Monte Carlo simulations of hadronic collisions

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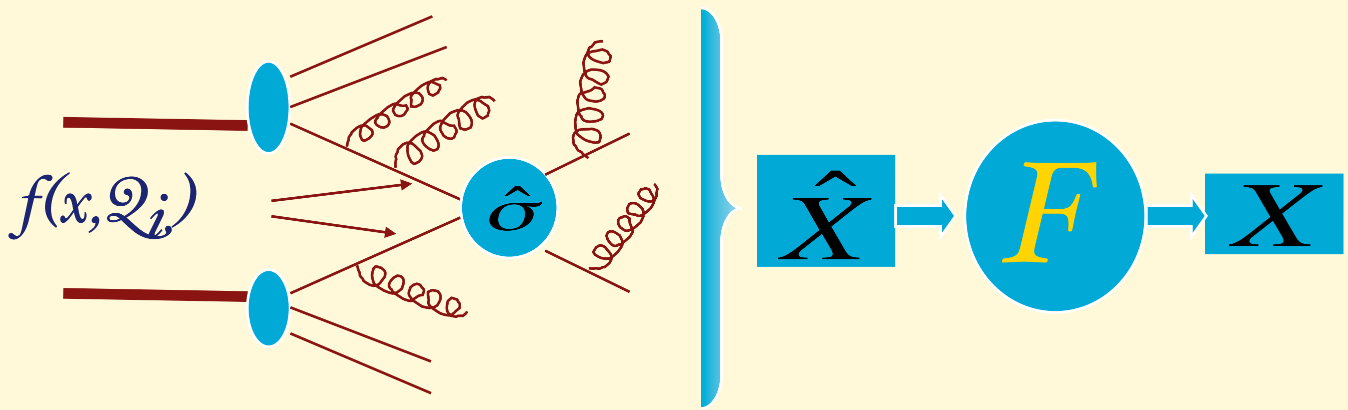
Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1, Q_i)\, f_k(x_2, Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i, Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$f_j(x, Q)$ Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

$$\vec{p}_j = x\, \vec{P}_{proton}$$

$F(\hat{X} \to X; Q_i, Q_f)$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

- The possible histories of initial and final state, and their relative probabilities, are in principle independent of the hard process (they only depend on the flavours of partons involved and on the scales Q)
- Once an algorithm is developed to describe initial (IS) and final (FS) state evolution, it can be applied to partonic IS and FS arising from the calculation of an arbitrary hard process
- Depending on the extent to which different possible FS and IS histories affect the value of the observable $\boldsymbol{X}$, different realizations of the factorization theorem can be implemented, and 3 different tools developed:

    1. **Cross-section evaluators**
    2. **Parton-level Monte Carlos**
    3. **Shower Monte Carlos**

1: Cross-section evaluators

- Only some component of the final state is singled out for the measurement, all the rest being ignored (i.e. integrated over). E.g. $pp \to e^+e^- + X$
- No 'events' are 'generated', only cross-sections are evaluated:

$$\sigma\left(pp \to Z^0\right), \quad \frac{d\sigma}{dM(e^+e^-)\,dy(e^+e^-)}, \quad \ldots$$

Experimental selection criteria (e.g. jet definition or acceptance) are applied on parton-level quantities. Provided these are infrared/collinear finite, it therefore doesn't matter what **F(X)** is, as we assume (*fact. theorem*) that:

$$\sum_X F(\hat{X}, X) = 1 \quad \forall \hat{X}$$

- Thanks to the inclusiveness of the result, it is `straightforward' to include higher-order corrections, as well as to resum classes of dominant and subdominant logs

State of the art

- NLO available for:
  - jet and heavy quarks production
  - prompt photon production
  - gauge boson pairs
  - most new physics processes (e.g. SUSY)
- NNLO available for:
  - W/Z/DY production $(q\bar{q} \to W)$
  - Higgs production $(gg \to H)$

2: Parton-level (*aka* matrix-element) MC's

- Parton level configurations (i.e. sets of quarks and gluons) are generated, with probability proportional to the respective perturbative M.E.
- Transition function between a final-state parton and the observed object (jet, missing energy, lepton, etc) is unity
- No need to expand $f(x)$ or $F(X)$ in terms of histories, since they all lead to the same observable
- Experimentally, equivalent to assuming
  - perfect jet reconstruction ($\mathbf{P}_\mu{}^{parton} \rightarrow \mathbf{P}_\mu{}^{jet}$)
  - linear detector response

State of the art

- W/Z/gamma + N jets ($N \le 6$)
- W/Z/gamma + Q Qbar + N jets ($N \le 4$)
- Q Qbar + N jets ($N \le 4$)
- Q Qbar Q' Q'bar + N jets ($N \le 2$)
- Q Qbar H + N jets ($N \le 3$)
- nW + mZ + kH + N jets ($n+m+k+N \le 8$, $N \le 2$)
- N jets ($N \le 8$)

ALPGEN: MLM, Moretti, Piccinini, Pittau, Polosa
MADGRAPH: Maltoni, Stelzer
CompHEP: Boos etal
VECBOS: Giele et al
NJETS: Giele et al
Kleiss, Papadopoulos
......

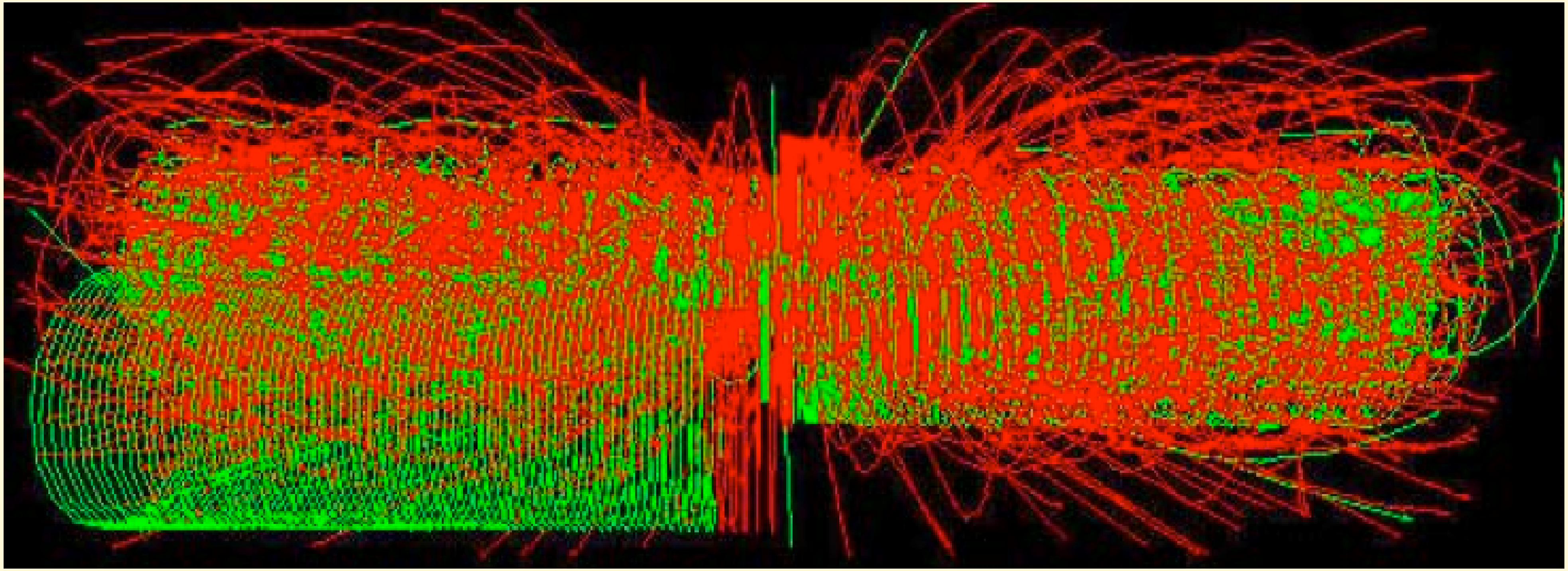
Example of complexity of the calculations, for gg-> N gluons:

Njets	2	3	4	5	6	7	8
# diag's	4	25	220	2485	34300	5×10^5	10^7

For each process, flavour state and colour flow (leading 1/Nc) are calculated on an event-by-event basis, to allow QCD-coherent shower evolution

3: Shower Monte Carlos

Goal: complete description of the event, at the level of individual hadrons

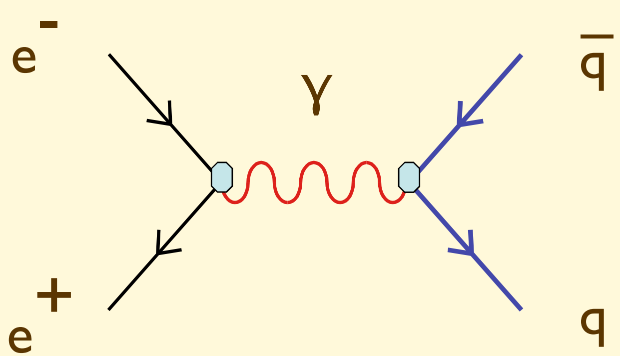


Evolution of hadronic final states

Asymptotic freedom implies that at E_{CM} >> 1 GeV

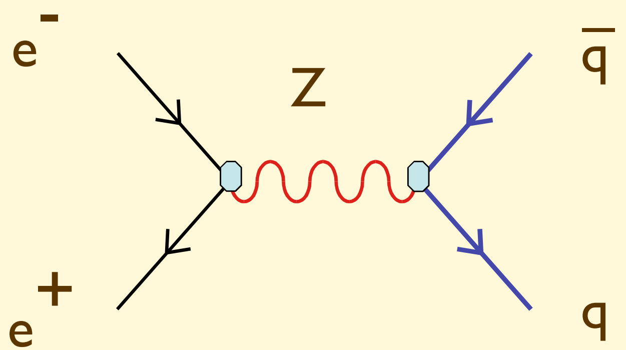
$\sigma(e^+ e^- \rightarrow \text{hadrons}) \longleftrightarrow \sigma(e^+ e^- \rightarrow \text{quarks/gluons})$

At the Leading Order (LO) in PT:



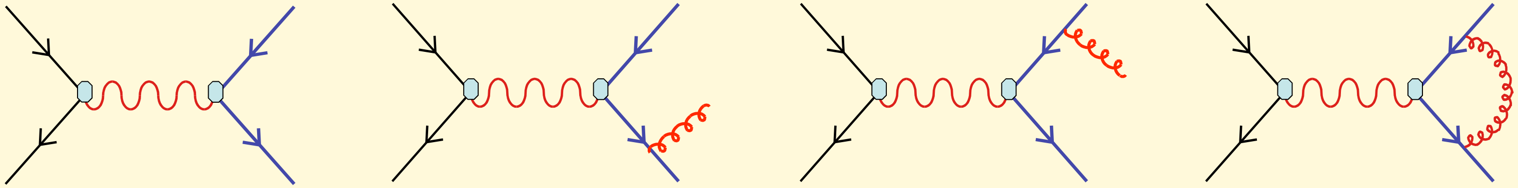
$$\sigma_0(e^+e^- \to q\bar{q}) = \frac{4\pi\alpha^2}{9s} N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$

$$\frac{\sigma_0(e^+e^- \to q\bar{q})}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{f=u,d,\ldots} e_{q_f}^2$$



$$\frac{\sigma_0(e^+e^- \to Z \to q\bar{q})}{\sigma_0(e^+e^- \to Z \to \mu^+\mu^-)} = N_c \frac{\sum_{f=u,d,\ldots}\left(v_{q_f}^2 + a_{q_f}^2\right)}{\left(v_\mu^2 + a_\mu^2\right)}$$

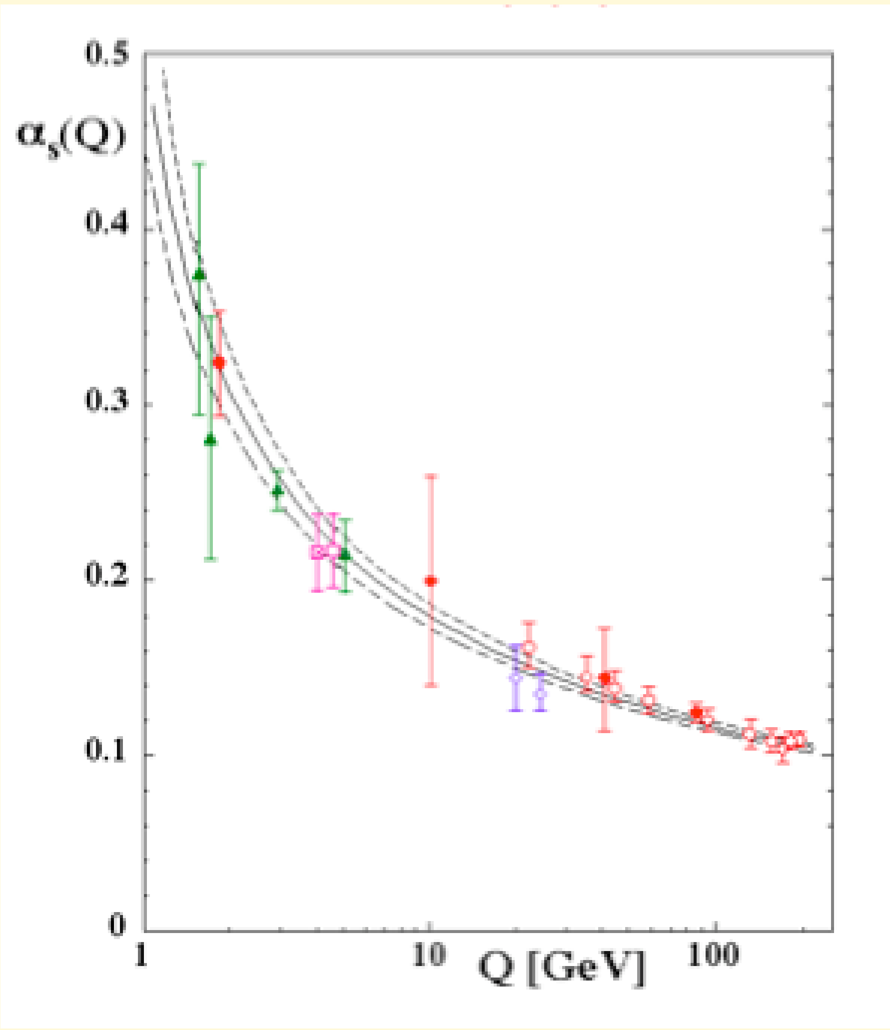
Adding higher-order perturbative terms:



+ $\geq$2-gluon emissions

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

O(3%) at M_Z

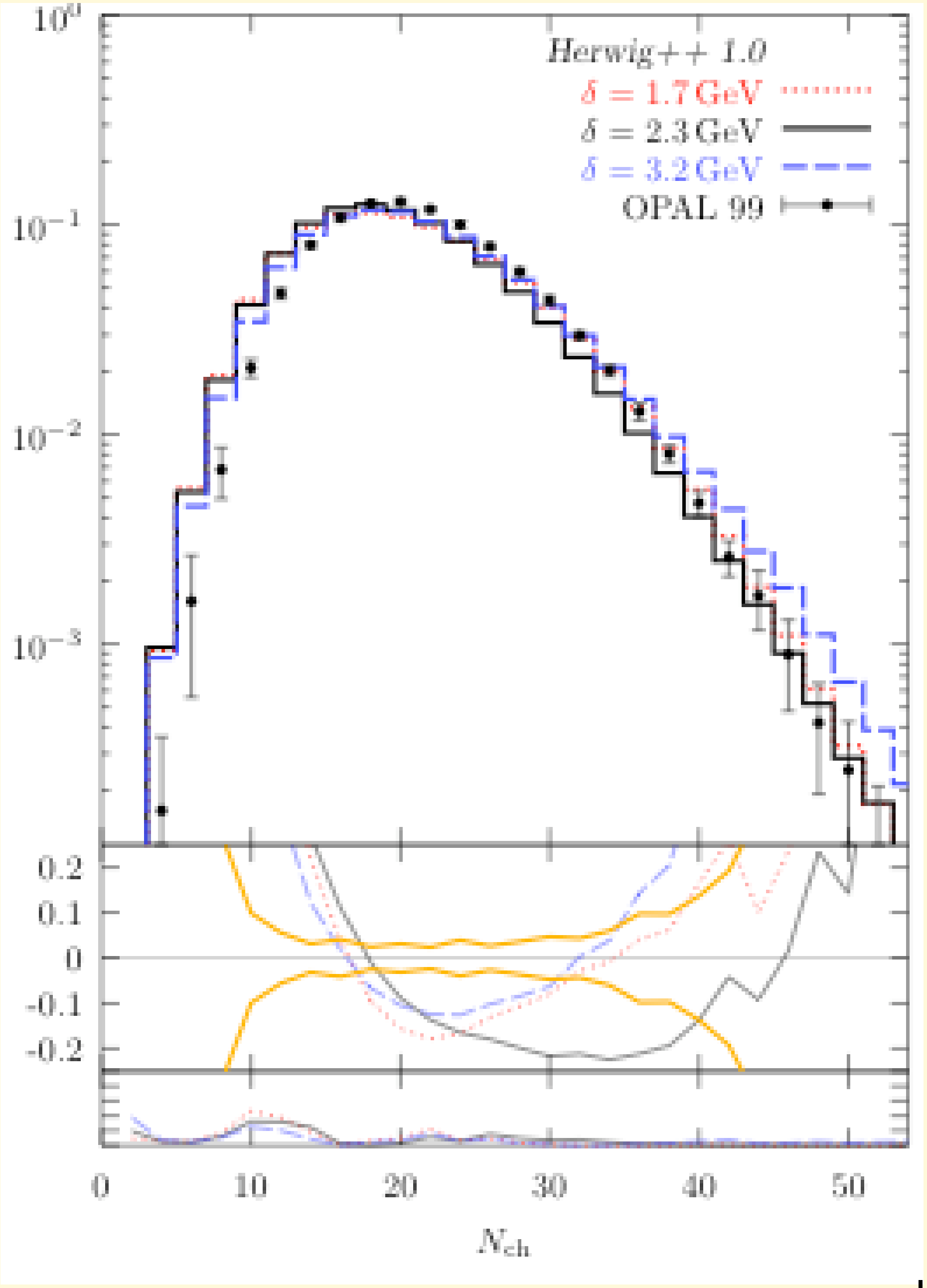


Excellent agreement with data,
provided N_c=3
Extraction of αs consistent with the Q evolution predicted by QCD

Experimentally, the final states contain a large number of particles, not the 2 or 3 which apparently saturate the perturbative cross-section.

Experimental multiplicity distribution

$\langle n_{charged} \rangle = 20.9$



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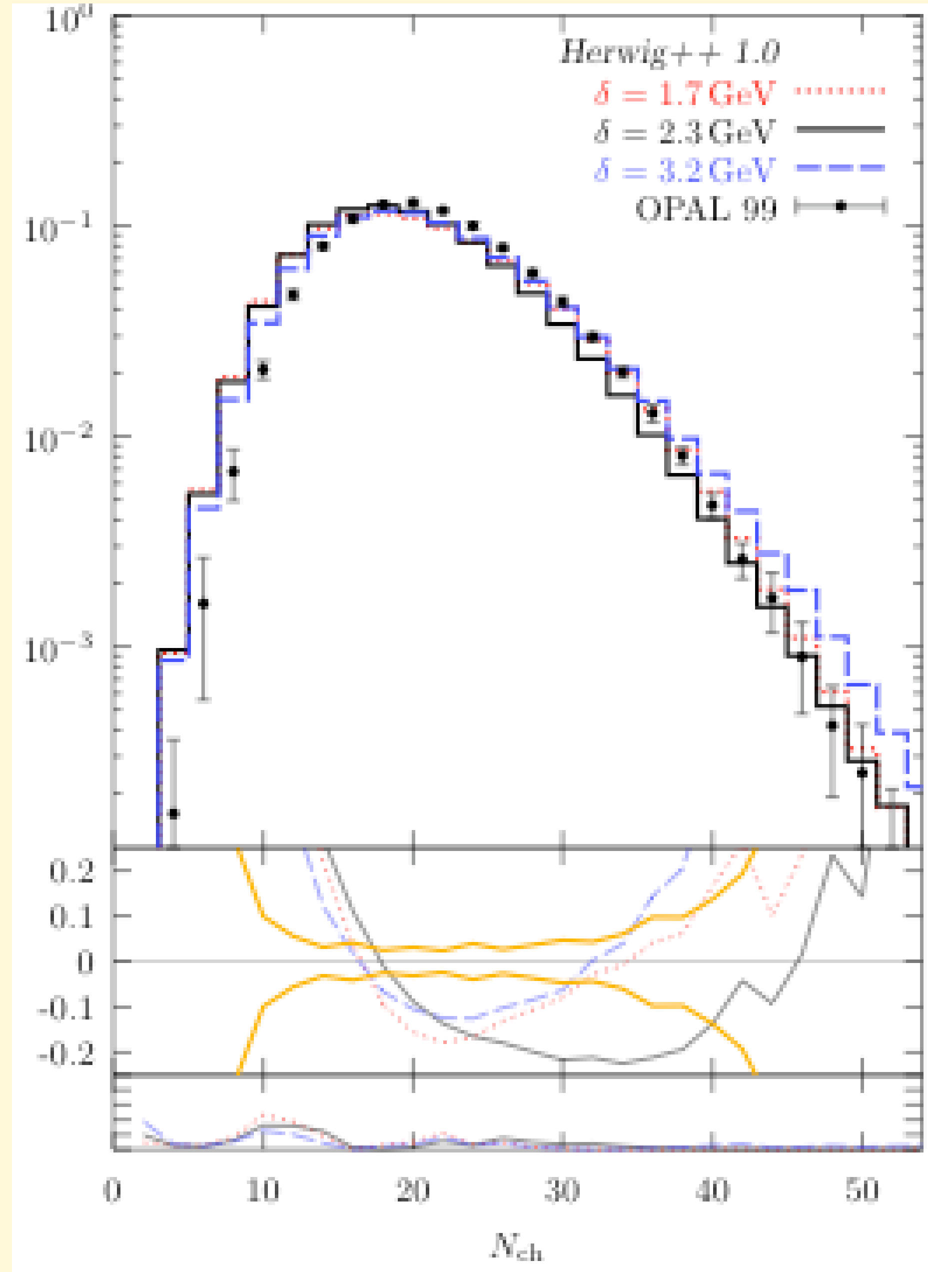
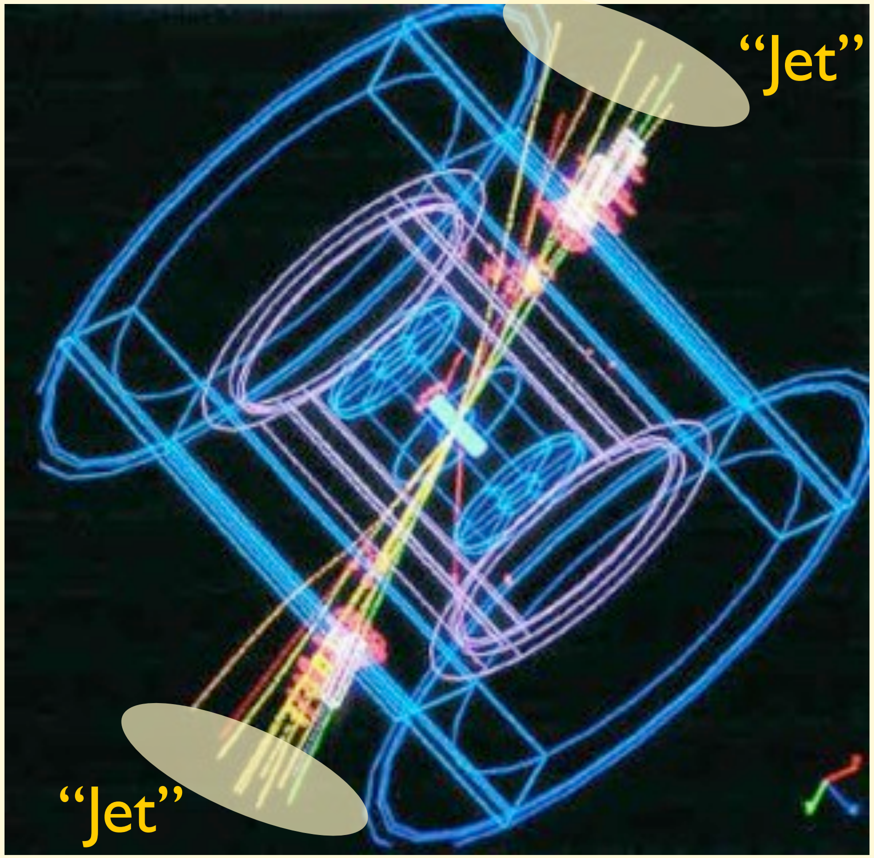
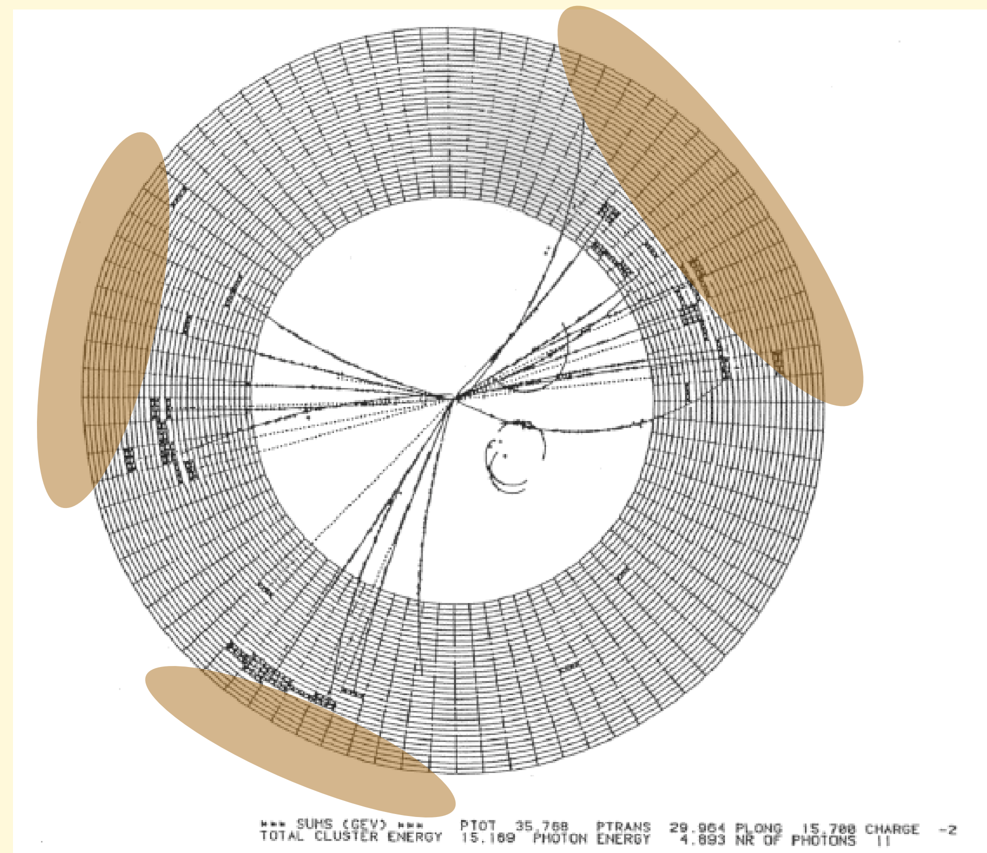


$\langle n_{charged} \rangle = 20.9$

Isn't this bizarre?

Look more closely at the structure of these events:





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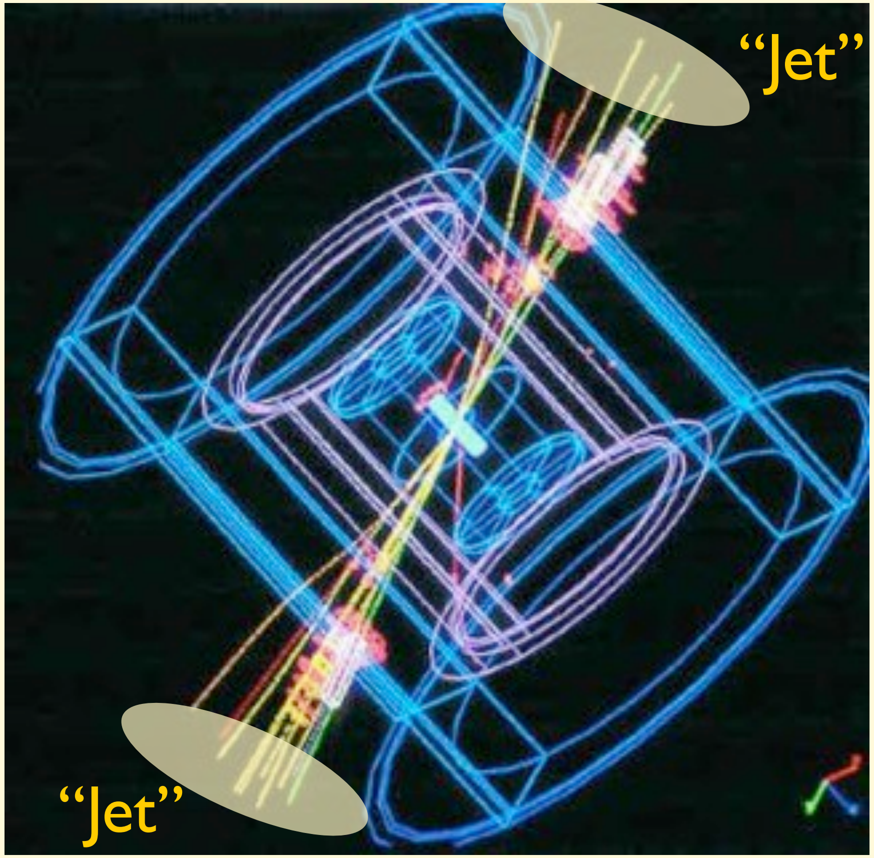
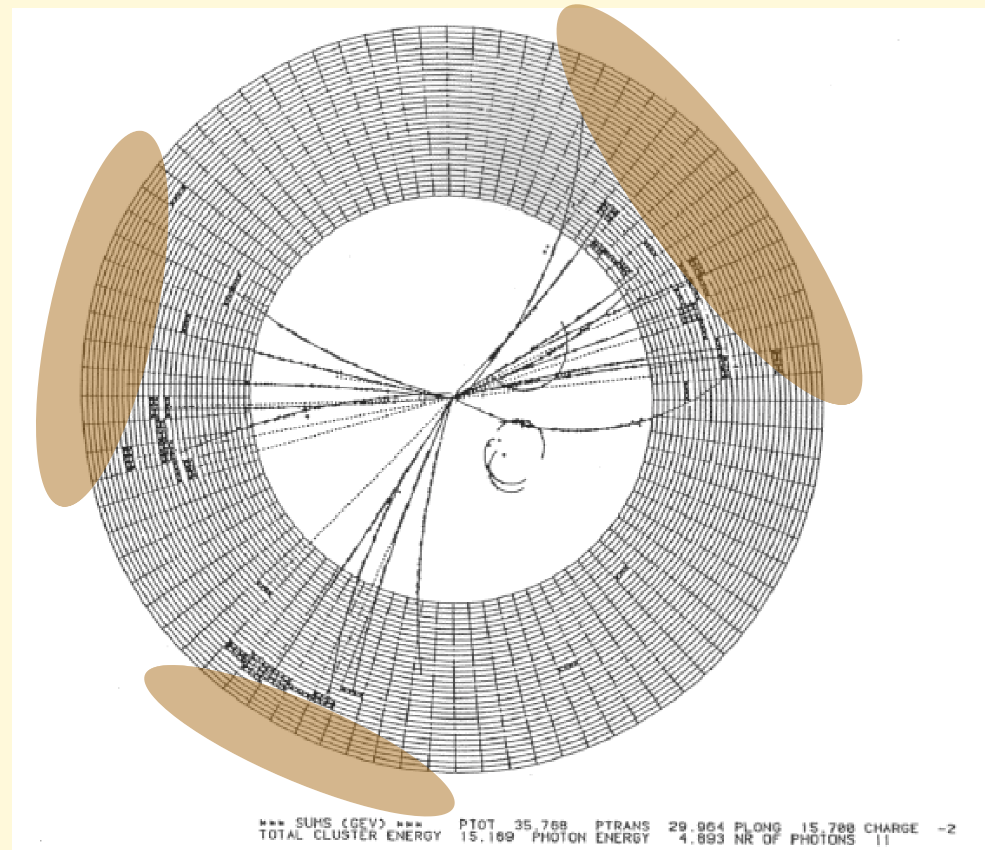


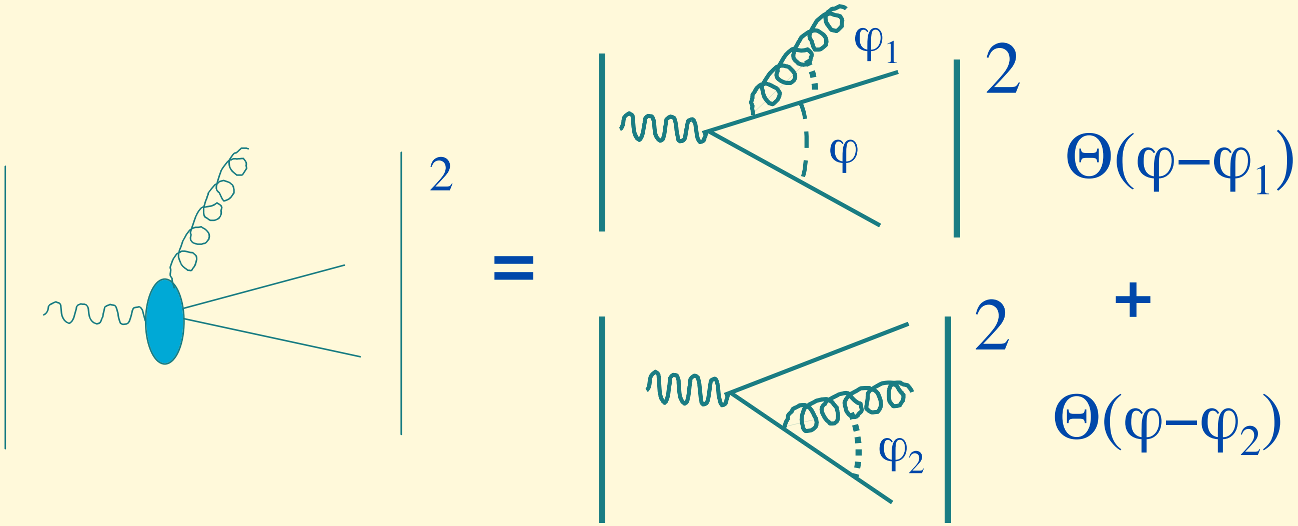


$e^+ e^- \to qq \;\Rightarrow\; e^+ e^- \to 2$ jets

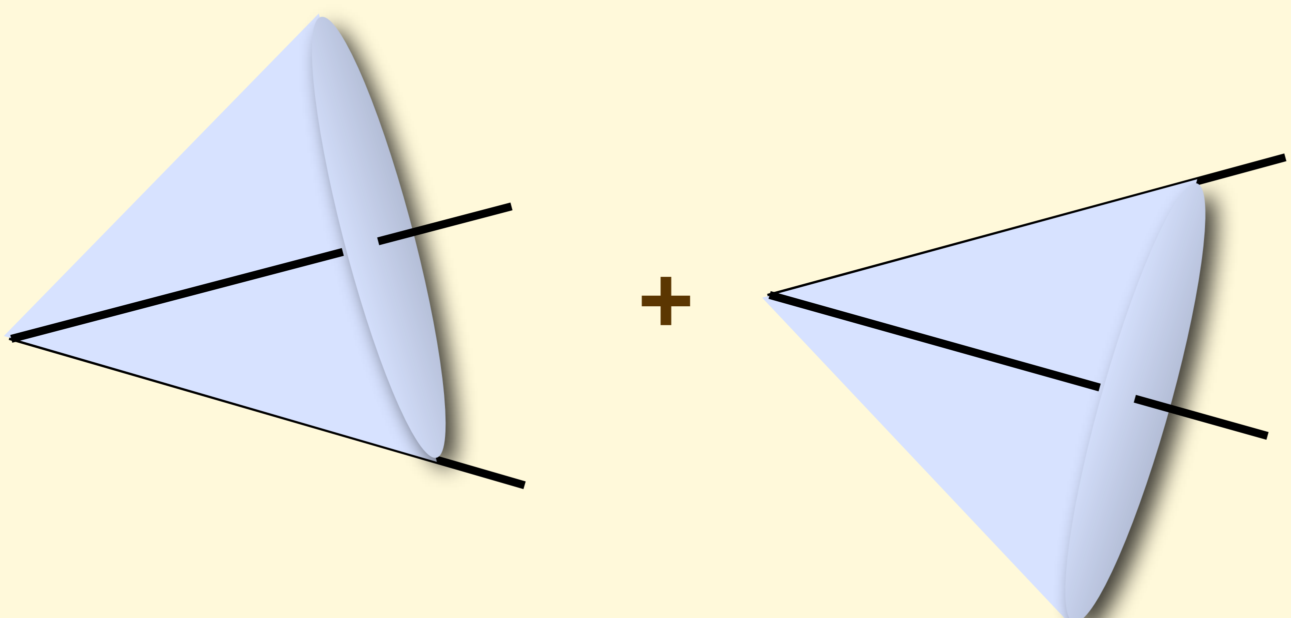
$e^+ e^- \to qqg \;\Rightarrow\; e^+ e^- \to 3$ jets

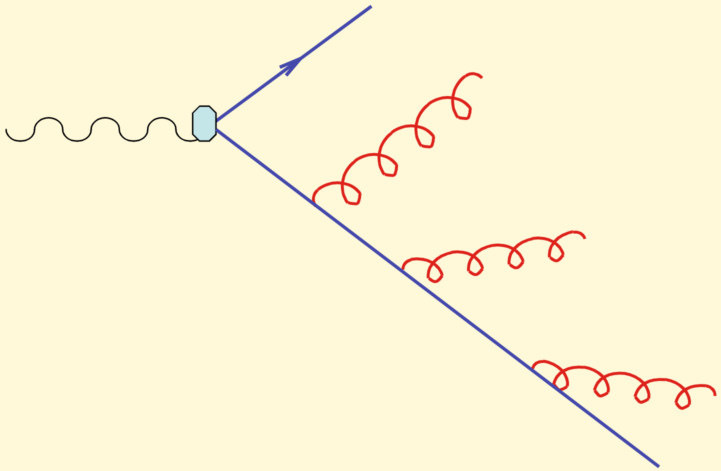
The puzzle is solved by associating partons to collimated “**jets**” of hadrons

Angular ordering

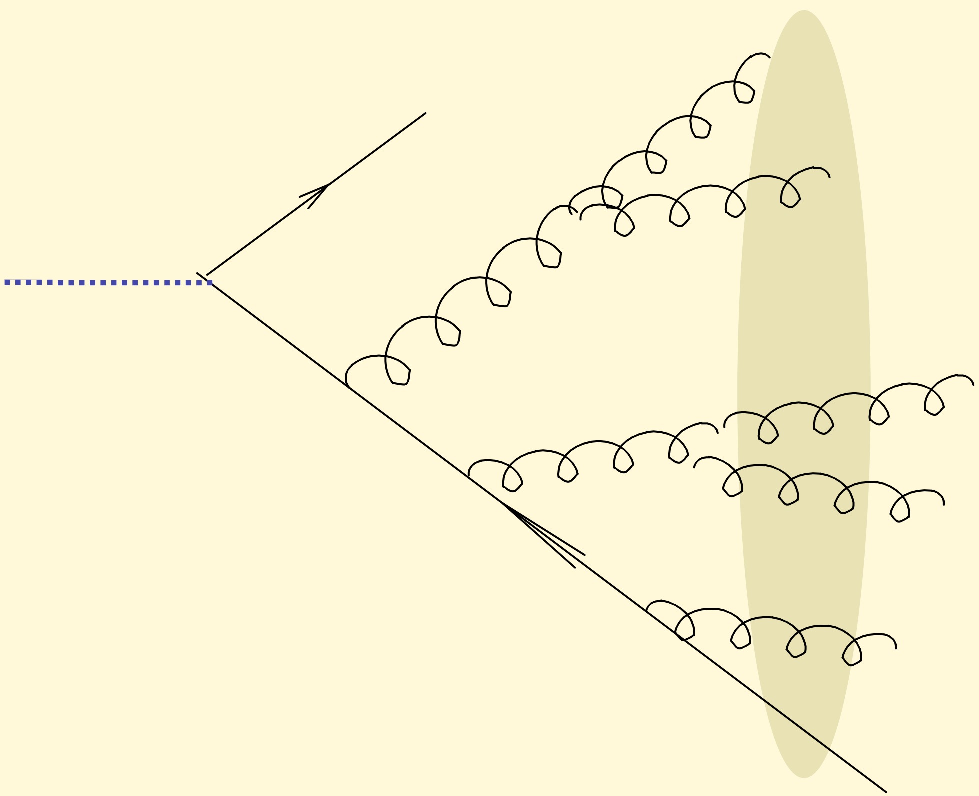


Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



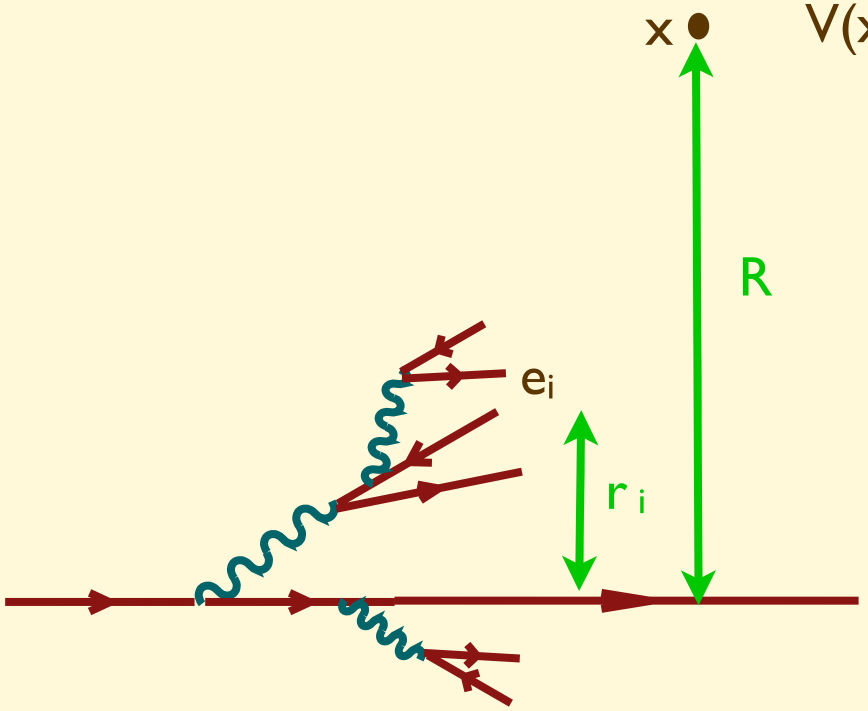


The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**

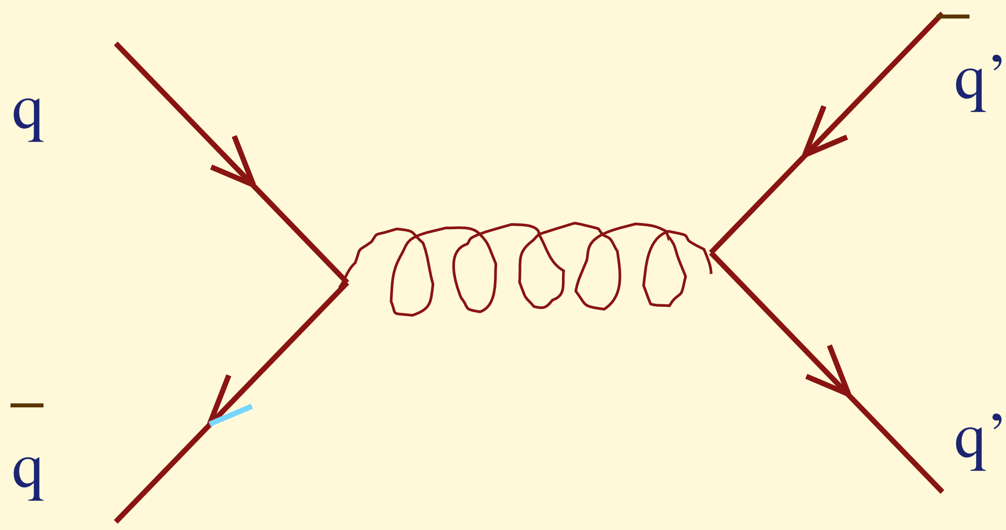


Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of N gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.

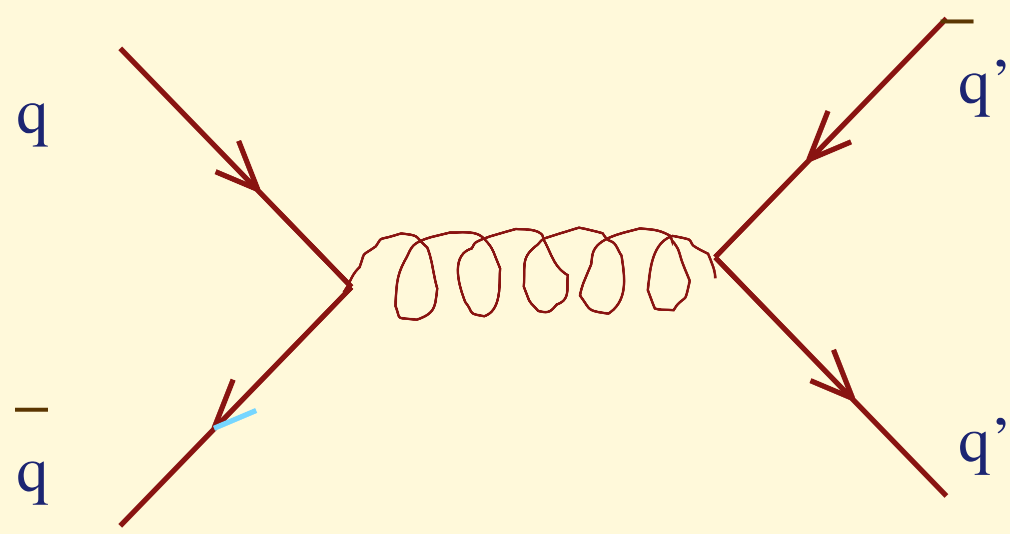
$$V(x) = \frac{e}{R} + \frac{1}{R^2}\left|\sum e_i \boldsymbol{r}_i\right|$$



I: Generate the parton-level hard event

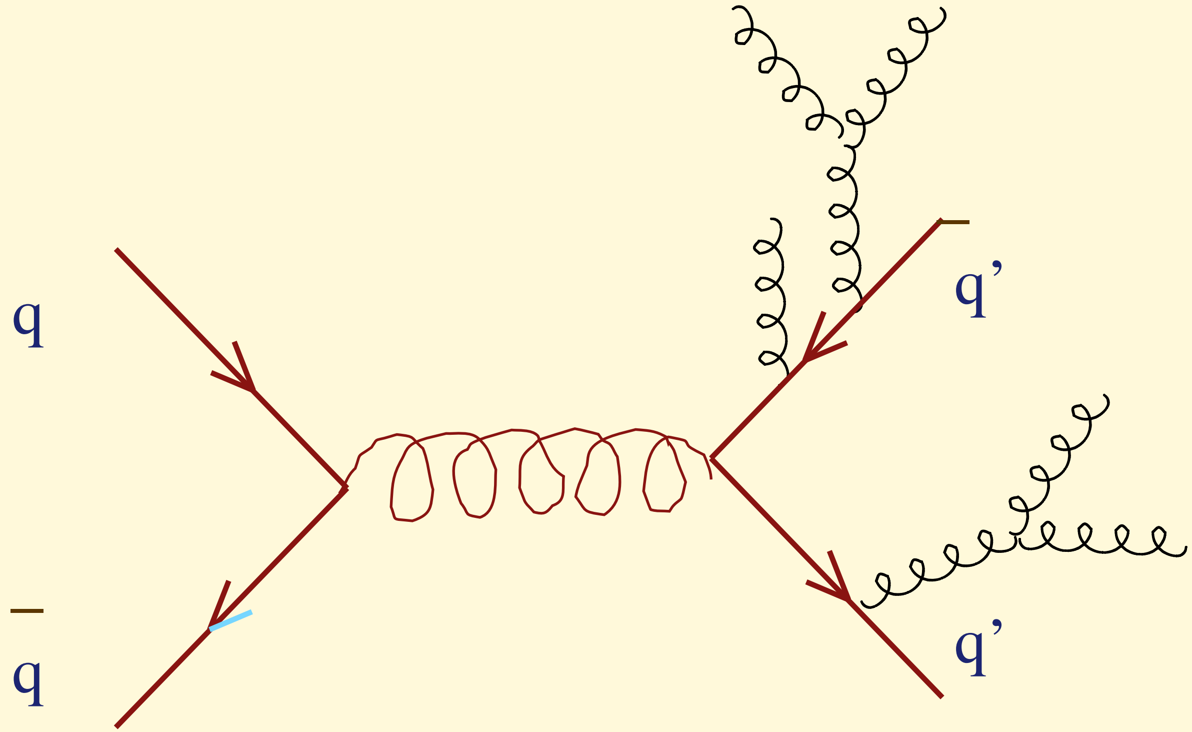


II: Develop the parton shower



II: Develop the parton shower

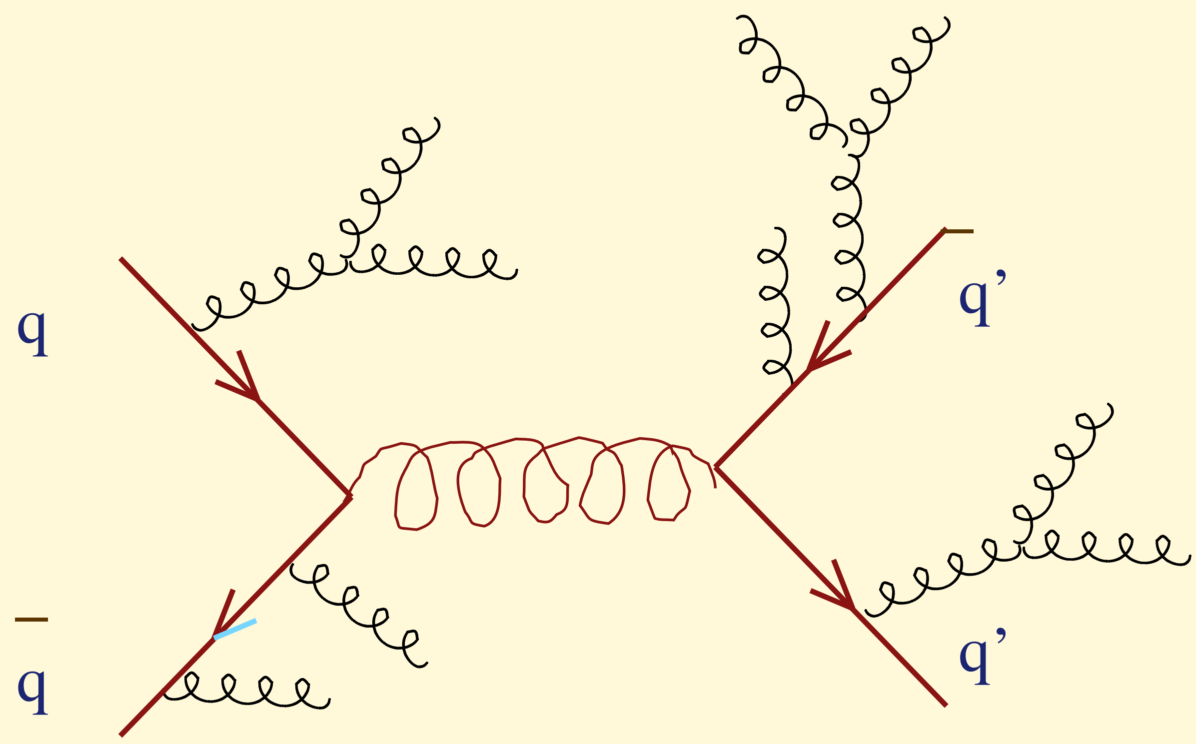
1. Final state



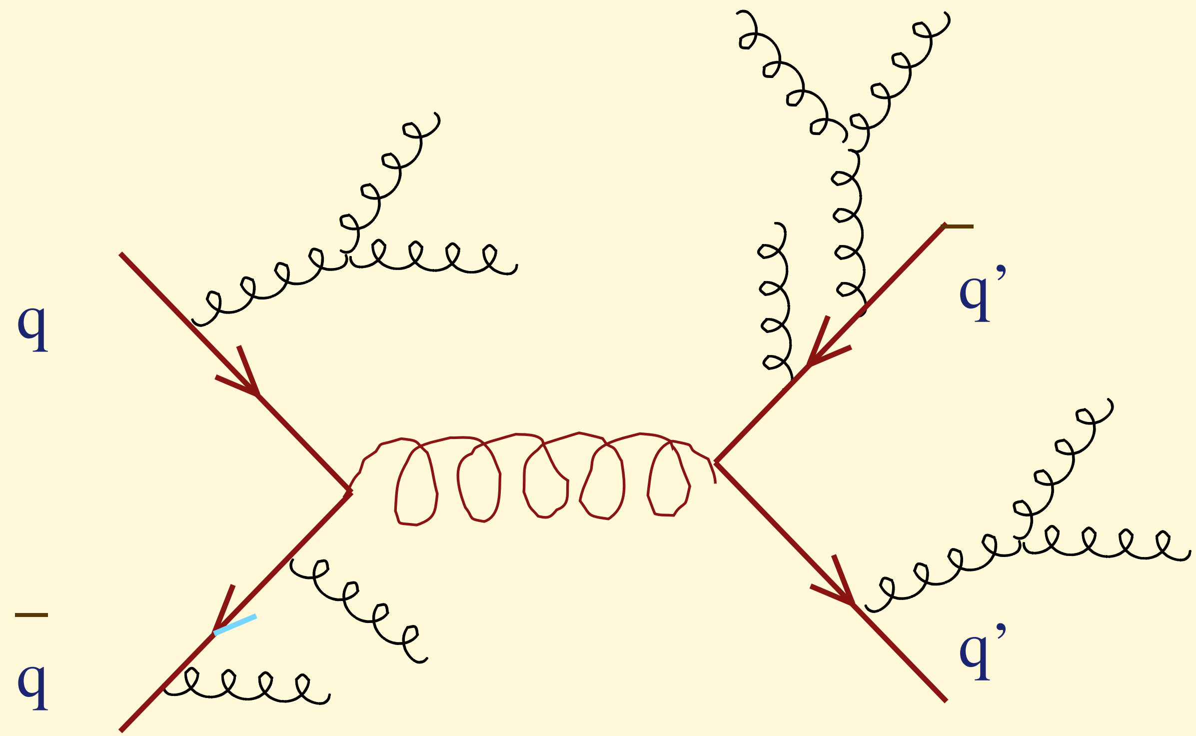
II: Develop the parton shower

1. Final state

2. Initial state

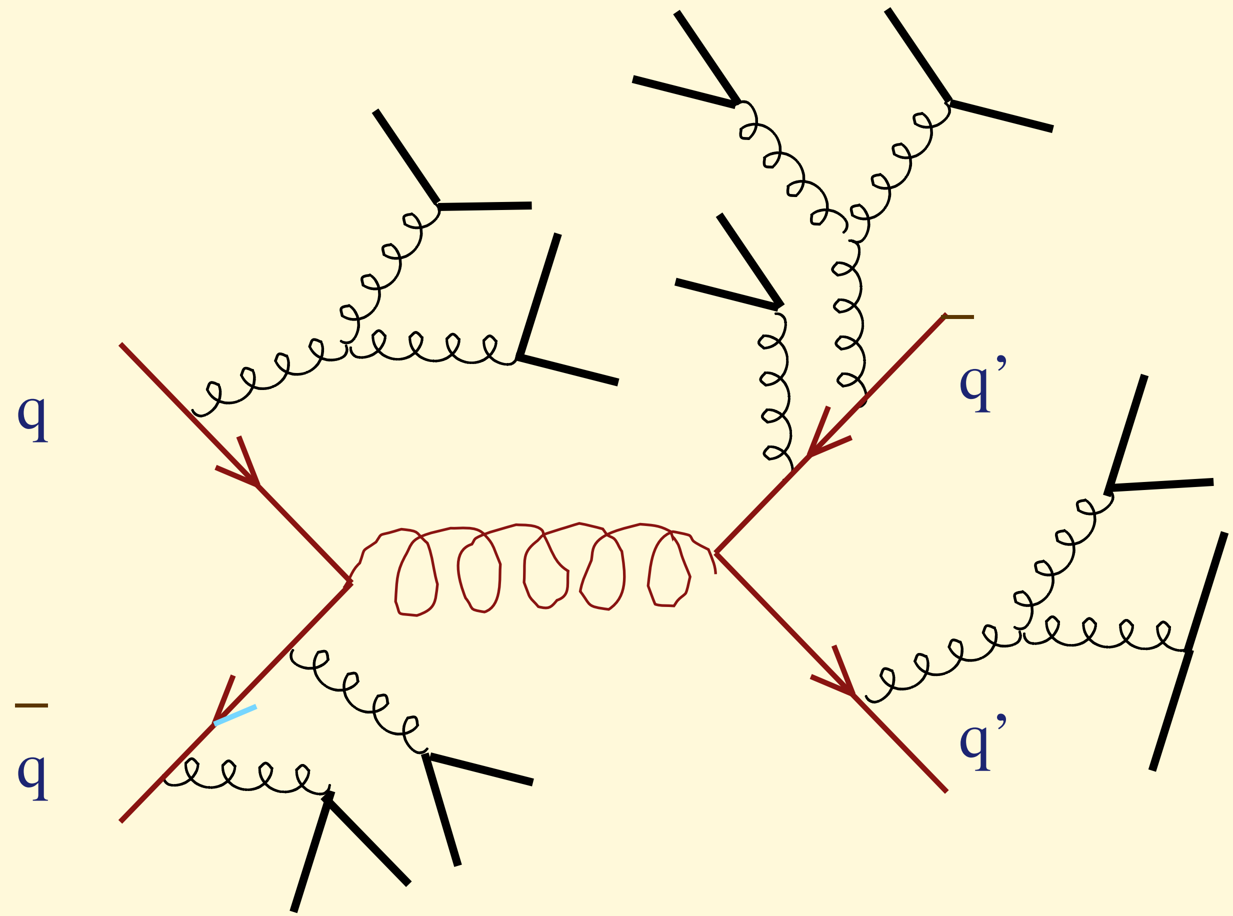


III: Partons Hadronize



III: Partons Hadronize

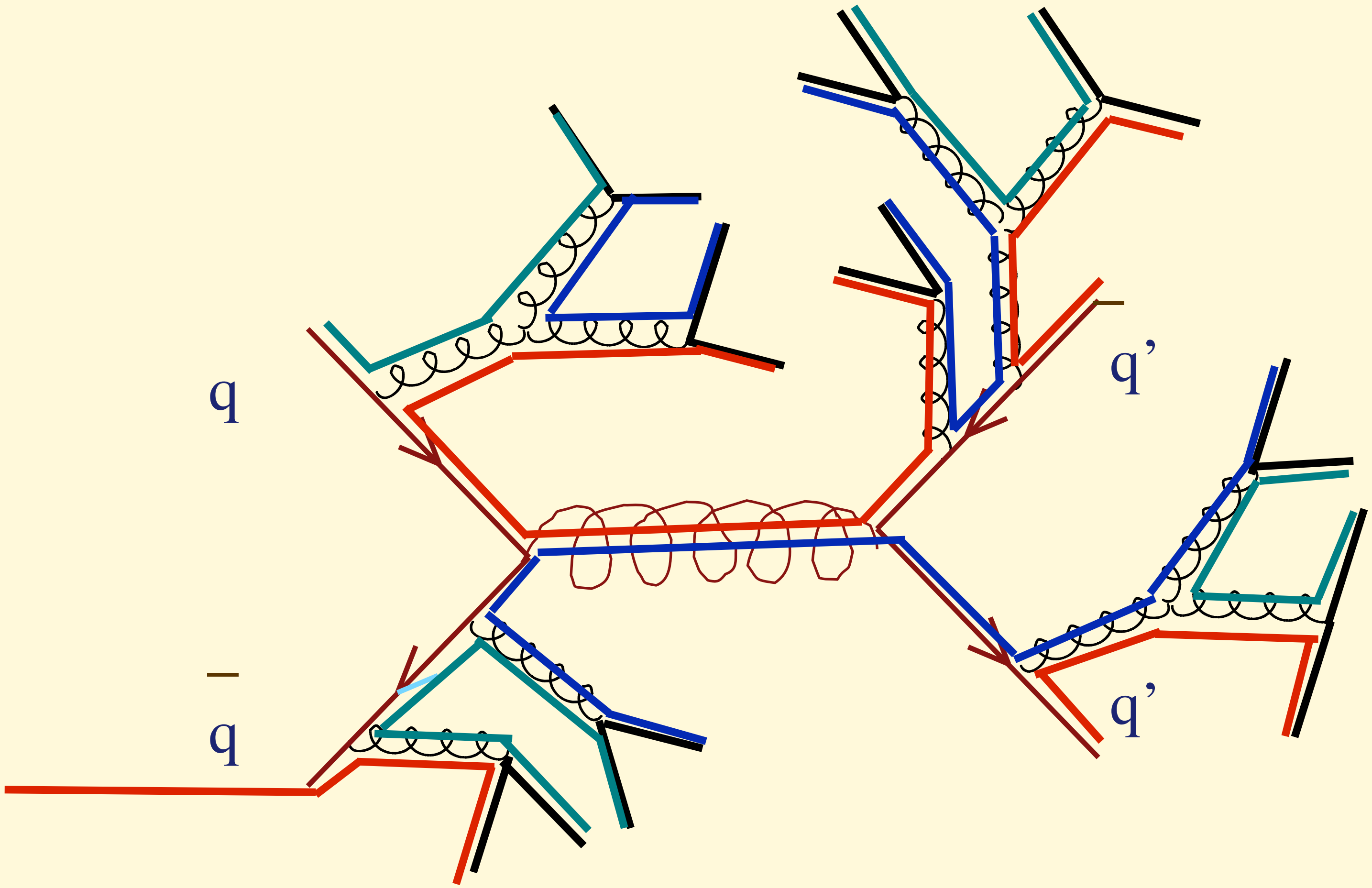
1. Split gluons into q-qbar pairs



III: Partons Hadronize

1. Split gluons into q-qbar pairs

2. Connect colour-singlet pairs



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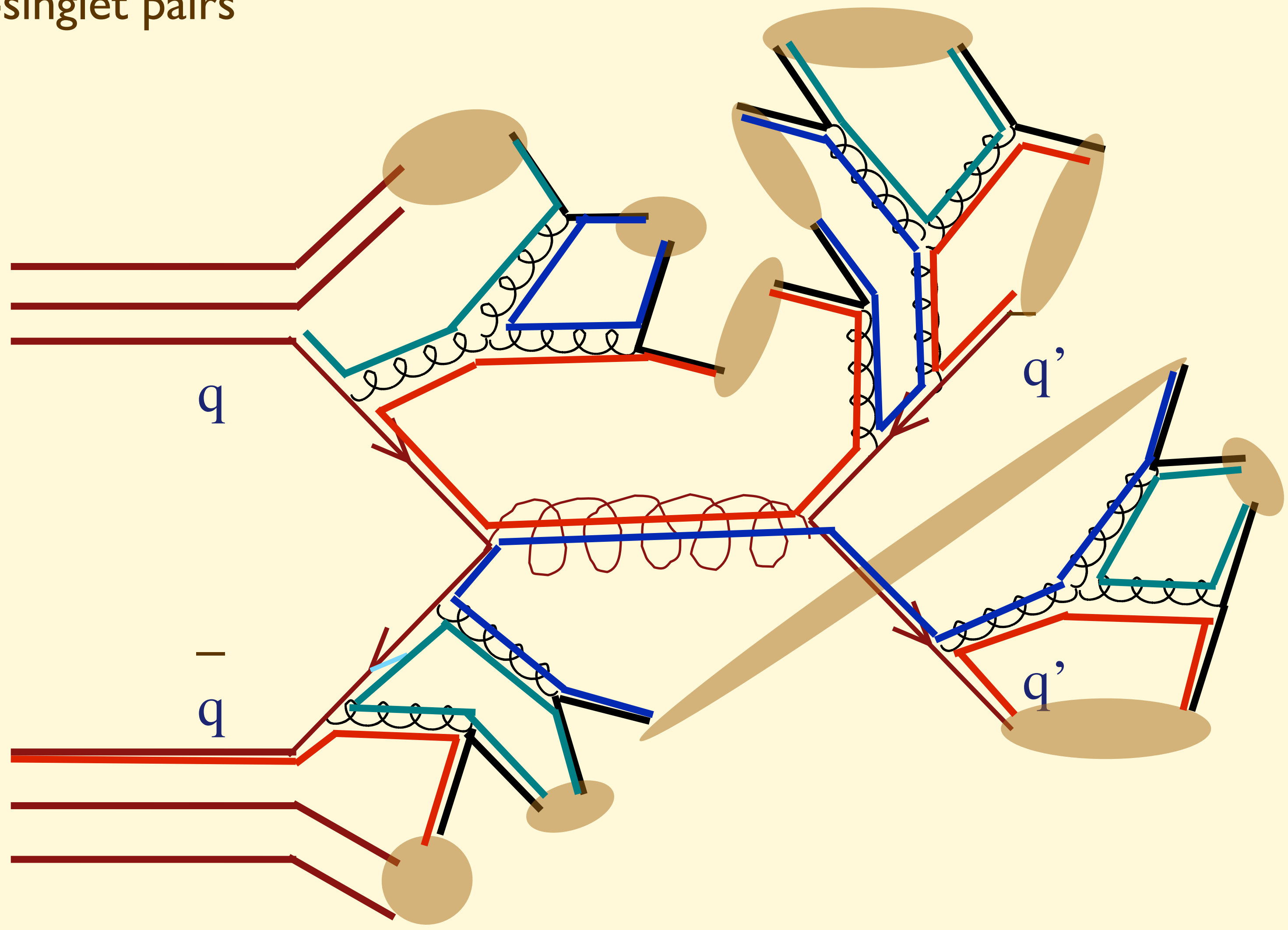
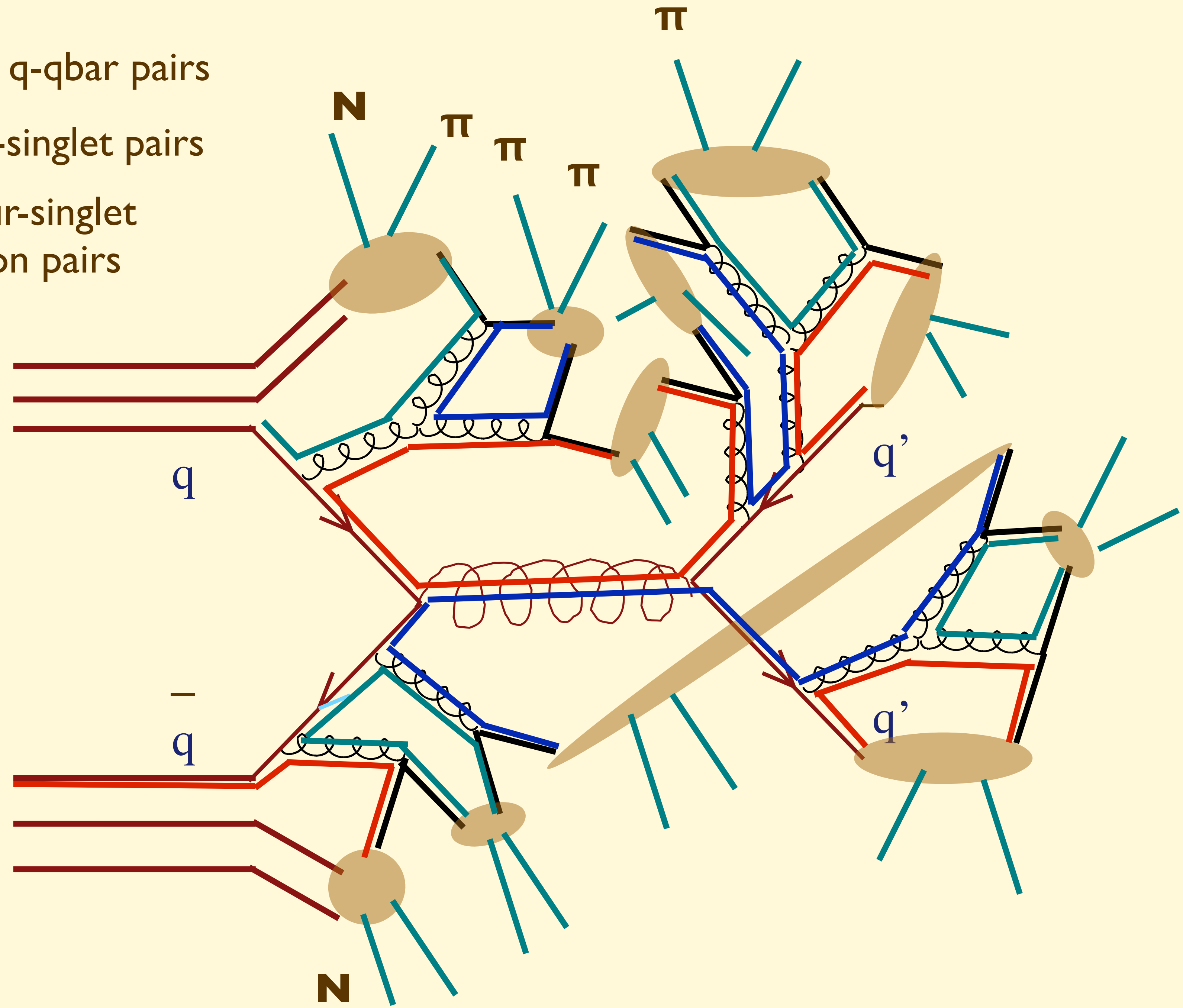
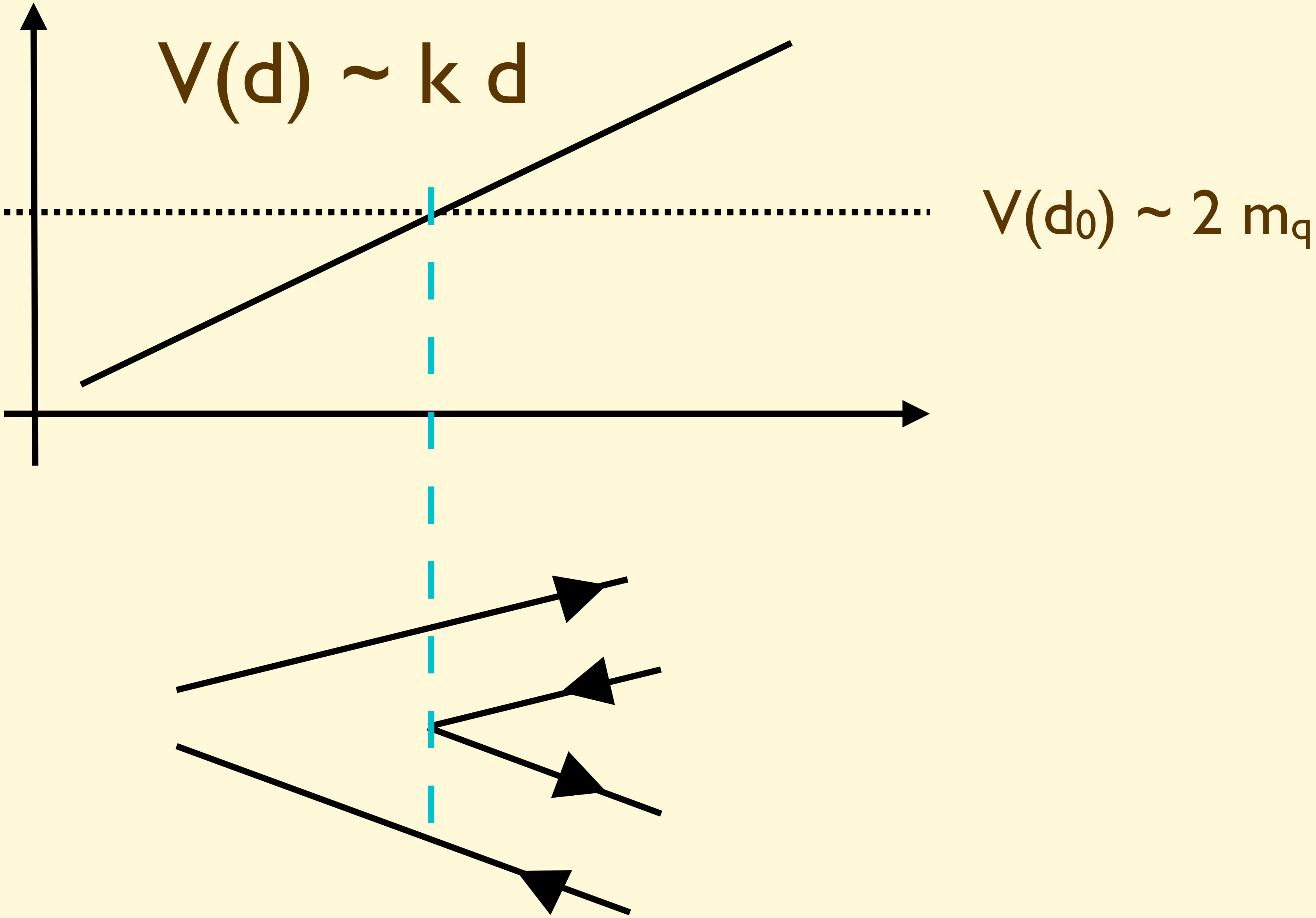


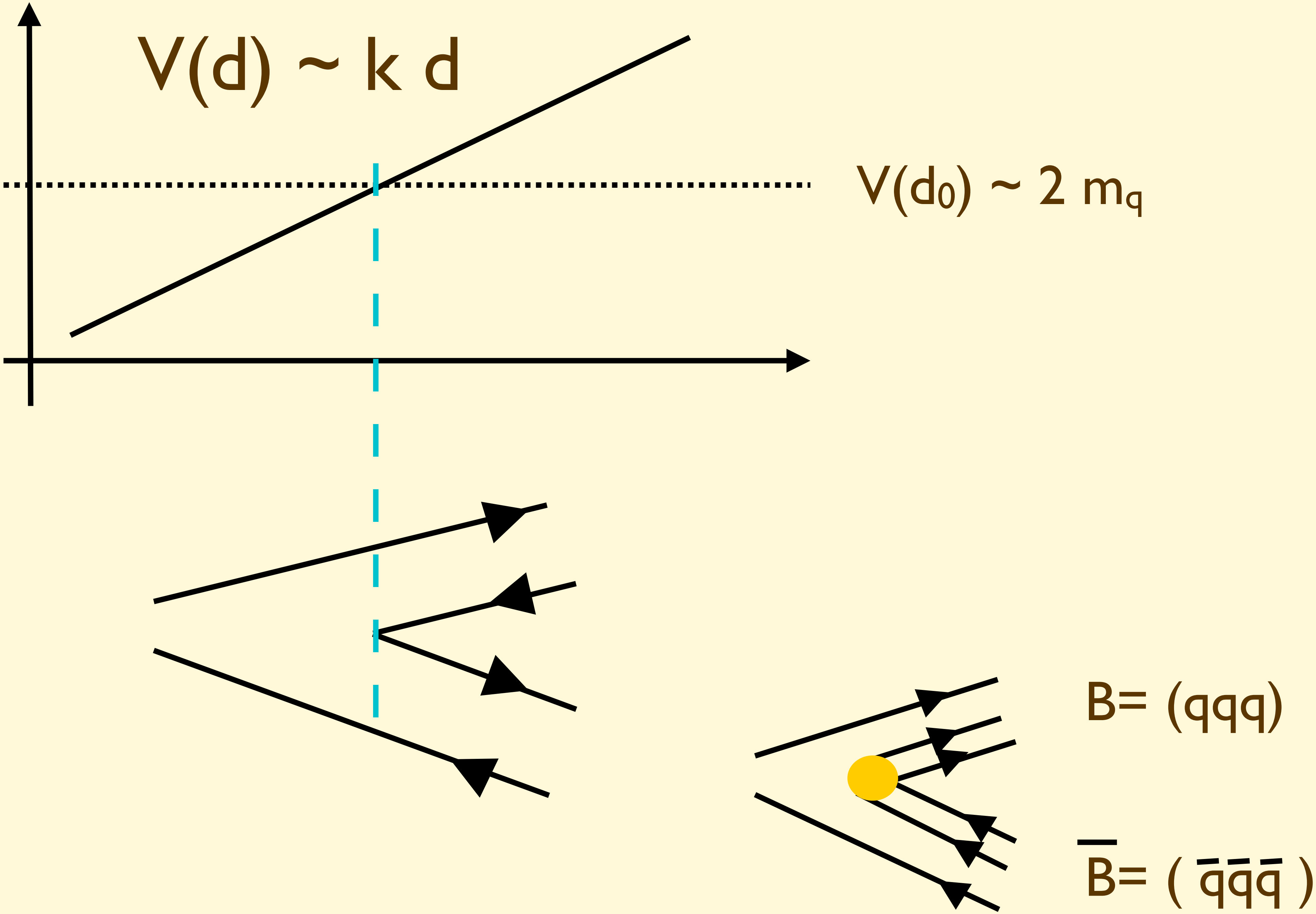
III: Partons Hadronize

1. Split gluons into q-qbar pairs
2. Connect colour-singlet pairs
3. Decay the colour-singlet clusters into hadron pairs



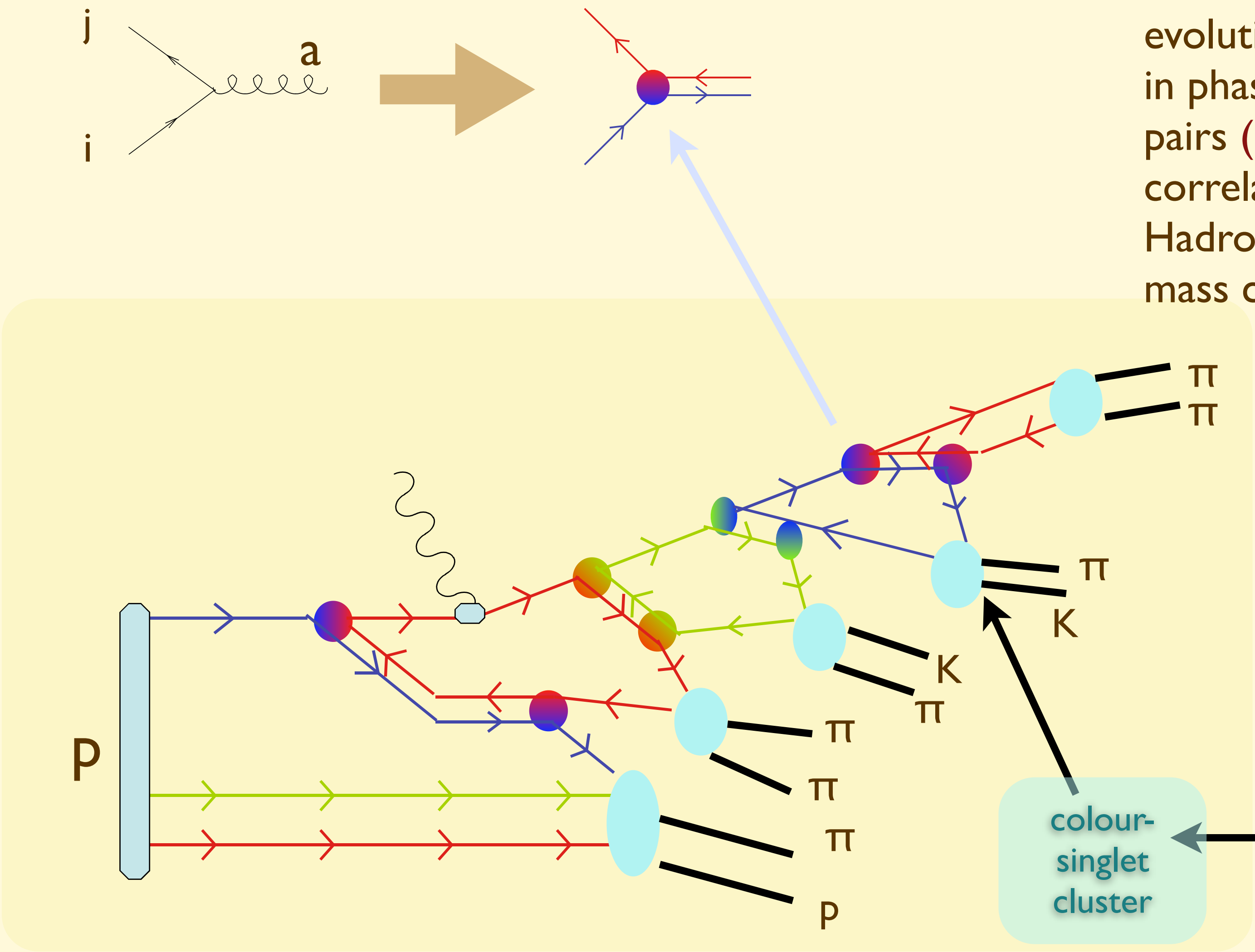


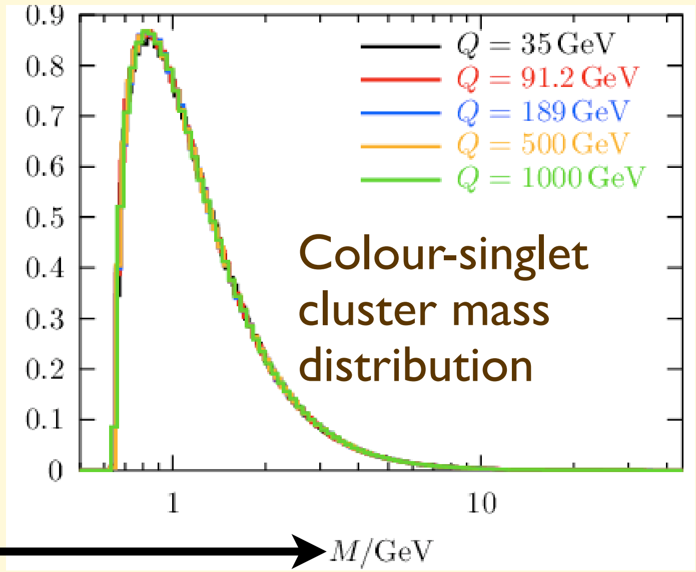
V(d) ~ k d
V(d₀) ~ 2 m_q

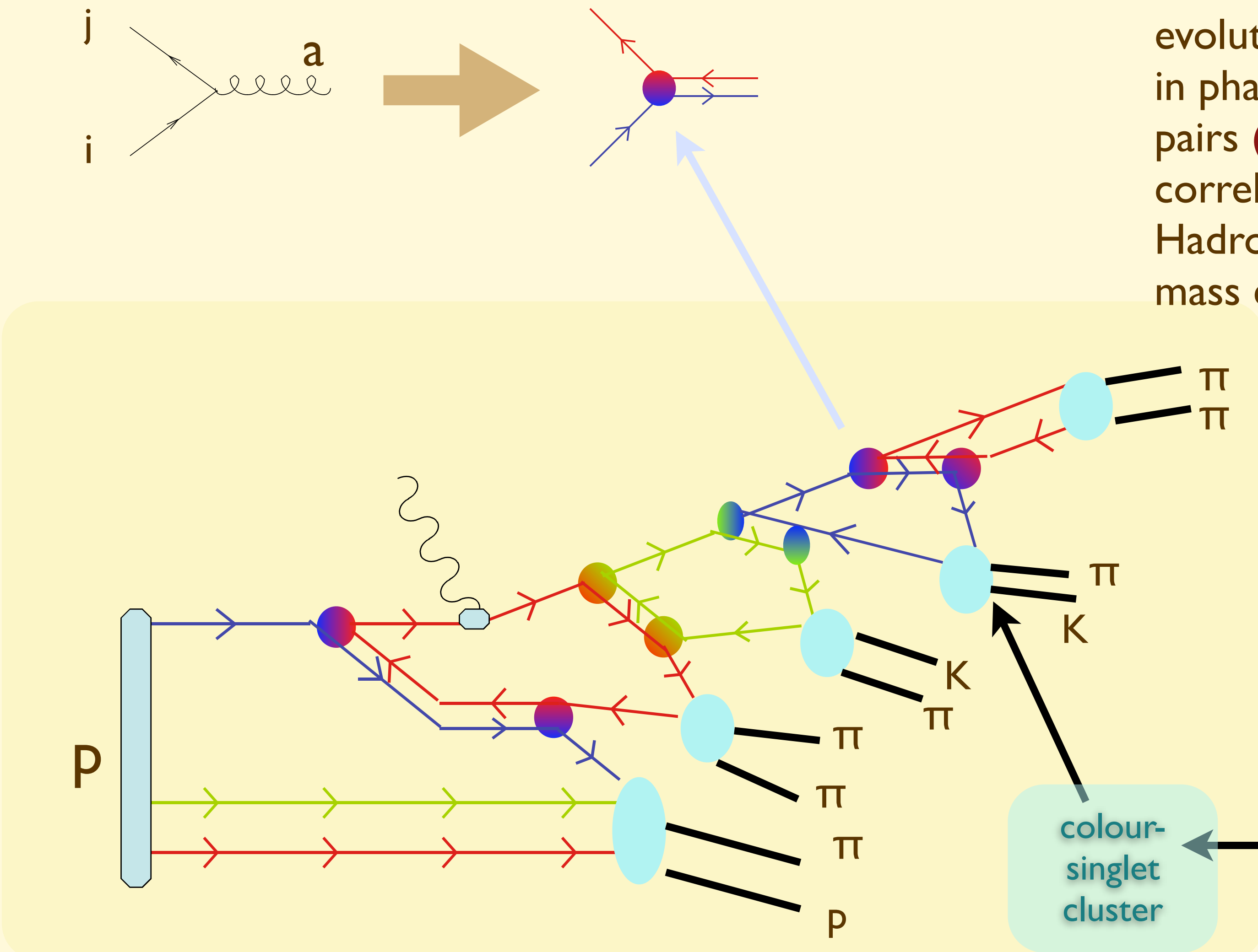


$V(d) \sim k\,d$
$V(d_0) \sim 2\,m_q$
$B = (qqq)$
$\overline{B} = (\bar{q}\bar{q}\bar{q})$

The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs ("preconfinement"). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.







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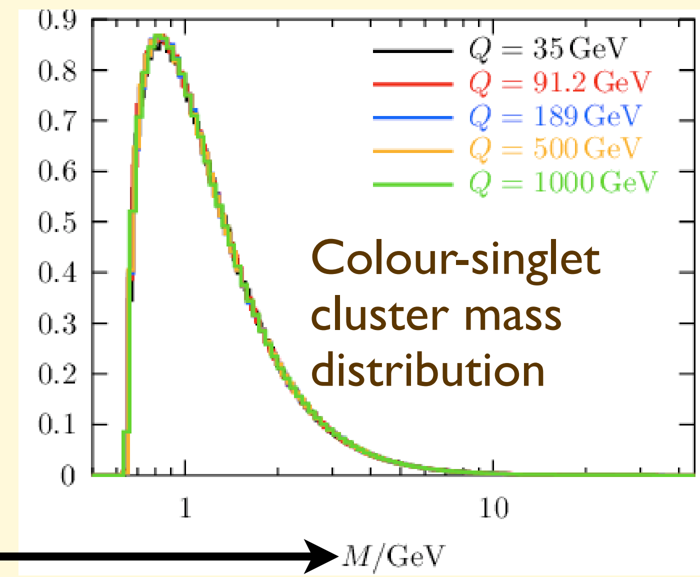
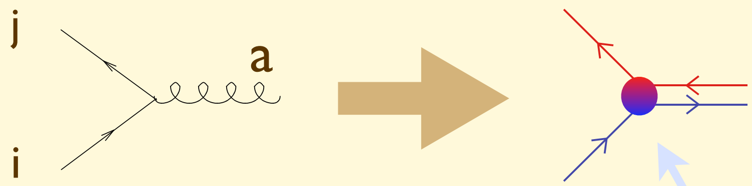


Colour is left "behind" by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.



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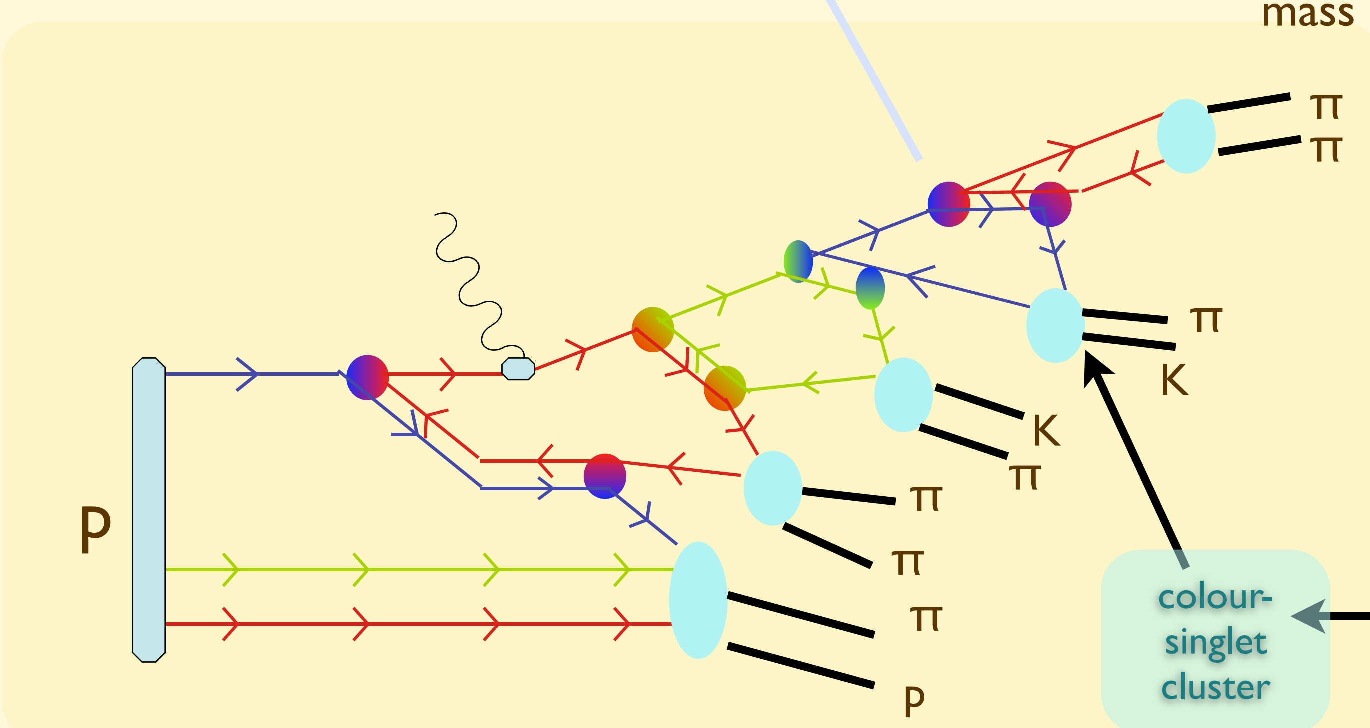

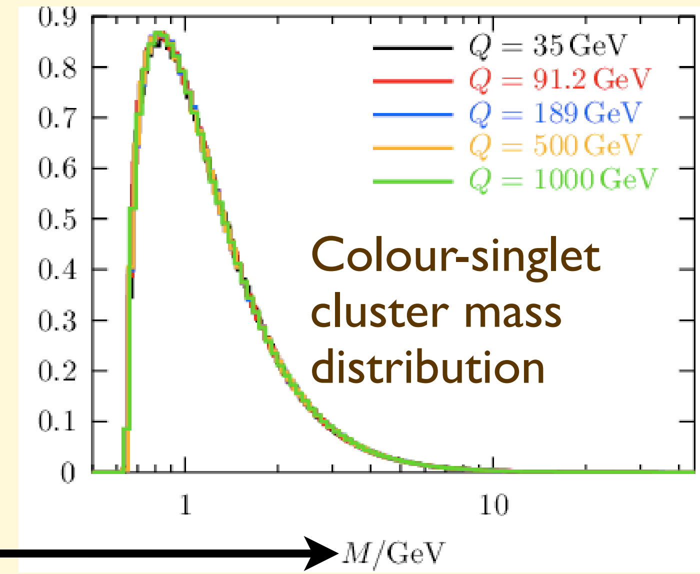

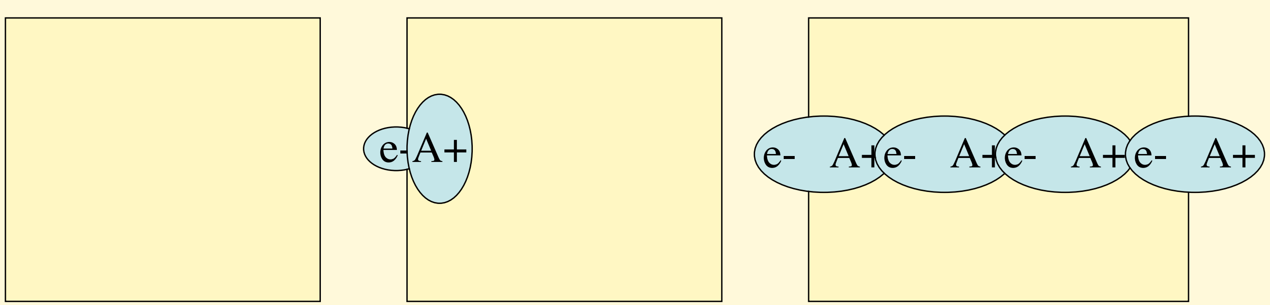

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The shower algorithm

Sequential probabilistic evolution (Markov chain)

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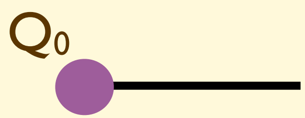

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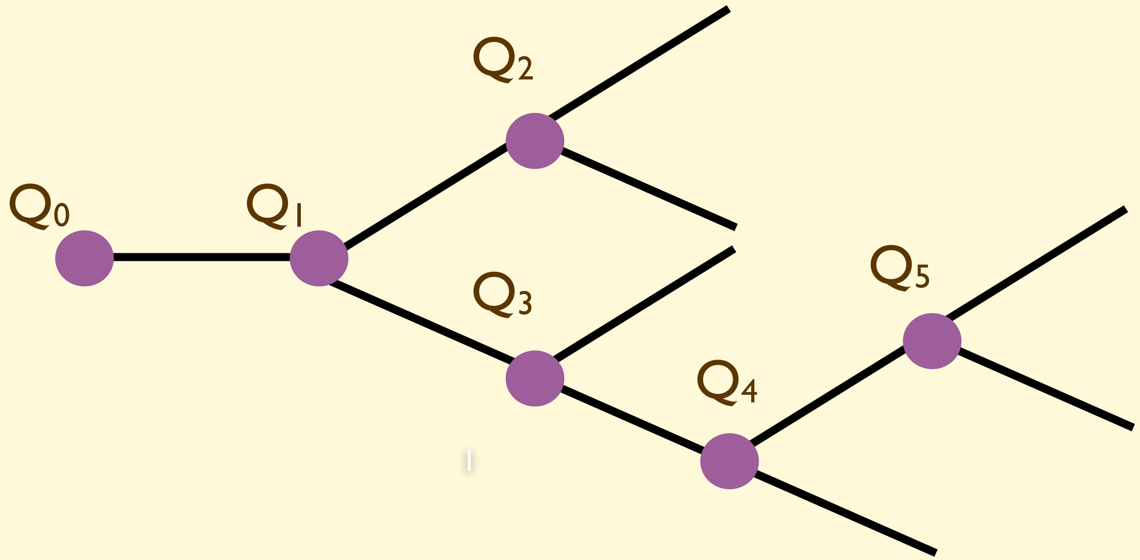

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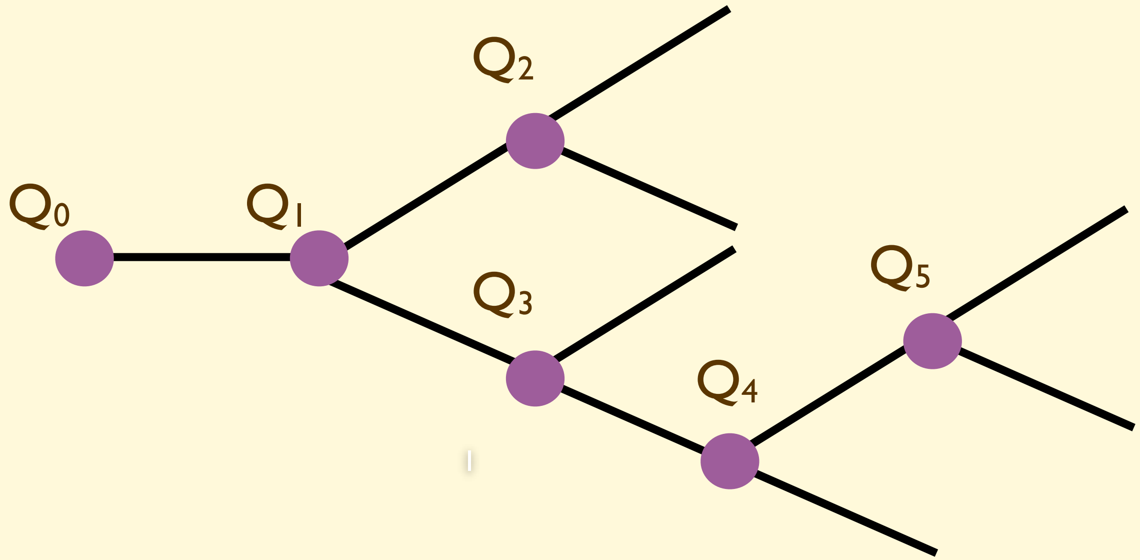

The probability of each emission only depends on the state of the splitting parton, and of the daughters. The QCD dynamics is encoded in these splitting probabilities.

The shower algorithm

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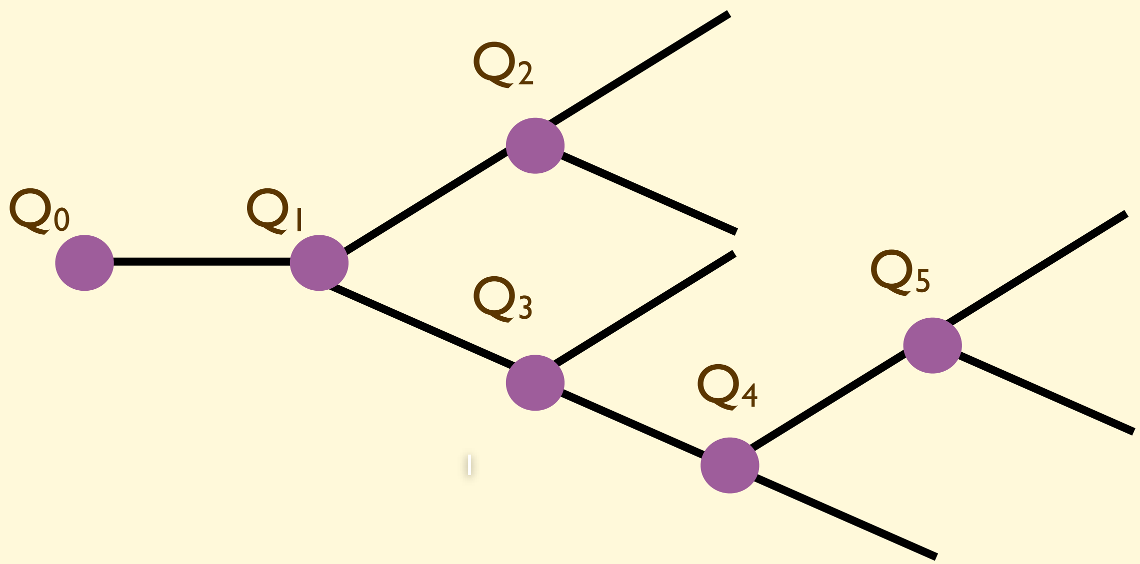

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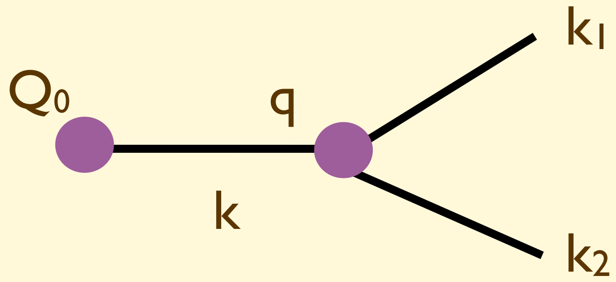
The total probability of all possible evolutions is **1** (unitary evolution).

- The shower evolution does not change the event rate inherited from the parton level, matrix element computation.
- No K-factors from the shower, even though the shower describes higher-order corrections to the leading-order process

Single emission



$$\frac{d\mathrm{Prob}(Q_0 \to q^2)}{dq^2\, dz\, d\phi} = P_0 \frac{\alpha_s(\mu)}{2\pi} \frac{1}{q^2} P(z)$$

$P_0 \Rightarrow \int d\,\mathrm{Prob} = 1$

q^2 $\approx$ virtuality scale of the branching:

- **$(k_1 + k_2)^2$**
- **$k_1 \cdot k_2$**
- **$k_\perp^2$**
- **....**

ϕ = azimuth

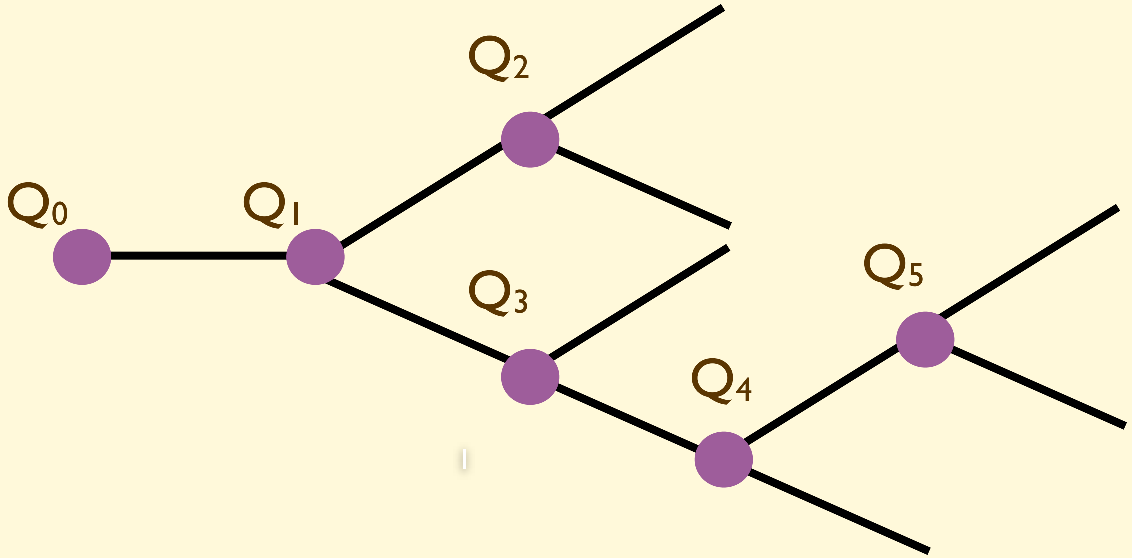
$z=P(k_2)/P(k)$ $\approx$ energy/ momentum fraction carried by one of the two partons after splitting

- **$P=k^0$**
- **$P=k_{//}$**
- **$P=k_{//} + k^0$**
- **...**

$\mu = f(z,q)$

While at leading-logarithmic order (LL) all choices of evolution variables and of scale for α_s are equivalent, specific choices can lead to improved description of NLL effects and allow a more accurate and easy-to-implement inclusion of angular-ordering constraints and mass effects, as well as to a better merging of multijet ME's with the shower

Multiple emission



$$\mathrm{Prob}(Q_0 \to Q_1) = P_0 \frac{\alpha_s}{2\pi} \int_{Q_1}^{Q_0} \frac{dq^2}{q^2} dz P(z) d\phi$$

$$\mathrm{Prob}(Q_0 \to Q_1 \to Q_2) = P_0 \frac{\alpha_s}{2\pi} \int_{Q_1}^{Q_0} \frac{dq^2}{q^2} dz P(z) d\phi \frac{\alpha_s}{2\pi} \int_{Q_2}^{Q_1} \frac{dq^2}{q^2} dz P(z) d\phi$$

$$\sim P_0 \frac{1}{2!} [\frac{\alpha_s}{2\pi} \int_{Q_2}^{Q_0} \frac{dq^2}{q^2} dz P(z) d\phi]^2$$

$$\mathrm{Prob}(Q_0 \to X) = P_0 \times \sum \frac{1}{n!} [\frac{\alpha_s}{2\pi} \int_{\Lambda}^{Q_0} \frac{dq^2}{q^2} dz P(z) d\phi]^n = 1$$

Λ=infrared cutoff

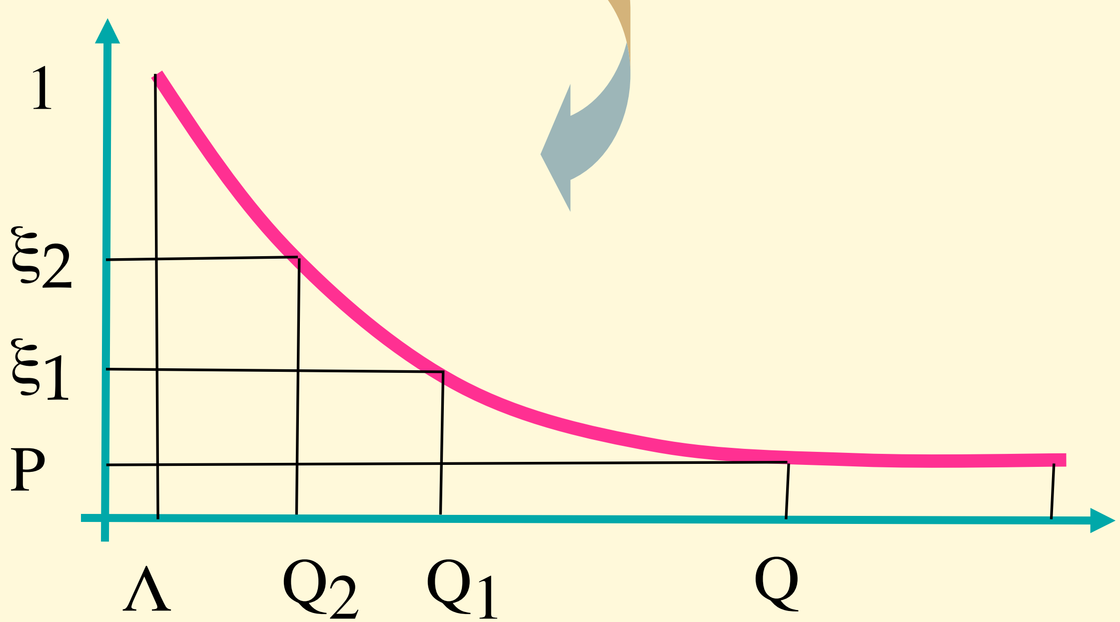
$$P_0 = \exp\{-\frac{\alpha_s}{2\pi} \int_{\Lambda}^{Q_0} \frac{dq^2}{q^2} dz P(z) d\phi\}$$

P_0 = Sudakov form factor
~ probability of no emission between the scale Q_0 and Λ

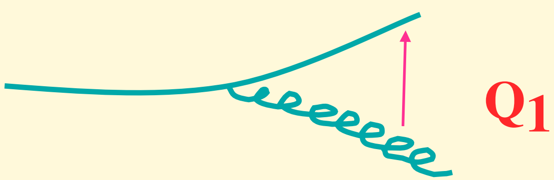
Generation of splittings

$$P(Q,\Lambda) = exp\left[-\int_{\Lambda}^{Q} \frac{dq^2}{q^2}\frac{\alpha_s(q)}{2\pi}P(z)dz\right]$$

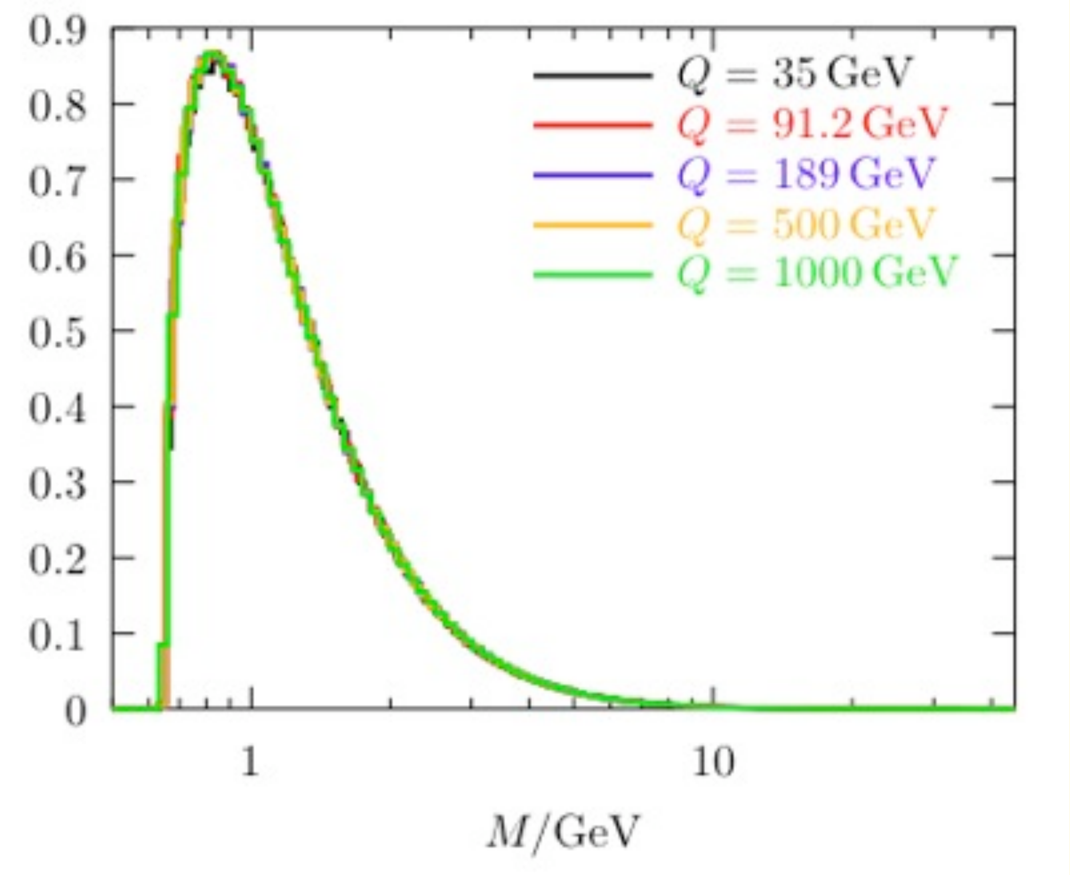
prob. of no radiation between Q and Λ



1. Generate $0< \xi_1 <1$
2. If $\xi_1 < P(Q, \Lambda)$ ⇒ no radiation, q' goes directly on-shell at scale Λ≈GeV
3. Else
   1. calculate Q_1 such that $P(Q_1,\Lambda)= \xi_1$
   2. emission at scale Q_1:



4. Select z according to P(z)
5. Reconstruct the full kinematics of the splitting
6. Go back to 1) and reiterate, until shower stops in 2). At each step the probability of emission gets smaller and smaller



The existence of high-mass clusters, however rare, is unavoidable, due to IR cutoff which leads to a non-zero probability that no emission takes place. This is particularly true for evolution of massive quarks (as in, e.g. $Z \to bb$ or cc). Prescriptions have to be defined to deal with the "evolution" of these clusters. **This has an impact on the $z \to 1$ behaviour of fragmentation functions.**

Phenomenologically, this leads to uncertainties, for example, in the background rates for $H \to \gamma\gamma$ (jet$\to\gamma$).

This approach is extremely successful in describing the properties of hadronic final states!

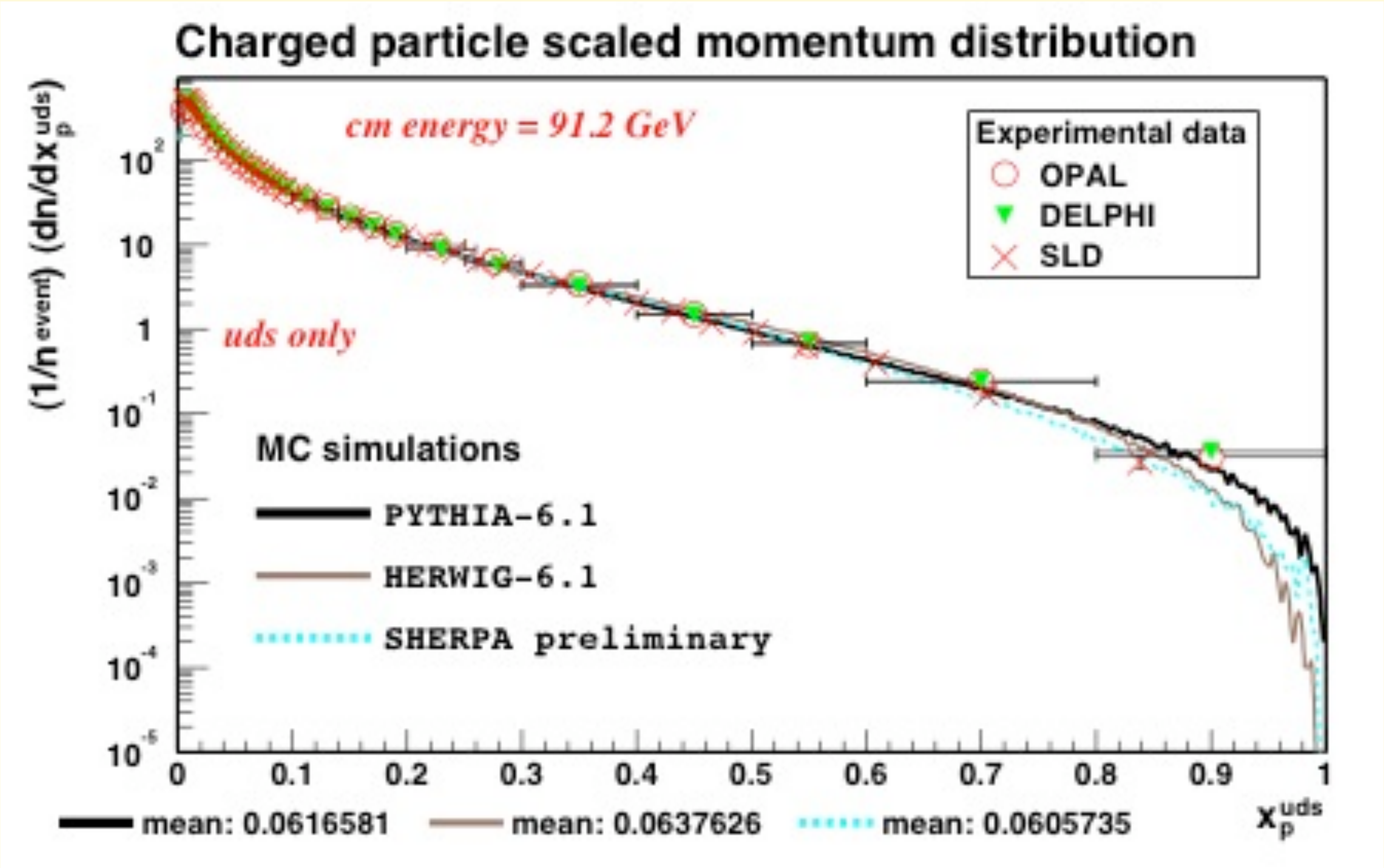
Ex: Particle multiplicities:

Particle	Experiment	Measured	Old Model	Herwig++	Fortran
All Charged	M,A,D,L,O	20.924 ± 0.117	20.22*	20.814	20.532*
γ	A,O	21.27 ± 0.6	23.03	22.67	20.74
π^0	A,D,L,O	9.59 ± 0.33	10.27	10.08	9.88
$\rho(770)^0$	A,D	1.295 ± 0.125	1.235	1.316	1.07
$\pi^\pm$	A,O	17.04 ± 0.25	16.30	16.95	16.74
$\rho(770)^\pm$	O	2.4 ± 0.43	1.99	2.14	2.06
η	A,L,O	0.956 ± 0.049	0.886	0.893	0.669*
$\omega(782)$	A,L,O	1.083 ± 0.088	0.859	0.916	1.044
$\eta'(958)$	A,L,O	0.152 ± 0.03	0.13	0.136	0.106
K^0	S,A,D,L,O	2.027 ± 0.025	2.121*	2.062	2.026
$K^*(892)^0$	A,D,O	0.761 ± 0.032	0.667	0.681	0.583*
$K^*(1430)^0$	D,O	0.106 ± 0.06	0.065	0.079	0.072
$K^\pm$	A,D,O	2.319 ± 0.079	2.335	2.286	2.250
$K^*(892)^\pm$	A,D,O	0.731 ± 0.058	0.637	0.657	0.578
$\phi(1020)$	A,D,O	0.097 ± 0.007	0.107	0.114	0.134*
p	A,D,O	0.991 ± 0.054	0.981	0.947	1.027
Δ^{++}	D,O	0.088 ± 0.034	0.185	0.092	0.209*
Σ^-	O	0.083 ± 0.011	0.063	0.071	0.071
Λ	A,D,L,O	0.373 ± 0.008	0.325*	0.384	0.347*
Σ^0	A,D,O	0.074 ± 0.009	0.078	0.091	0.063
Σ^+	O	0.099 ± 0.015	0.067	0.077	0.088
$\Sigma(1385)^\pm$	A,D,O	0.0471 ± 0.0046	0.057	0.0312*	0.061*
Ξ^-	A,D,O	0.0262 ± 0.001	0.024	0.0286	0.029
$\Xi(1530)^0$	A,D,O	0.0058 ± 0.001	0.026*	0.0288*	0.009*
Ω^-	A,D,O	0.00125 ± 0.00024	0.001	0.00144	0.0009
$f_2(1270)$	D,L,O	0.168 ± 0.021	0.113	0.150	0.173
$f_2'(1525)$	D	0.02 ± 0.008	0.003	0.012	0.012
$D^\pm$	A,D,O	0.184 ± 0.018	0.322*	0.319*	0.283*
$D^*(2010)^\pm$	A,D,O	0.182 ± 0.009	0.168	0.180	0.151*
D^0	A,D,O	0.473 ± 0.026	0.625*	0.570*	0.501
$D_s^\pm$	A,O	0.129 ± 0.013	0.218*	0.195*	0.127
$D_s^{*\pm}$	O	0.096 ± 0.046	0.082	0.066	0.043
J/Ψ	A,D,L,O	0.00544 ± 0.00029	0.006	0.00361*	0.002*
Λ_c^+	D,O	0.077 ± 0.016	0.006*	0.023*	0.001*
$\Psi'(3685)$	D,L,O	0.00229 ± 0.00041	0.001*	0.00178	0.0008*

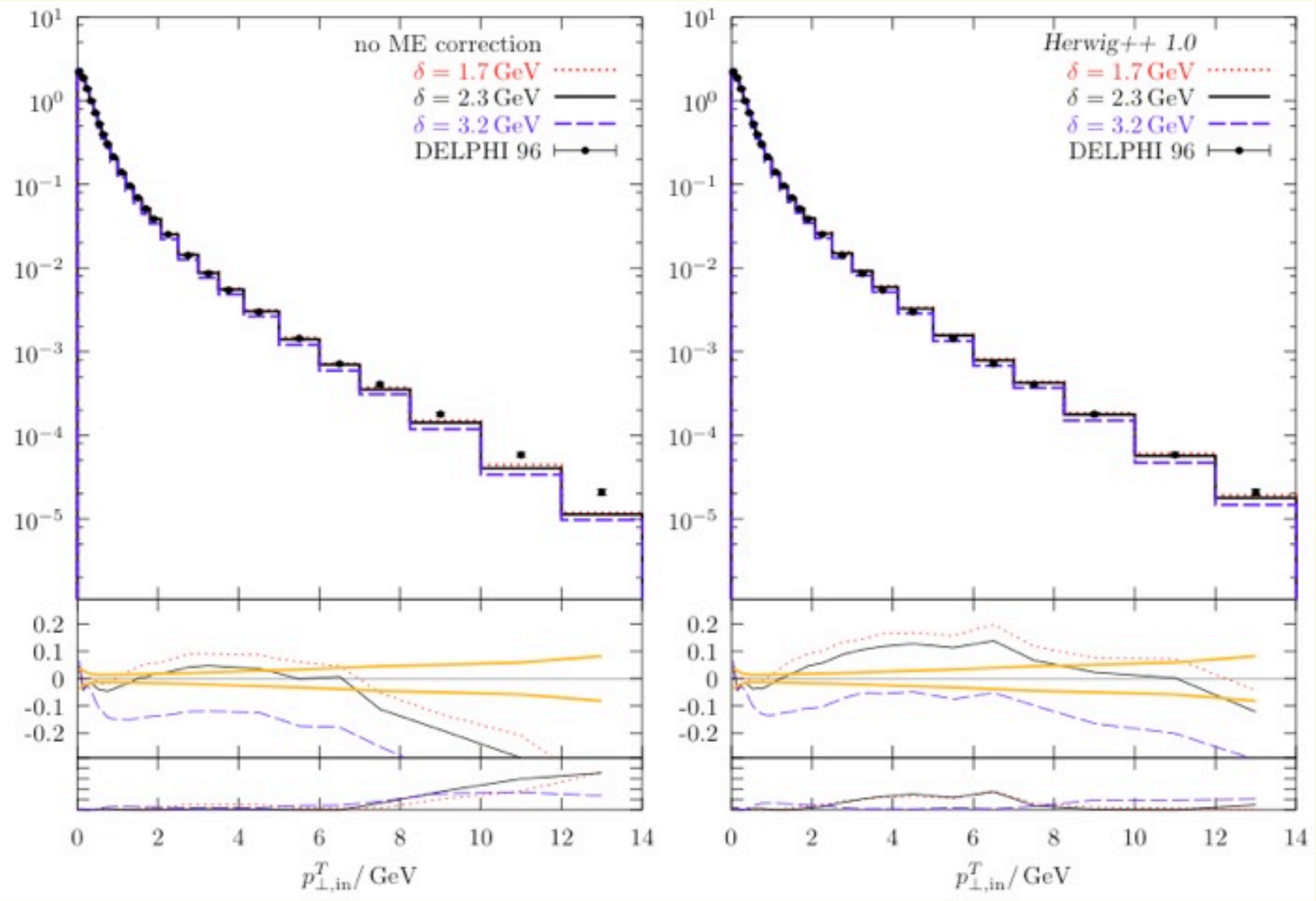
Table 2: Multiplicities per event at 91.2 GeV. We show results from Herwig++ with the implementation of the old cluster hadronization model (Old Model) and the new model (Herwig++), and from HERWIG 6.5 shower and hadronization (Fortran). Parameter values used are given in table 1. Experiments are Aleph(A), Delphi(D), L3(L), Opal(O), Mk2(M) and SLD(S). The * indicates a prediction that differs from the measured value by more than three standard deviations.

Ex: Energy distributions

(Winter, Krauss, Soff,
hep-ph/0311085)

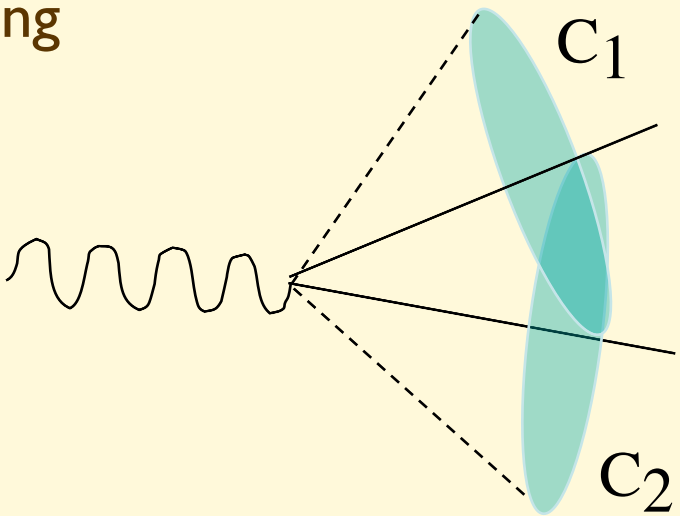


Ex: Transverse momenta w.r.t. thrust axis:



Main limitation of shower approach:

Because of angular ordering



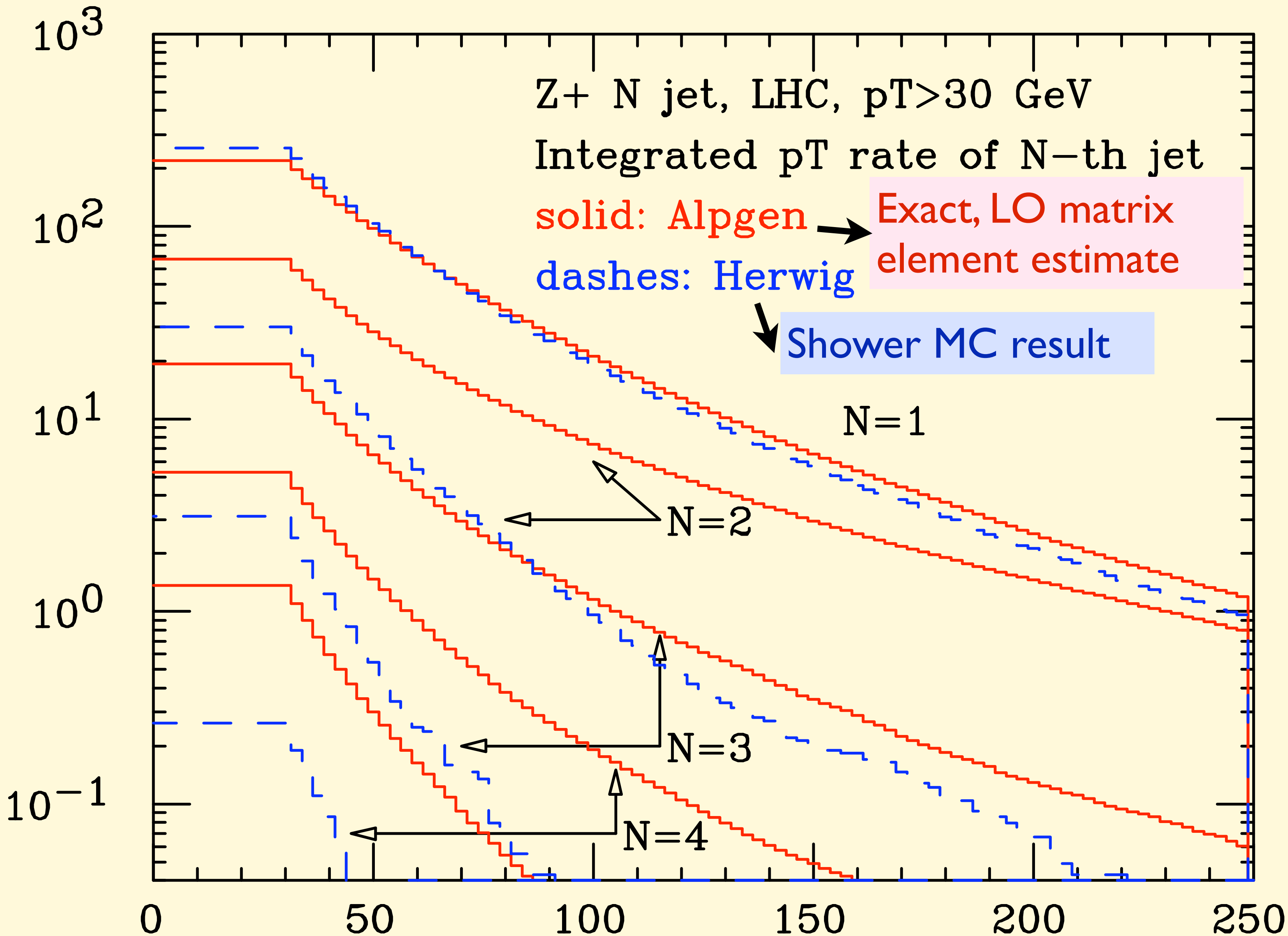
no emission outside $C_1 \oplus C_2$:

- lack of hard, large-angle emission
- poor description of multijet events

incoherent emission inside $C_1 \oplus C_2$:

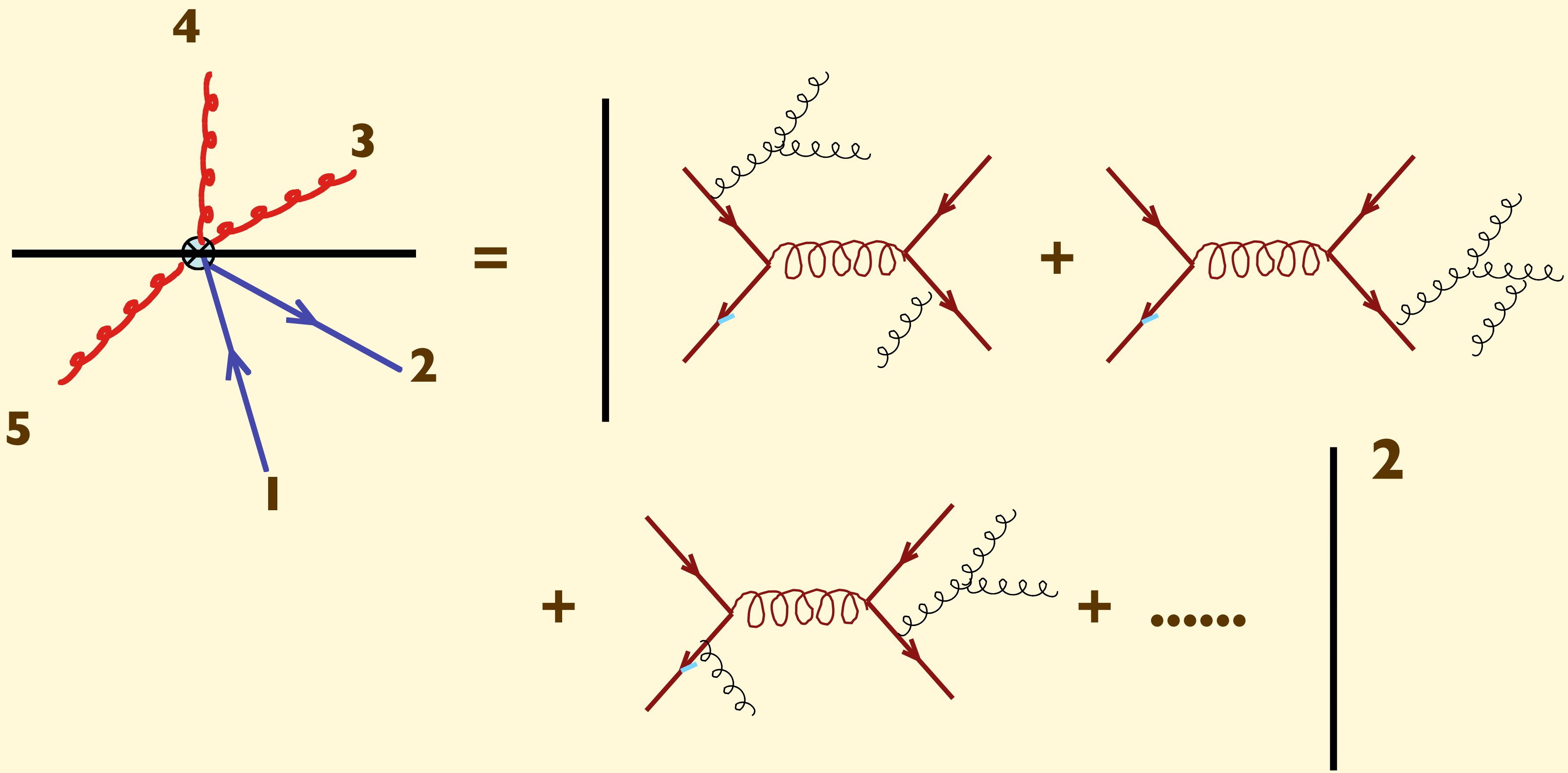
- loss of accuracy for intrajet radiation

Example



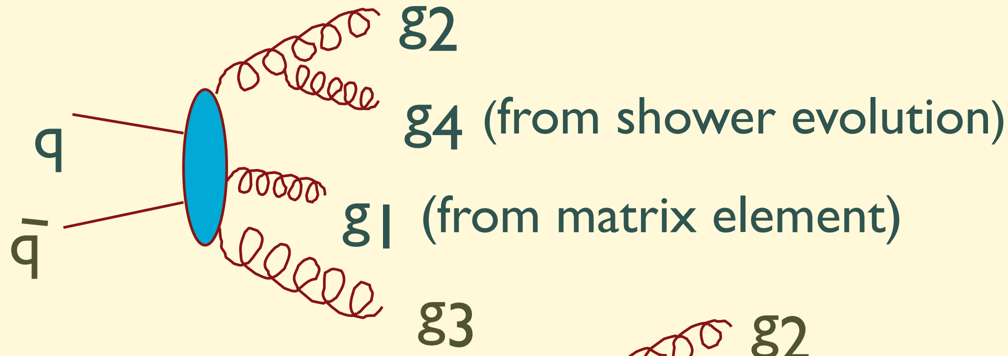
Z+ N jet, LHC, pT>30 GeV
Integrated pT rate of N−th jet
solid: Alpgen
dashes: Herwig
Exact, LO matrix element estimate
Shower MC result
N=1
N=2
N=3
N=4
10^3
10^2
10^1
10^0
10^{-1}
0
50
100
150
200
250

The obvious solution is to start the shower from a higher-order process calculated at the parton level with the exact LO matrix element:



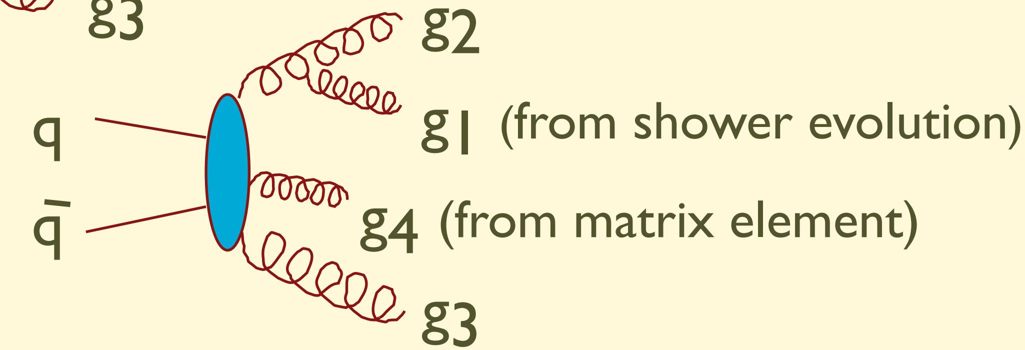
Each hard parton then undergoes the shower evolution according to the previous prescription.

This approach is also afflicted by difficulties:



with $PT1 << PT4 << PT2, PT3$

versus



⇒ **double counting of the same phase-space points**

Recent work started providing solutions to these problems, and new generations of MC codes successfully combine higher-order ME and shower evoloution ("CKKW", "MLM matching")

A useful ref:

Hard Interactions of Quarks and Gluons: a Primer for LHC Physics

http://arXiv.org/abs/hep-ph/0611148