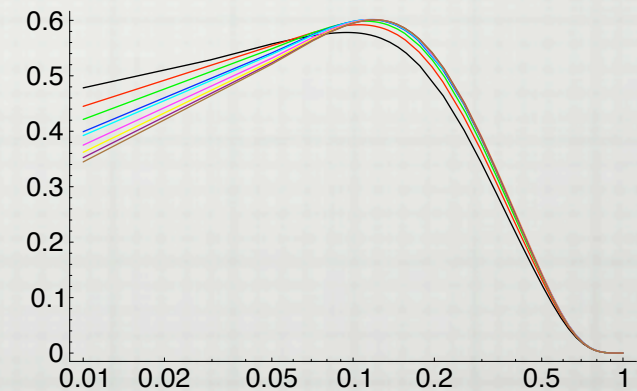
$x f_k^A(x, Q)$ for $A = (1, 2, 4, 9, 12, 27, 56, 108, 207)$
 black yellow brown
 red purple



gluon



total up-quark

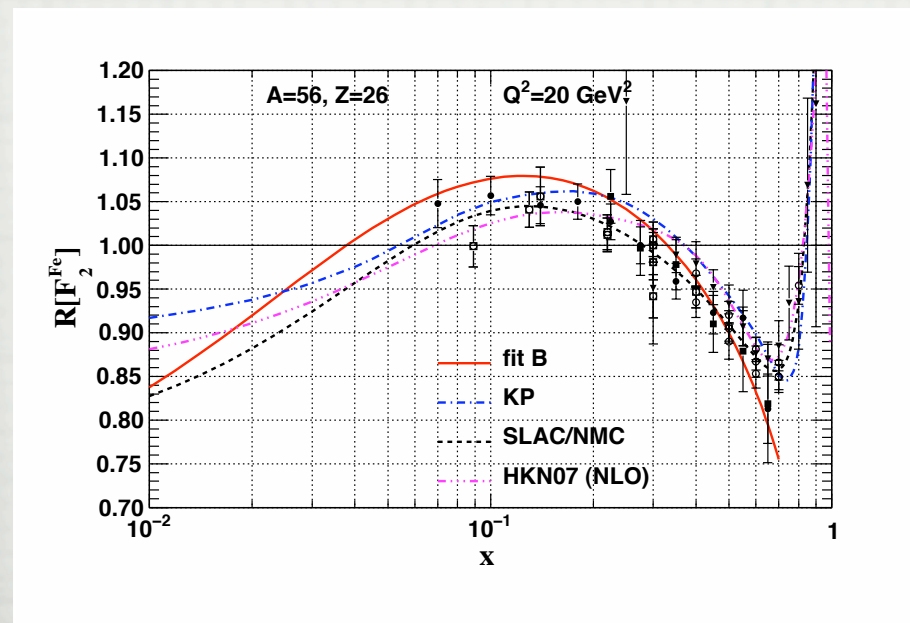
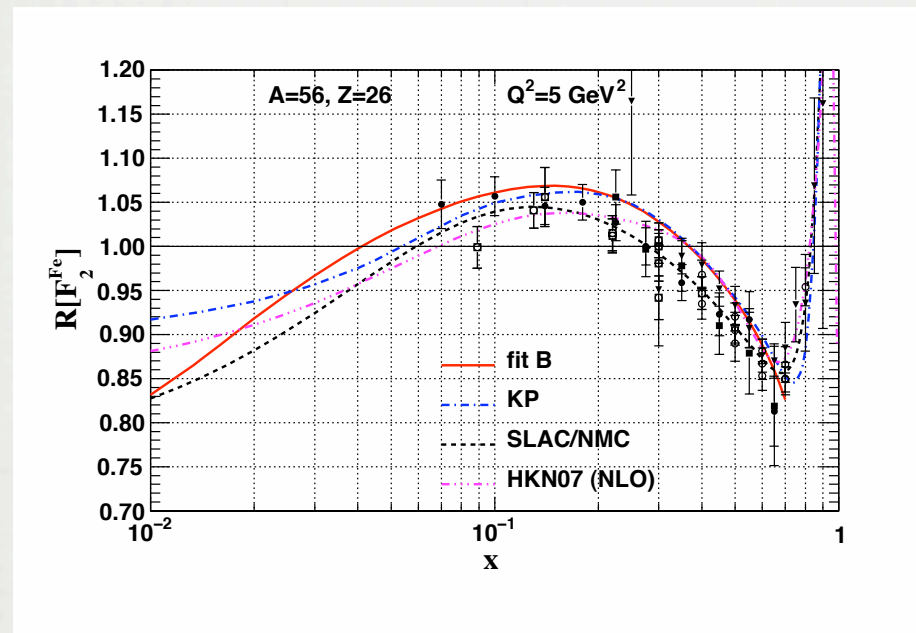


total down-quark

NEUTRINO DIS

- Comparison of iron F_2 from neutrino and charged lepton DIS

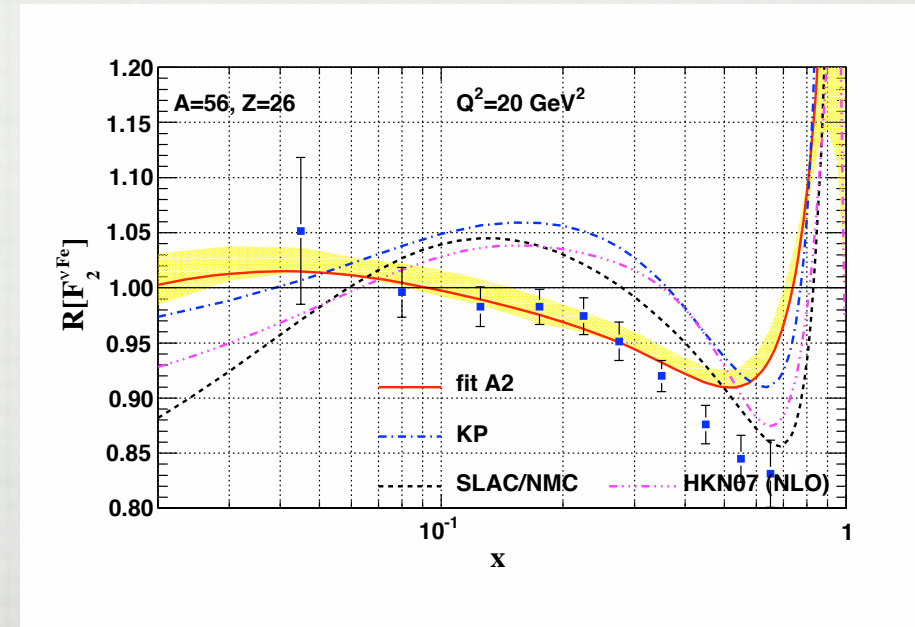
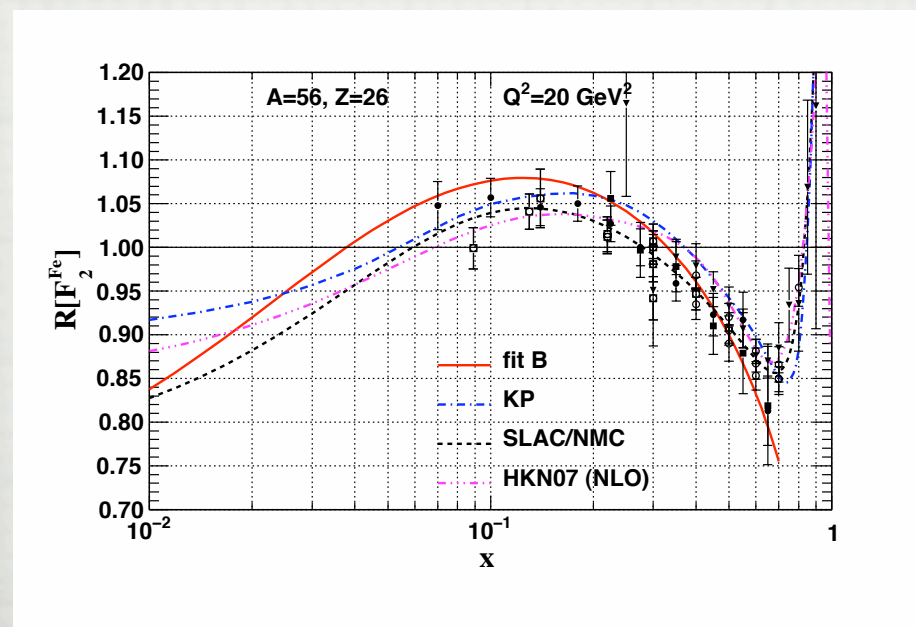
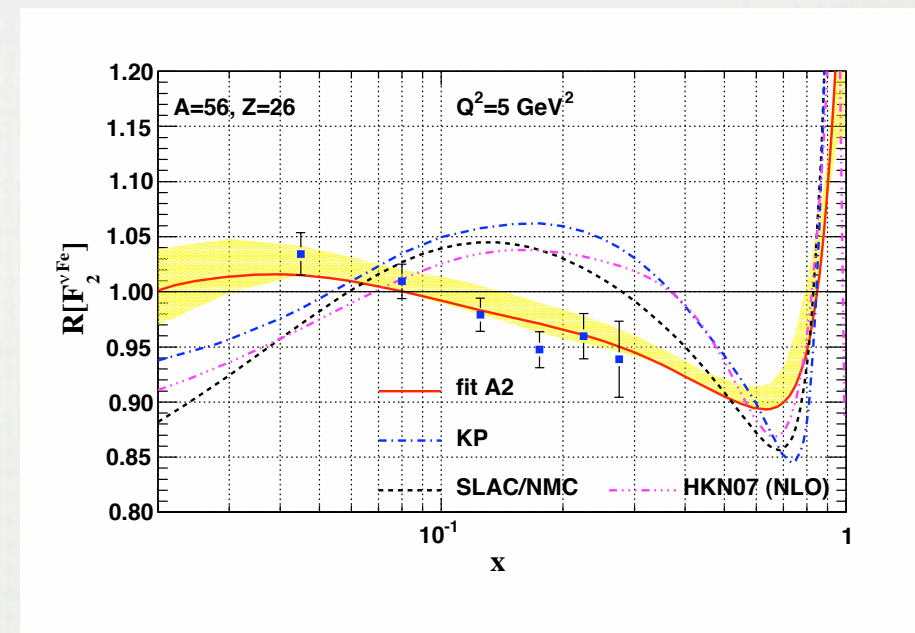
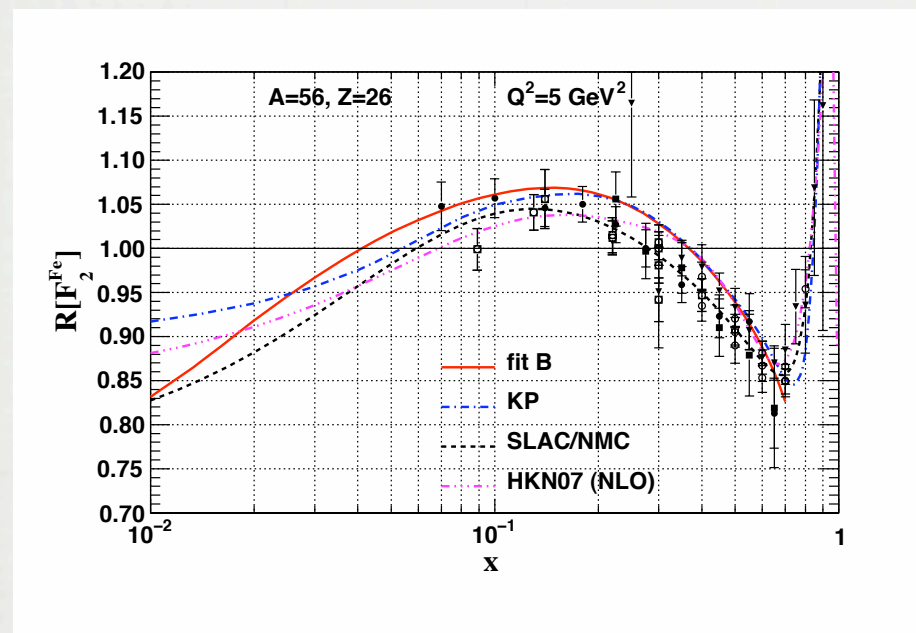
$$R[F_2^{Fe}] = F_2^{Fe} / F_2^D$$



NEUTRINO DIS

Comparison of iron F_2 from neutrino and charged lepton DIS

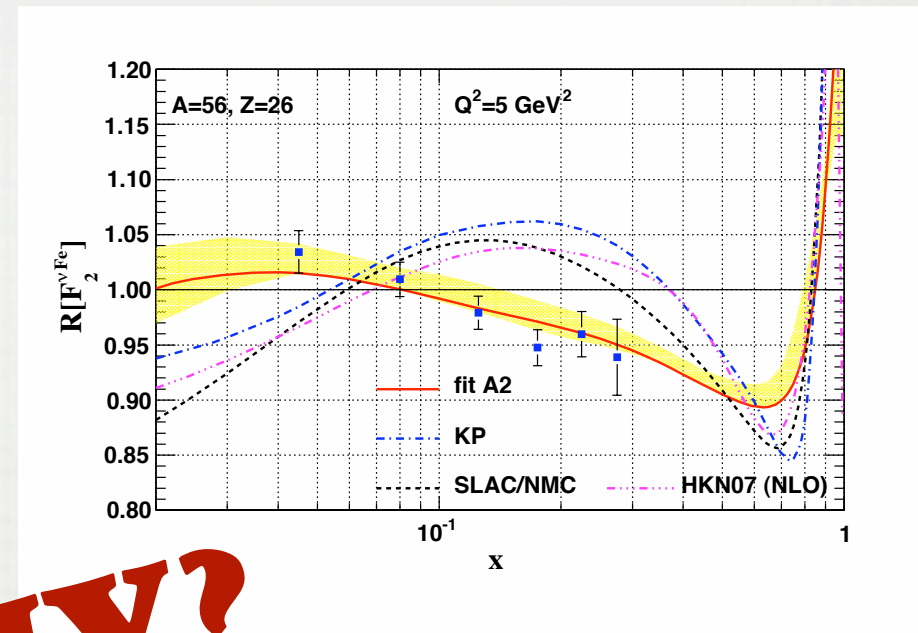
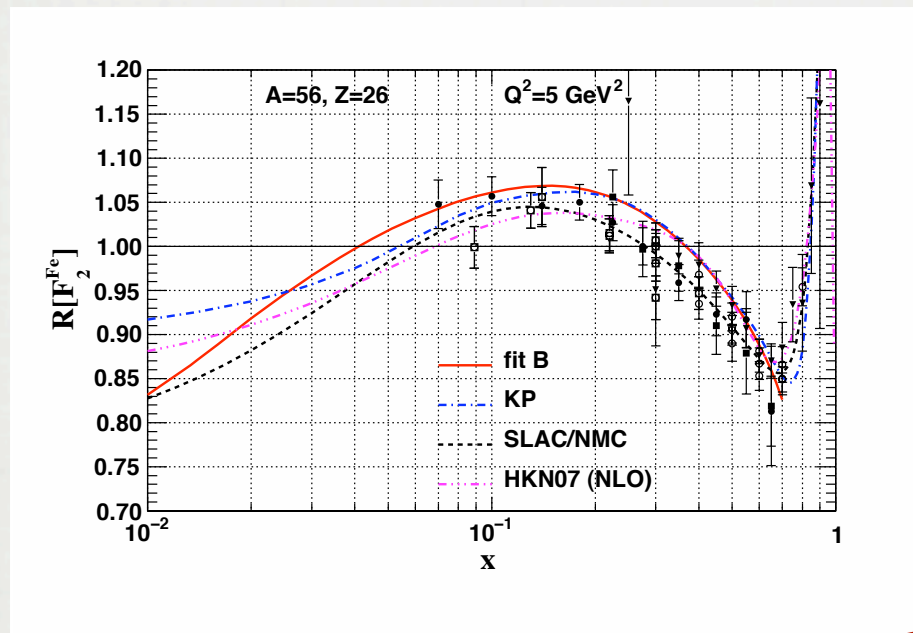
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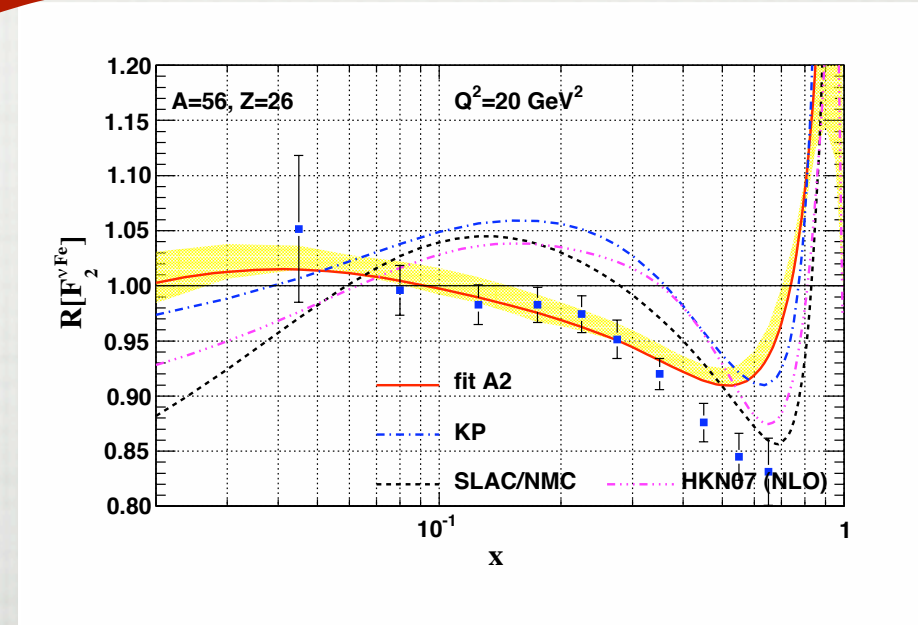
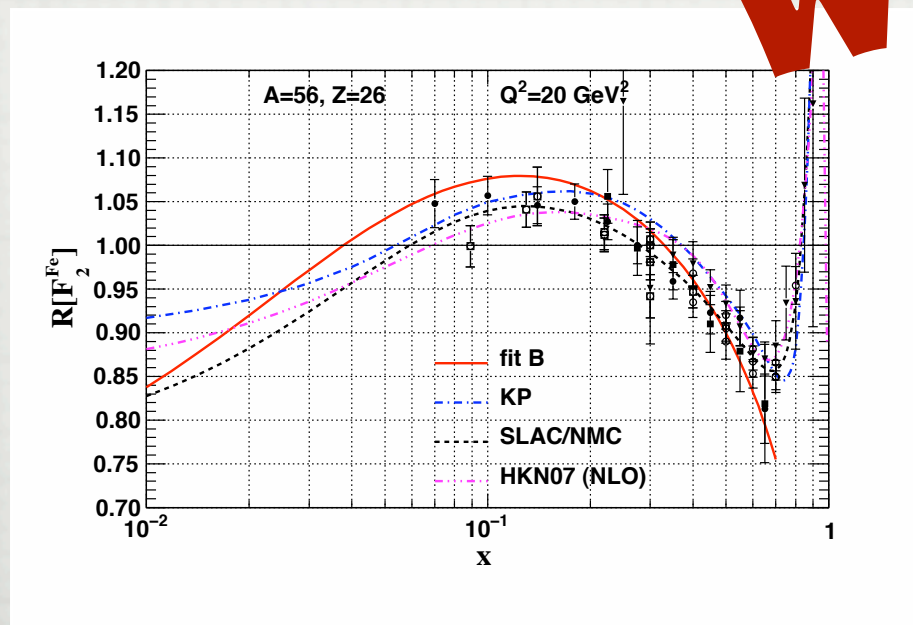
NEUTRINO DIS

Comparison of iron F_2 from neutrino and charged lepton DIS

$$R[F_2^{Fe}] = F_2^{Fe} / F_2^D$$



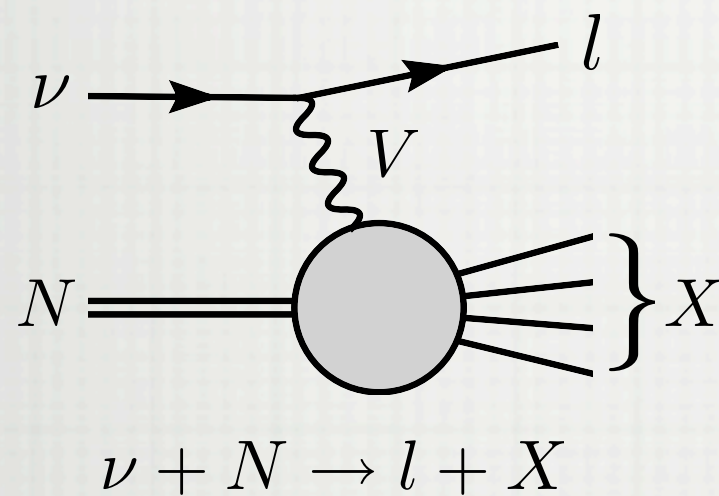
WHY?



NEUTRINO DIS

- Re-analyze neutrino data within the same framework as for charged lepton

- Neutrino DIS data



NuTeV & di-muon

$$N = \text{Fe}$$

→ 2310 data points

CHORUS

$$N = \text{Pb}$$

→ 824 data points

All charged lepton DIS & Drell-Yan data

→ 708 data points

- Challenges in combining the neutrino & charged lepton data

- deal with the disparity of number of data points - assigning weights to neutrino data
- neutrino DIS data only with 2 heavy nuclei - insufficient to get a reliable A -dependence
- do all neutrino data show the different behavior or only NuTeV ?

NEUTRINO DIS

● Properties of neutrino fits

- CHORUS data are in good agreement with the charged lepton data

combined: $\chi^2/pt=1.03$

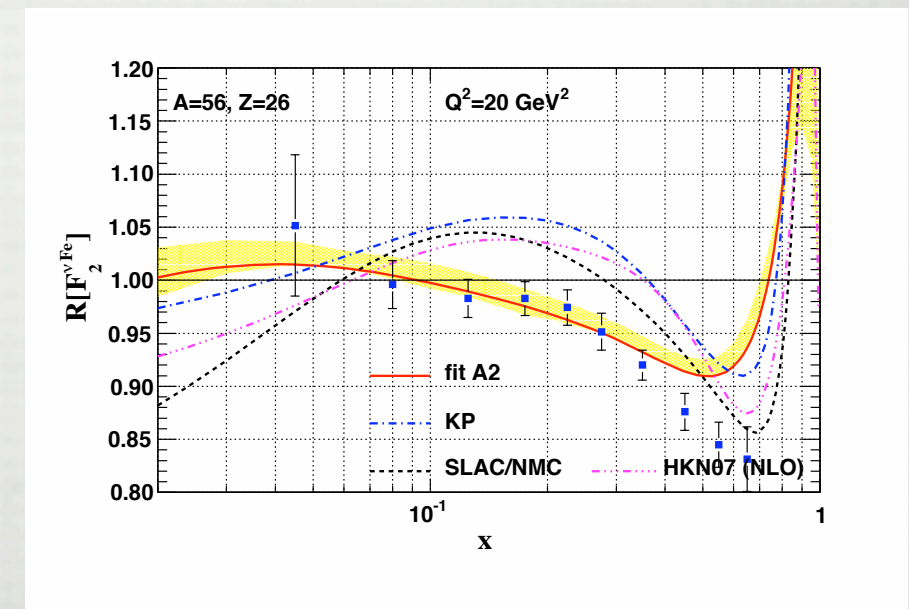
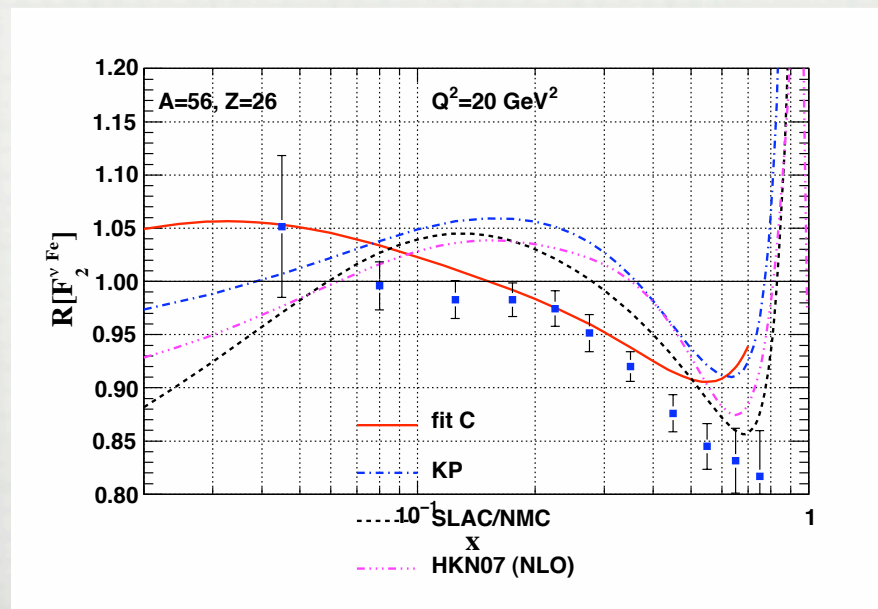
- NuTeV data difficult to fit alone or together with the charged lepton data

alone: $\chi^2/pt=1.35$

combined: $\chi^2/pt=1.33$

- Neutrino data dominate the combined fit without re-weighting - final result depends from the weight chosen

Consistency check
with only NuTeV

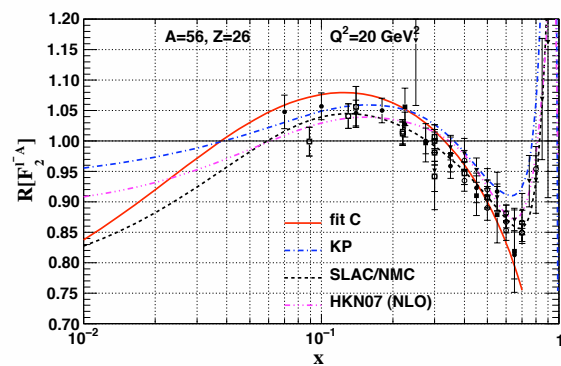


NEUTRINO DIS

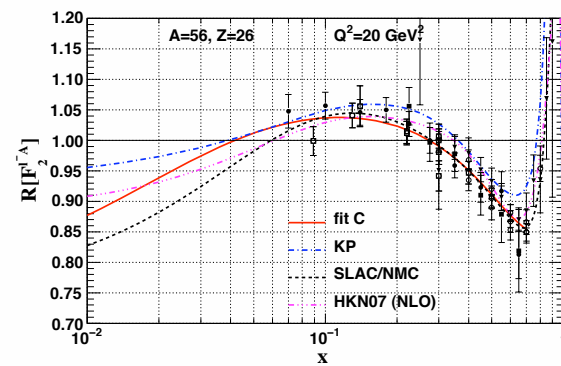
Analysis of fits with different weights of neutrino DIS

- Nuclear correction factors - $R = F_2^{Fe} / F_2^{Fe,0}$

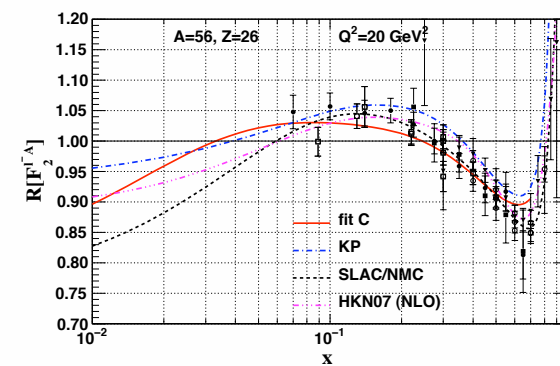
Weight	Name of fit	$l^\pm A$ data	χ^2 (/pt)	νA data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	638 (0.90)	-	-	638 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nuanual	-	-	3134	4192 (1.33)	4192 (1.33)



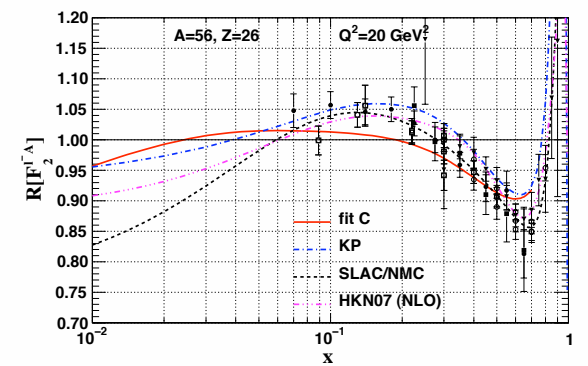
decut3 ($w=0$)



glofac1a ($w=1/7$)



global2b ($w=1$)



nuanual - only neutrino DIS

NEUTRINO DIS

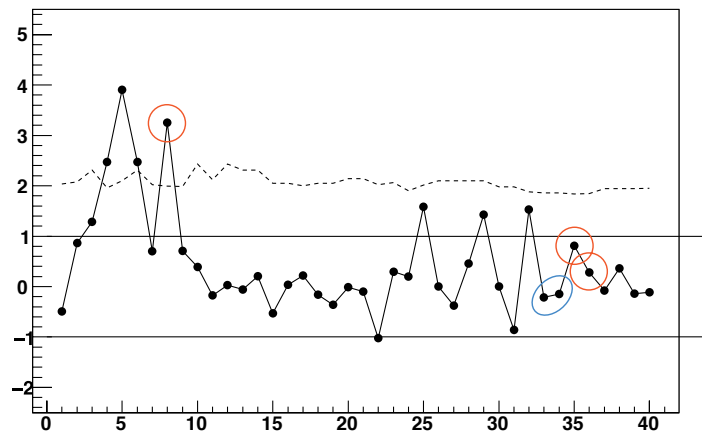
Analysis of fits with different weights of neutrino DIS

- χ^2 - distribution criterion

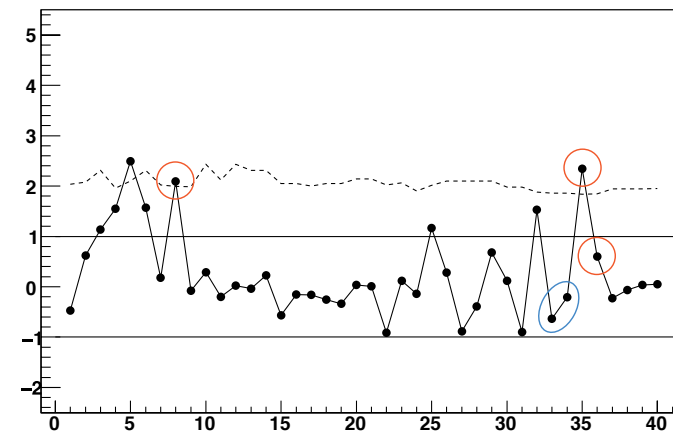
$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

error PDFs defined as 90% C.L. $\rightarrow \int_0^{\xi_{90}} P(\chi^2, N) d\chi^2 = 0.90$

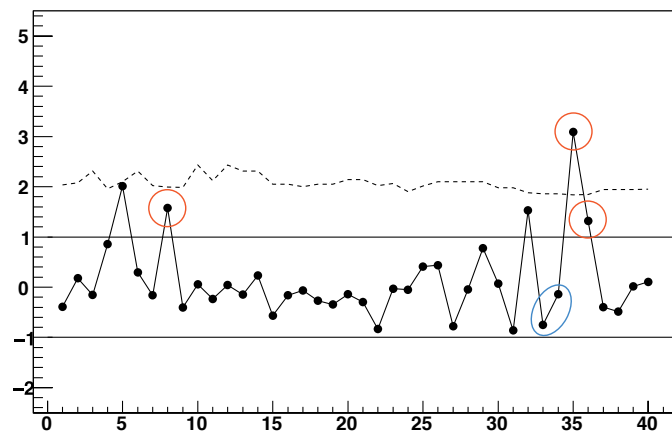
(w=1)



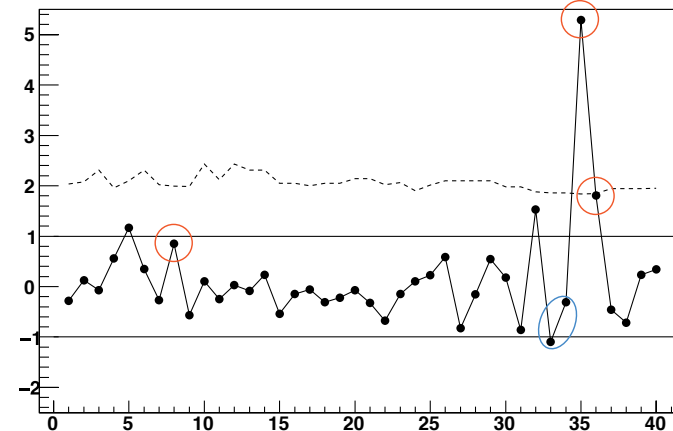
(w=1/2)



(w=1/4)



(w=1/7)



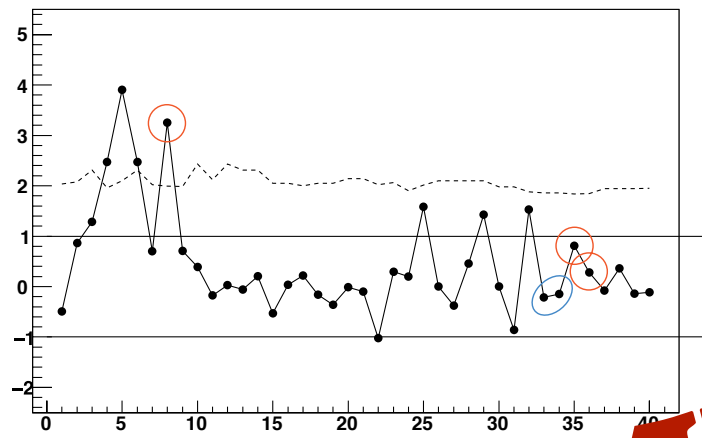
NEUTRINO DIS

● Analysis of fits with different weights of neutrino DIS

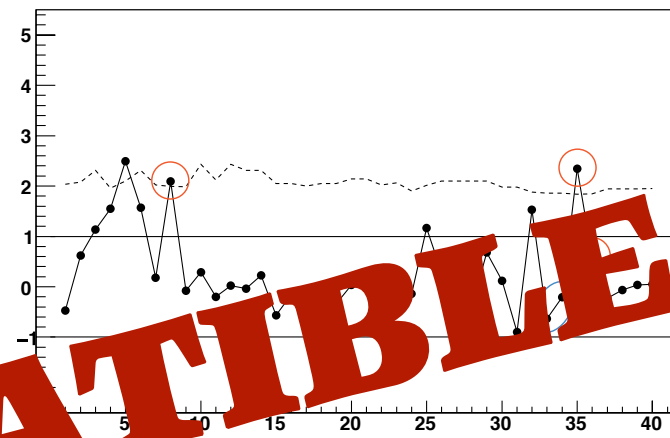
- χ^2 - distribution criterion $P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$

error PDFs defined as 90% C.L. $\rightarrow \int_0^{\xi_{90}} P(\chi^2, N) d\chi^2 = 0.90$

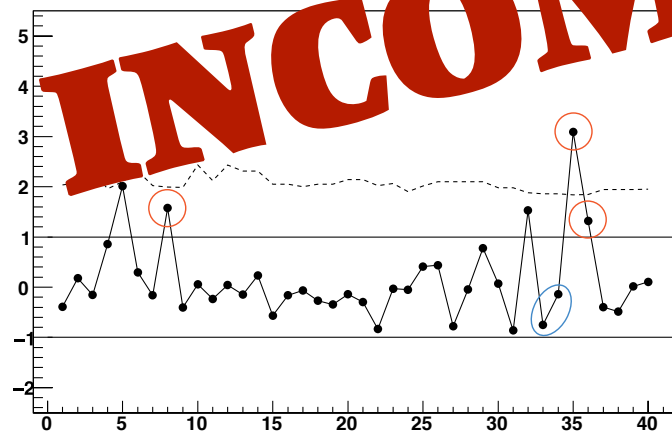
(w=1)



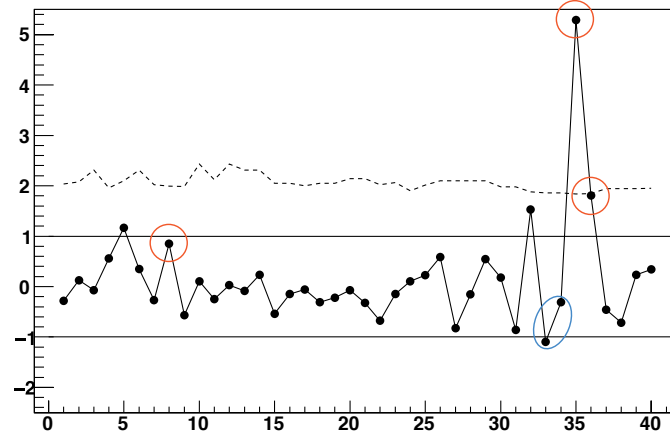
(w=1/2)



(w=1/4)



(w=1/7)



INCOMPATIBLE

CONCLUSIONS

- Global nuclear CTEQ fit is able to describe the charged lepton data well
 - hope to release a first public nPDF set in not so distant future
- Greatest challenges on the way to public nPDFs
 - relax kinematical cuts and fit Fermi motion peak in a natural way
 - error PDFs and an error estimate of PDF
 - find a way how to constrain the gluon
- Incompatibility of neutrino DIS with charged lepton DIS
 - incompatibility a "precision" effect - the result changes when using uncorrelated errors
 - understand the high χ^2 of the NuTeV data
 - can NOMAD data help ?