Diphoton signals in TeV scale gravity models to NLO QCD

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- **TeV scale gravity models**
- **Direct probe of extra dimensions**
- **Collider searches**
- **diphoton process to NLO-QCD**
- **Summary**

Phys. Lett. B 672 (2009) 45; Nucl. Phys. B 818 (2009) 28

with M C Kumar, V Ravindran & A Tripathi

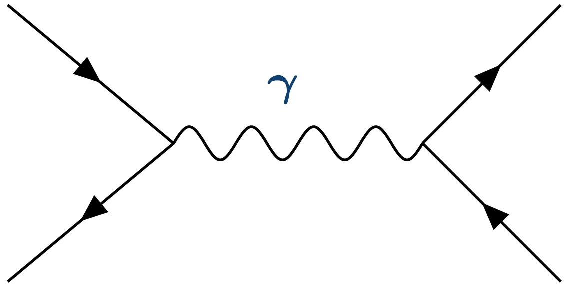
Extra spacial dimensions

- Extra spacial dimensions were first introduced by
  - G Nordström (1914)
  - T Kaluza (1921)
  - O Klein (1926)
- An attempt to unify interactions then known— EM & gravity, by assuming that the photon field originates from the fifth component of a 5-dimensional metric tensor $g_{\mu 4}$
- Developments in string theory has led to new phenomenological ideas which addressed long standing problems in particle physics— Hierarchy Problem
- Two different fundamental energy scales observed in nature
  - EW scale ($M_{EW} \sim 1$ TeV)
  - Planck scale ($M_P = 10^{16}$ TeV)
- SM successfully explains particle physics up to M_{EW}
  - GAUGE
  - MATTER
  - HIGGS
- Central problem is to understand the physics of EWSB— the SM Higgs mechanism jeopardises our current understanding of the SM at the quantum level and EW precision measurements seriously contrive any extension beyond it

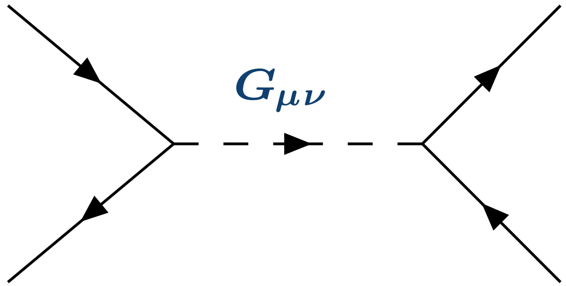
WHAT ABOUT GRAVITY

Gravity ?

- Gravity neglected compared to gauge interaction when dealing with elementary particles— very weak



$$\sigma(E) \sim \alpha^2 \frac{1}{E^2}$$



$$\sigma(E) \sim {G_N}^2 E^2 \sim \frac{E^2}{M_P^4}$$

- Quantum gravity effects become important only at energies $\sim M_P$

- Non-renormalisable— dimensionless coupling constant $\sim G_N E^2$

Mass Hierarchy problem

- Why is $M_{EW} << M_P$
- Why is M_{EW} radiatively stable
- In a quantum theory, the hierarchy $\Longrightarrow$ severe fine tuning of fundamental parameters to keep masses of elementary particles at their observed value

$$m^2(p^2) = m_0^2(\Lambda^2) + C\, g^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

g: Coupling Λ: Cutoff C: Calculable Coefficient

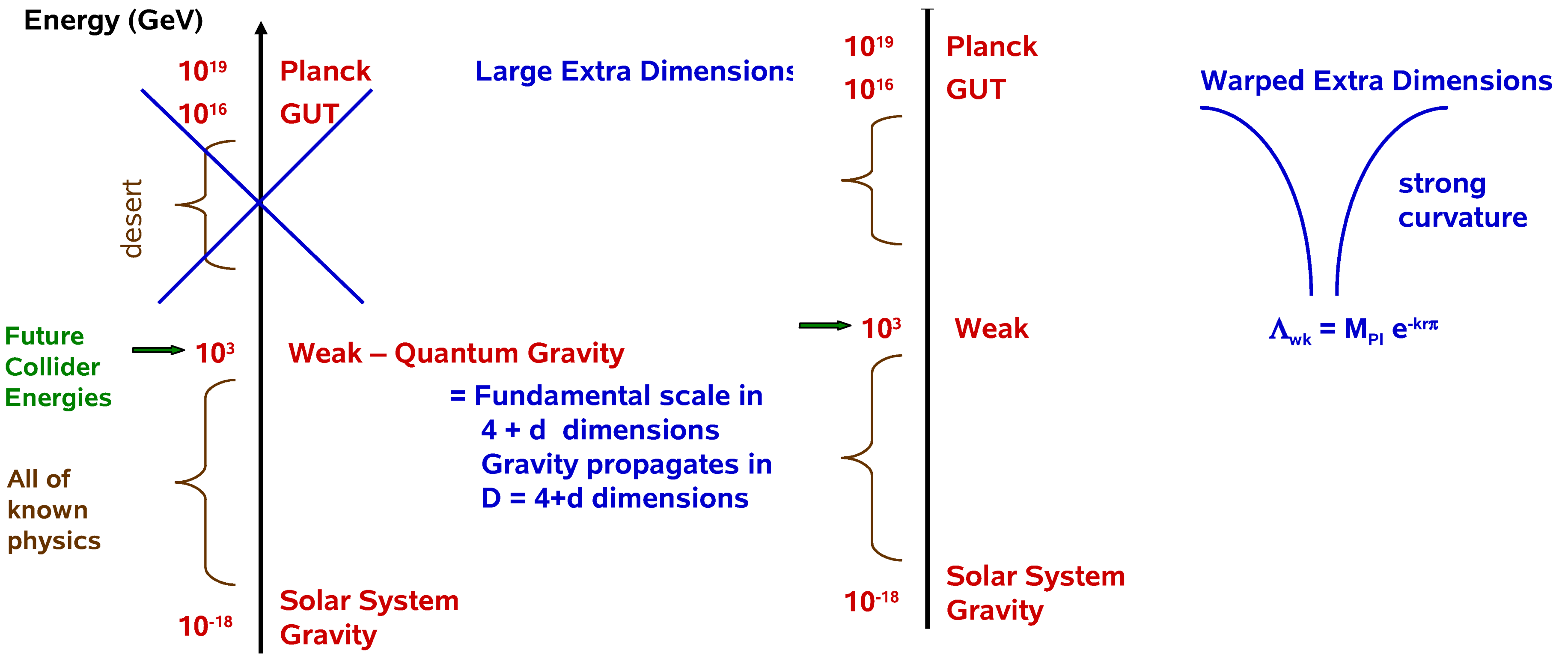
QUADRATICALLY DIVERGENT! $\Rightarrow$ Beyond SM Physics

Hierarchy problem has been the guiding principle to construct theories beyond the SM

- Protective Symmetry (SUSY $\sum_{i=F,B} C_i \int dk^2 = 0$, EXACT)
- New physics intervenes— Cuts off integral

Some non SUSY avenues beyond the SM

ADD was the first proposal to address the hierarchy problem in the context of extra dimensions. They provide an alternate view of the hierarchy between the EW (~ 1 TeV) and the Planck scale (10^{16} TeV)— additional structure in the gravity sector in contrast to previous approaches which introduced new structure in the particle physics



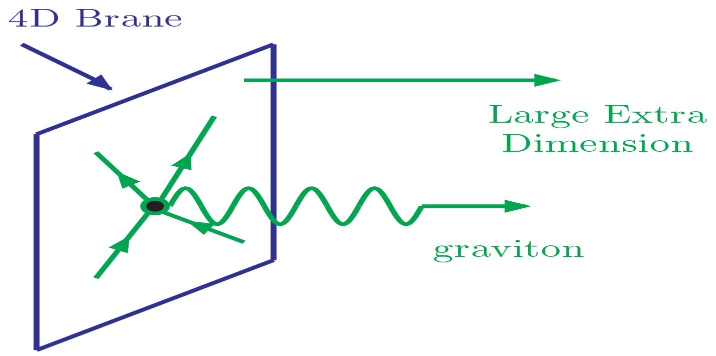
Geometry of extra spacial dimensions is responsible for the Hierarchy. These theories should have a viable mechanism (Brane world scenarios) to hide the extra dim such that space time is effectively four consistent with known physics

ADD Scenario

- **Space time** $\mathcal{M}_4 \times \mathcal{K}_d$ **FACTORISABLE GEOMETRY**

  $\mathcal{M}_4$ **: 3+1 dim space time** $\mathcal{K}_d$ **: Compact space of size** R

- **SM localised on a 3-brane embedded into the 4+d dim space time with d compact extra dimensions which can ONLY be probed by gravity**



Brane-world Picture

- **Gauss law in 4+d dim with d compact dim of radius R**

$r \ll R$ $r \gg R$

$$V(r) \sim \frac{m_1 m_2}{M_S^{d+2}} \frac{1}{r^{d+1}} \qquad V(r) \sim \frac{m_1 m_2}{M_S^{d+2} R^d} \frac{1}{r}$$

Large Extra Dimensions

- r $\gg$ R gravitational flux lines in 4+d dim are constrained in the compact dim and hence the potential is effectively r^{-1} at large distances

$$\boxed{M_P^2 \sim M_S^{2+d} \, R^d}$$

R: radius of compactification, could be large compared to a TeV^{-1}

$R \sim 10^{\frac{30}{d}-17}$ cm $\qquad$ $R^{-1} \sim 10^{\frac{-30}{d}+3}$ GeV

d	R (cm)	R^{-1}
1	10^{13}	10^{-27} eV
2	10^{-2}	10^{-3} eV
3	10^{-7}	100 eV
...	...	...
6	10^{-12}	10 MeV

- Only Gravitational field can probe the full 4+d dim space, deviation from Newtonian gravity puts constraint on number of extra dim

$$d \geq 2 \;\; \textbf{Possible}$$

If Fundamental Planck Scale $M_S \sim 1$ TeV, "no Hierarchy problem".

Brane world scenarios

- Apparent weakness of gravity accounted for by
  - Large extra dimensions (ADD)
  - warped extra dimension (RS1)
- Only gravity allowed to propagate the compact extra spacial dimensions, SM is constrained on a 3-brane
- For ADD and RS models, the KK spectrum and their effective interactions with SM particles in 4-dim are very distinct
- Interaction of the KK tower with SM fields on the 3-brane

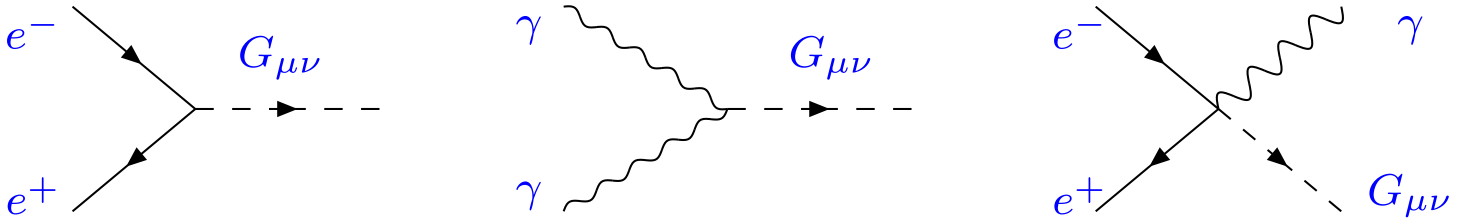
ADD $$\mathcal{L} \sim -\frac{1}{M_P} T^{\mu\nu}\, G^{(0)}_{\mu\nu} - \frac{1}{M_P} T^{\mu\nu} \sum_{n=1}^{\infty} G^{(n)}_{\mu\nu}$$

RS $$\mathcal{L} \sim -\frac{1}{M_P} T^{\mu\nu}\, G^{(0)}_{\mu\nu} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} G^{(n)}_{\mu\nu}$$

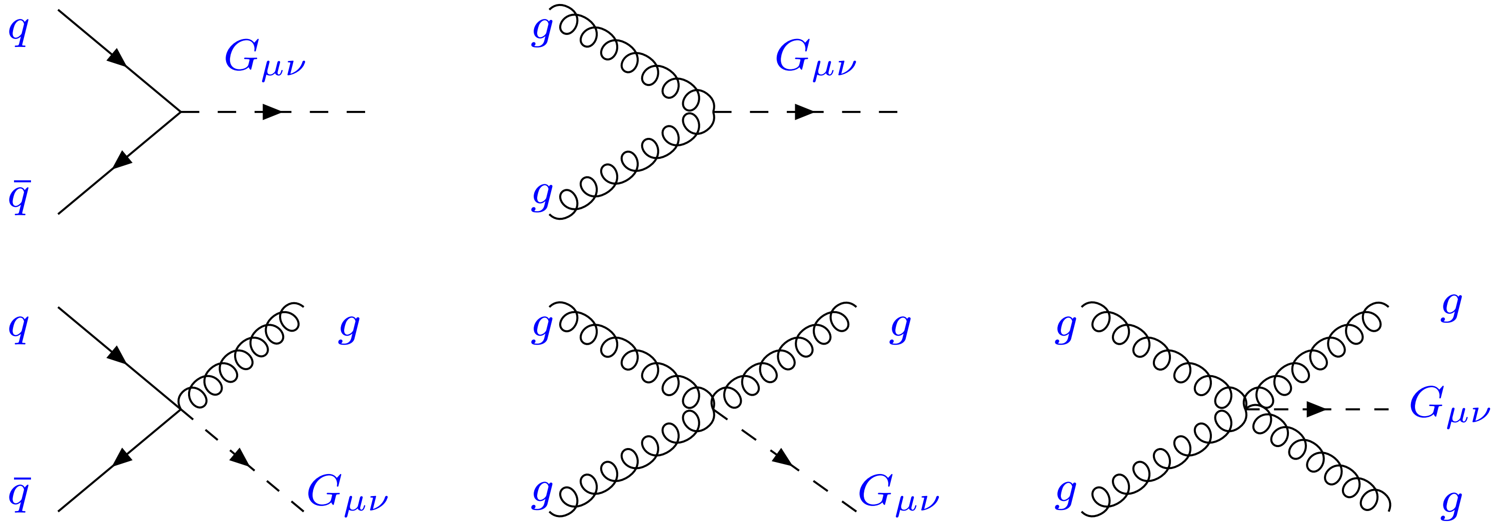
RULE OF THUMB: ATTACH A GRAVITON TO ANY SM LEG OR VERTEX

Feynman Rules

- QED

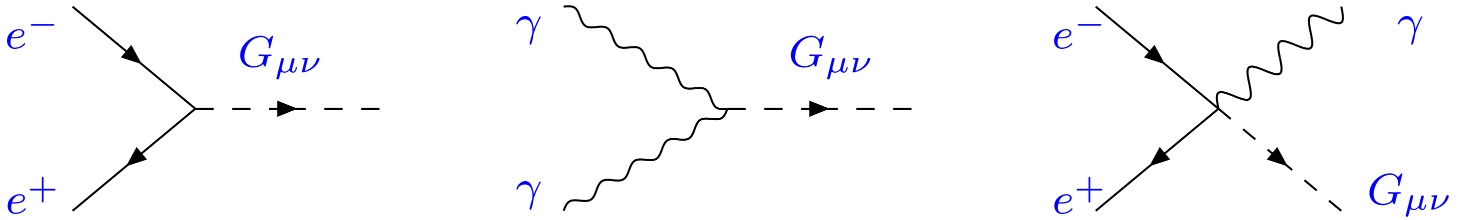


- QCD

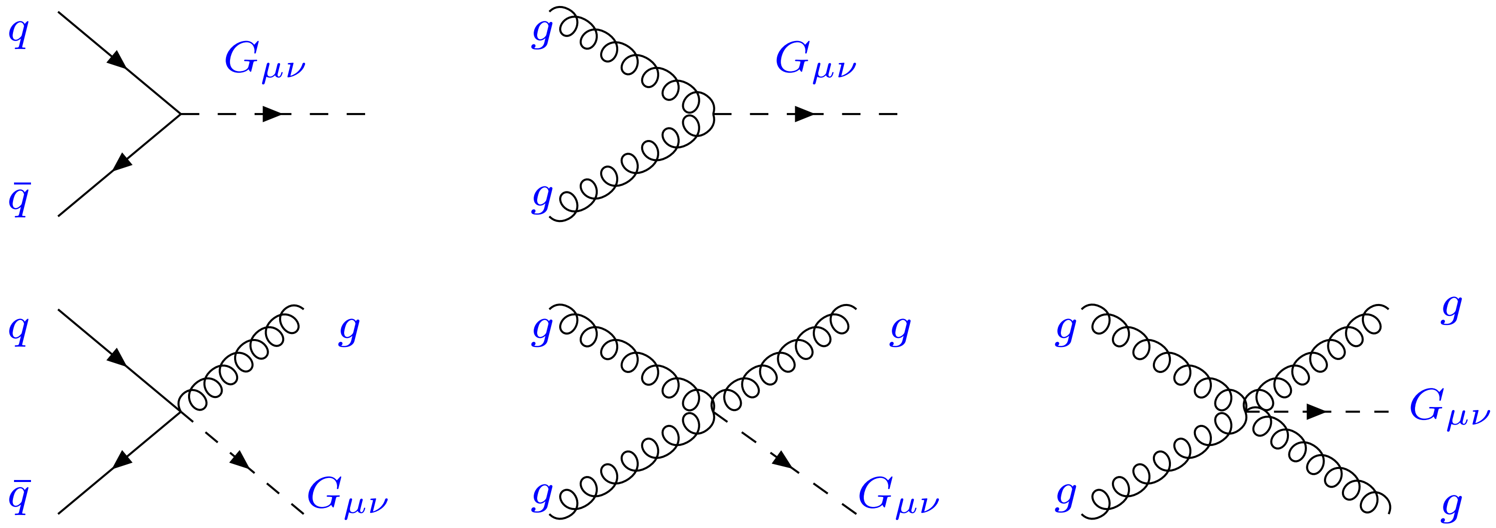


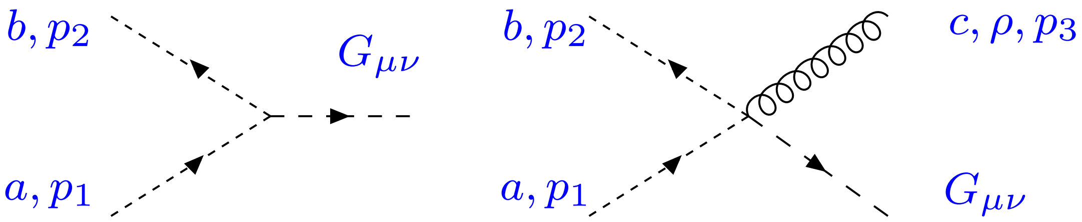
Feynman Rules

- QED



- QCD





Ghost Couplings

with V Ravindran et. al. JHEP 0408 (2004) 048

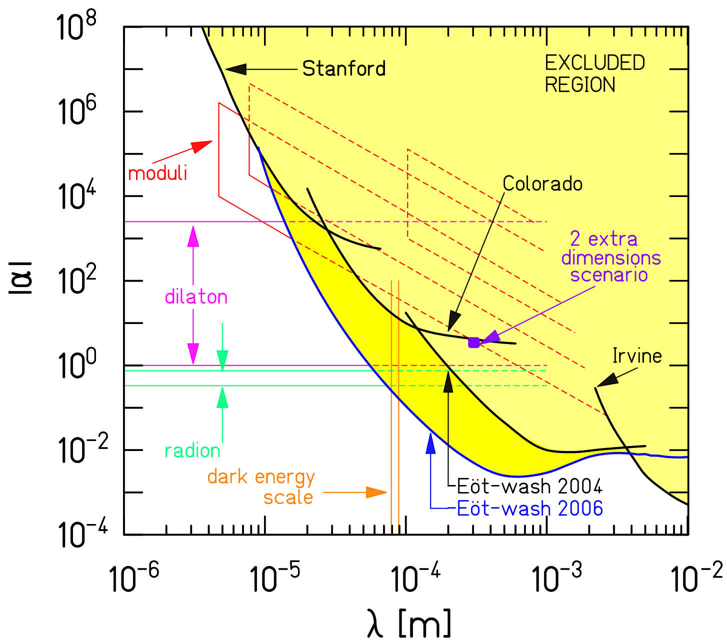
Direct probe of Large Extra dimension

- **Test Newtonian gravity (ISL) at length scales comparable to the size of extra dim**
- **Corrections to gravitational ISL due to compactified extra dims**

$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 + \alpha \exp\left(-\frac{r}{\lambda}\right)\right]$$

α strength of new potential as compared to the Newtonian potential & λ is its range

Kehagias & Sfetsos Phys. Lett. B472 (2000) 39



- ○ **Area above heavy curves excluded**
- ○ **No Yukawa type deviation from Newtonian gravity from light-year upto about 44 μm**
- ○ **hep-ph/0611184**

Probe Extra dimensions @ Colliders— ADD

Physics of extra dim is the physics of the KK spectrum

- Massless graviton and KK modes couple with SM fields with coupling M_P^{-1}
- Effects of KK modes
    - Real KK modes emission
    - Virtual KK modes exchange
- Since the KK modes are M_P^{-1} suppressed one has to sum over the tower of KK modes to get observable effect— an individual ADD KK mode can not be detected
    - Real case $\Rightarrow$ Inclusive production cross section of KK mode

      Phase space enhancement compensates M_P^{-1} suppression for production of a single KK mode. All states upto $m_n = \sqrt{s}$ can be emitted— integral cut off by kinematics

    - Virtual case $\Rightarrow$ contact interaction

      In contrast to real KK emission the summation of virtual KK modes depends on the UV cutoff

Virtual Exchange

- **Being virtual all states in fact contribute, not kinematically bound— but bounded by the validity of the effective theory**
- **KK density of state**

$$\rho(m_{\vec{n}}) = \frac{R^d m_{\vec{n}}^{d-2}}{(4\pi)^{d/2}\Gamma(d/2)}$$

- **Sum over KK mode propagator**

$$\sum_{\vec{n}} \frac{1}{s - m_{\vec{n}}^2 + i\epsilon} = \int_0^\infty dm_{\vec{n}}^2 \, \rho(m_{\vec{n}}) \, \frac{1}{s - m_{\vec{n}}^2 + i\epsilon}$$

- **Dominated by UV contribution:**

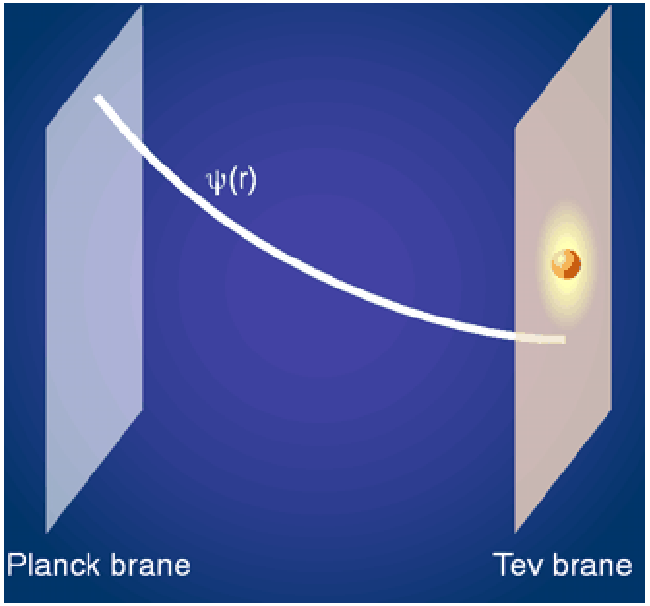
$d = 1$ **convergent**

$d = 2$ $\ln(\frac{s}{\Lambda_c^2})$

$d > 2$ $\frac{1}{M_S^4}\left(\frac{\Lambda_c}{M_S}\right)^{d-2} \Rightarrow \frac{1}{M_S^4}$

Warped Extra Dimension RS1

Non-Factorisable geometry, 5-dim AdS space— constant negative curvature



- $ds^2 = e^{-2\mathcal{K} r_c |\phi|}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + r_c^2\, d\phi^2$
- Consist of two branes at orbifold fixed points
- SM localised on the TeV brane
- Gravity localised on the Planck brane
- Gravity weak on the TeV brane due to the exponential warping
- Stabilisation of r_c with an additional scalar field whose energy is minimised for a particular value of extra dim — Goldberger & Wise

- Interaction of RS KK tower with SM fields on the TEV Brane

$$\mathcal{L} \quad \sim \quad -\frac{1}{M_P} T^{\alpha\beta}\, G^{(0)}_{\alpha\beta} - \frac{1}{\Lambda_\pi} T^{\alpha\beta} \sum_{n=1}^{\infty} G^{(n)}_{\alpha\beta}$$

$$\Lambda_\pi \quad \sim \quad M_P\, e^{-\mathcal{K} r_c \pi} \sim \mathcal{O}(\mathrm{TeV})$$

- Zero mode decouples (massless graviton) Newtonian Gravity intact M_P^{-1}

RS Scenario

- **Excited massive KK modes couple to SM with TeV^{-1} suppression**

$$M_n = x_n\, \mathcal{K}\, e^{-\mathcal{K} r_c \pi} \equiv x_n\, m_0$$

- **Two basic parameters of the RS model are**

$$m_0 = \mathcal{K}\, e^{-\mathcal{K} r_c \pi} \qquad\qquad c_0 = \frac{\mathcal{K}}{M_P}$$

- **Summing over RS KK modes**

$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) \quad = \quad \frac{c_0^2}{m_0^2}\sum_{n=1}^{\infty}\frac{1}{s - M_n^2 + iM_n\Gamma_n} \quad \equiv \frac{c_0^2}{m_0^4}\lambda\Big(\frac{Q}{m_0}\Big)$$

- **Phenomenological implication very distinctive compared to ADD**

ONLY MACHINES AVAILABLE TO PROBE BSM ARE QCD MACHINES— TEVATRON & LHC

Source of Theoretical Uncertainties

- **Renormalisation scale:**
  Due to UV divergence at beyond Leading Order

$$\alpha_s \longrightarrow \alpha_s(\mu_R^2)$$

- **Factorisation scale:**
  Originate from light quarks and massless gluon. Parton distribution functions are renormalised at the factorisation scale μ_F

$$f_a(x) \longrightarrow f_a(x, \mu_F^2) \qquad a = q, \bar{q}, g$$

- **Parton Distribution Functions:**
  Not calculable but extracted from experiments in some factorisation scheme by various groups by global fits to available data on DIS, DY and other hadronic process
- **Observables are "free" of μ_R and μ_F**
- **"Fixed order" perturbative results depend on μ_R and μ_F**
- **Can in principle give large uncertainties**

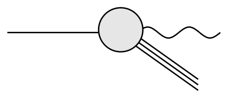
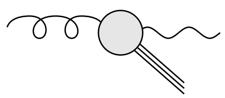
IT IS HENCE IMPORTANT FOR EXTRA DIMENSION SEARCHES TO HAVE BETTER CONTROL OVER THE THEORETICAL UNCERTAINTIES

Di-photon Process

- Prompt photons with large transverse momenta at hadron colliders is an interesting laboratory of the short distance dynamics of quarks and gluons and is an important channel for Higgs searches in the mass range 80 GeV $\leq m_H \leq$ 140 GeV and various BSM studies

- **Prompt photons** means they do not come from decay of hadron (π^0, η etc). Photon are faked by hadron, for eg: π^0s at large p_T could go into two nearly collinear photons which are difficult to distinguish from a single photon

- Prompt photon could be classified as (a) **direct**, both photons are not as a result of fragmentation and (b) **fragmentation**, atleast one of the photon is as a result of fragmentation

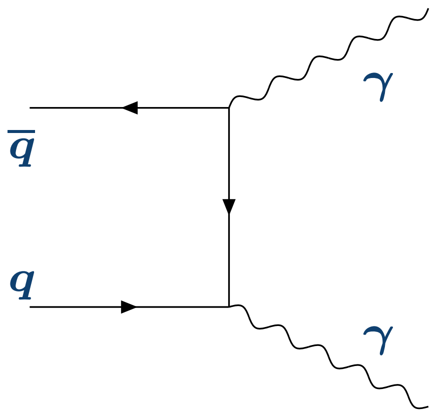
$D_{\frac{\gamma}{q}}(z)$  $D_{\frac{\gamma}{g}}(z)$ 

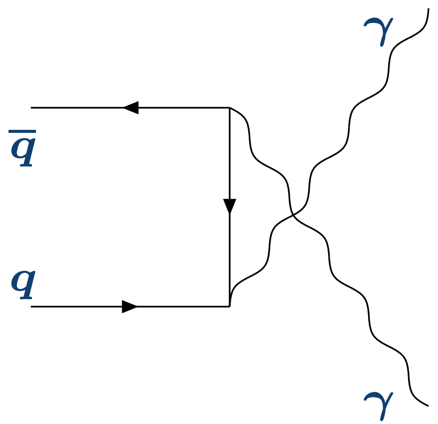
- At colliders **secondary photons** coming from the decay of hadron overwhelms the prompt photons signal but secondary photons can be rejected by experimental selection of prompt photons using isolation cuts

Di-photon Process

$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

Leading Order



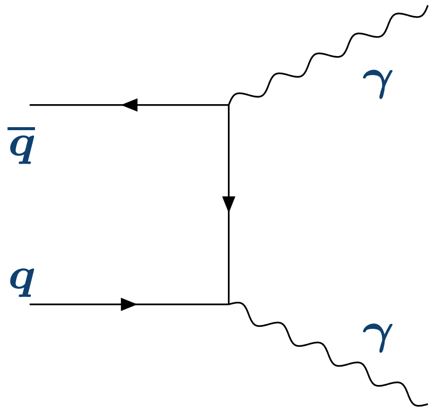


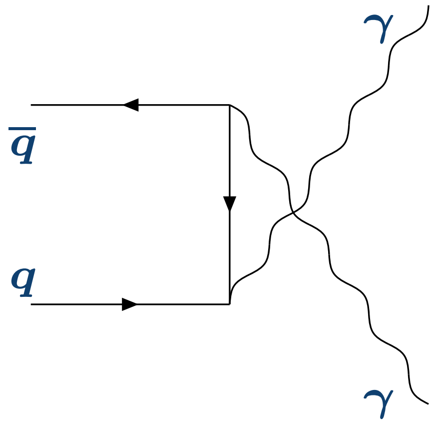
SM

Di-photon Process

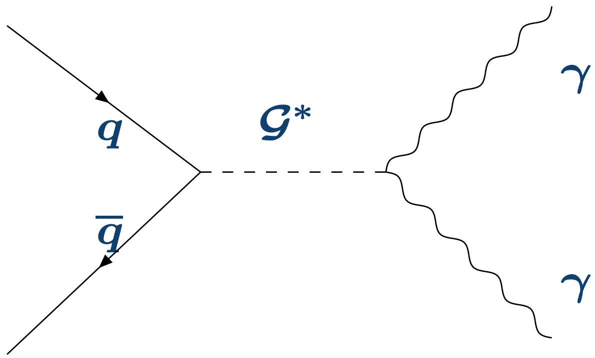
$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

Leading Order





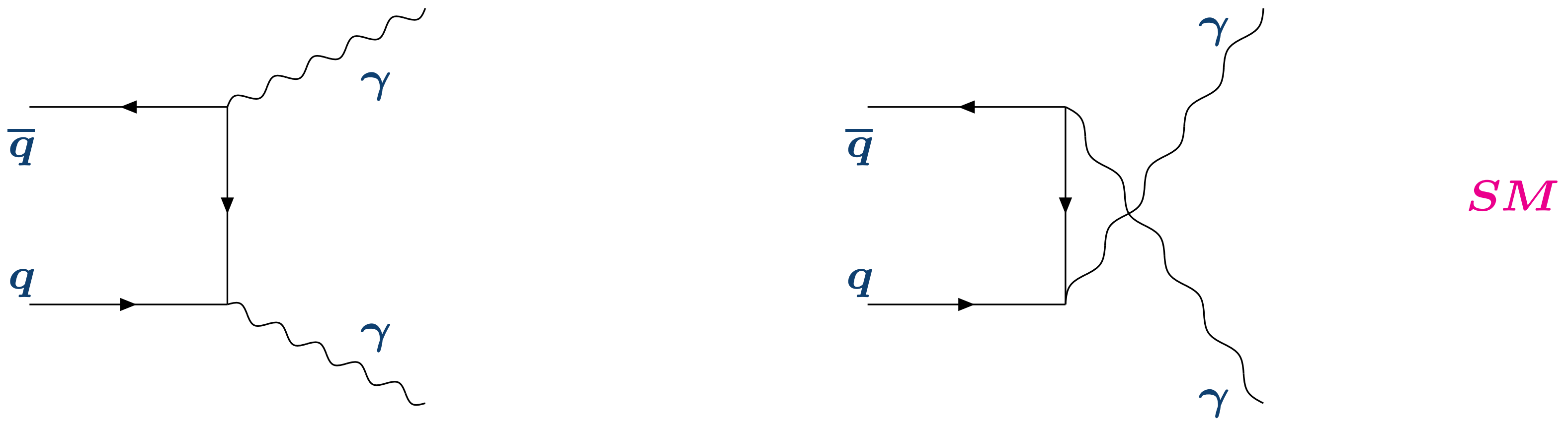
SM

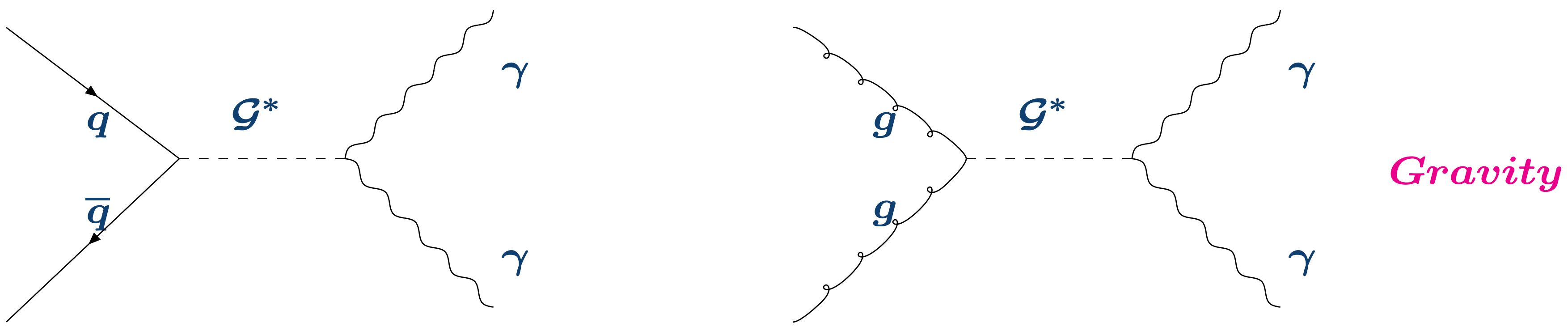


Di-photon Process

$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

Leading Order





Contributing Subprocess to digamma production

Leading Order:

Standard Model	KK-Modes
$q + \bar{q} \rightarrow \gamma\gamma$	$q + \bar{q} \rightarrow \mathcal{G}$ $g + g \rightarrow \mathcal{G}$

Next-to-Leading Order:

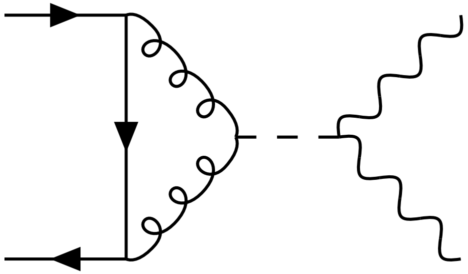
Standard Model	KK-Modes
$q + \bar{q} \rightarrow \gamma\gamma + g$, $q + \bar{q} \rightarrow \gamma\gamma$ + one loop $q + g \rightarrow \gamma\gamma + q$, $\bar{q} + g \rightarrow \gamma\gamma + \bar{q}$	$q + \bar{q} \rightarrow \mathcal{G} + g$, $q + \bar{q} \rightarrow \mathcal{G}$ + one loop $q + g \rightarrow \mathcal{G} + q$, $\bar{q} + g \rightarrow \mathcal{G} + \bar{q}$ $g + g \rightarrow \mathcal{G} + g$, $g + g \rightarrow \mathcal{G}$ + one loop

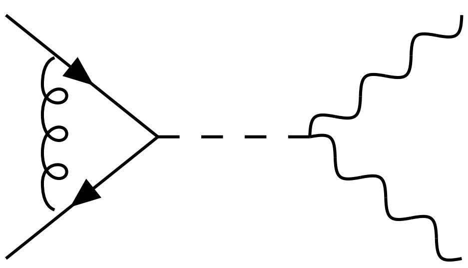
Phys. Lett. B 672 (2009) 45 *with* MC Kumar, V Ravindran & A Tripathi

Virtual Contributions $q\,\bar{q} \to \gamma\,\gamma$

$\mathcal{O}(\alpha_s)$ virtual corrections comes from the interference between the virtual graphs of the (SM + BSM) and the (SM + BSM) Born graphs

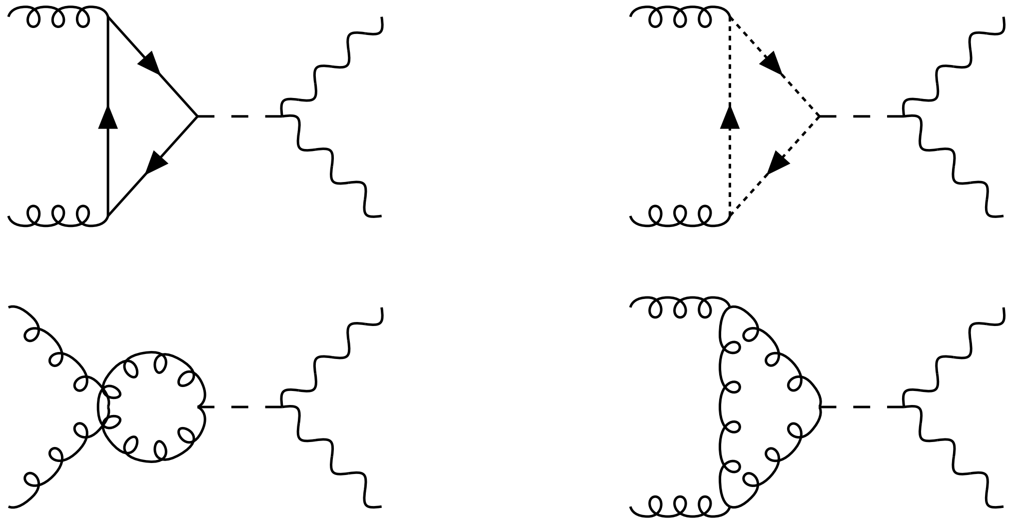
- $\boldsymbol{\mathcal{O}\Big(g_s^2(e_q^4 + e_q^2\kappa^2 + \kappa^4)\Big)}$



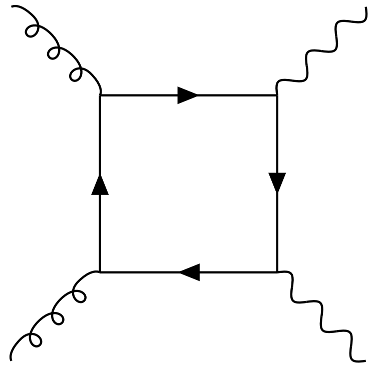


Virtual contributions $g\ g \rightarrow \gamma\ \gamma$

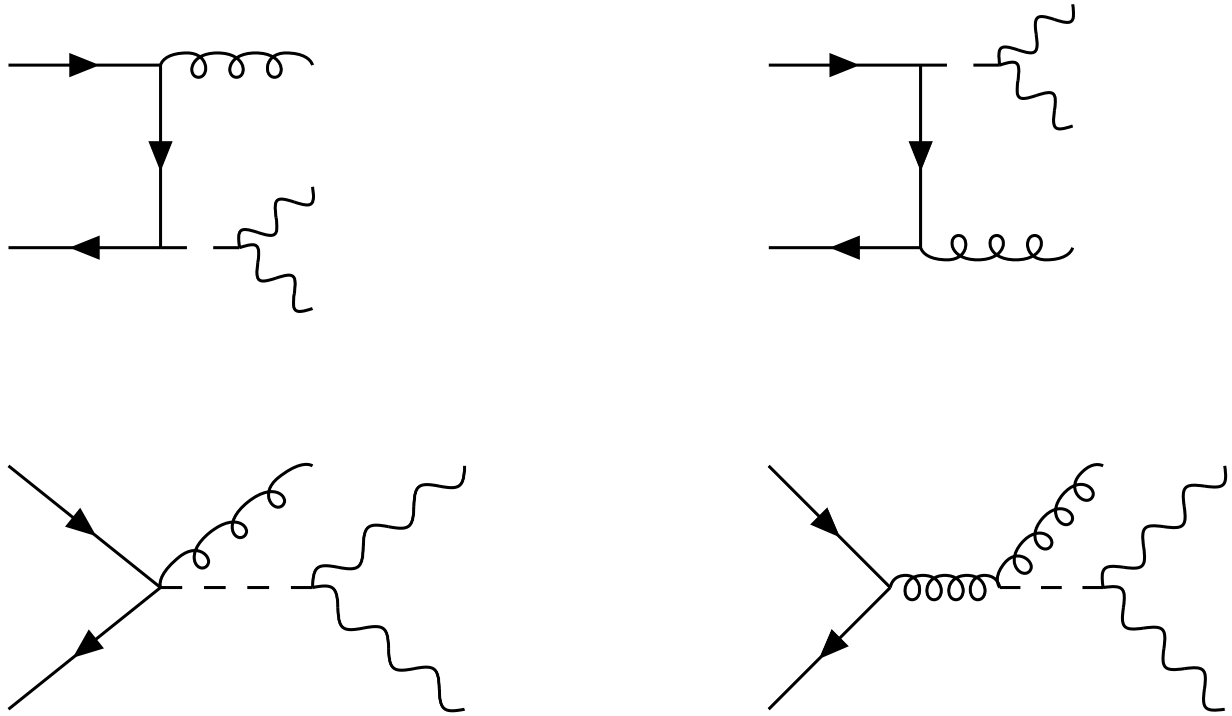
- $\mathcal{O}\left(g_s^2\ \kappa^4\right)$



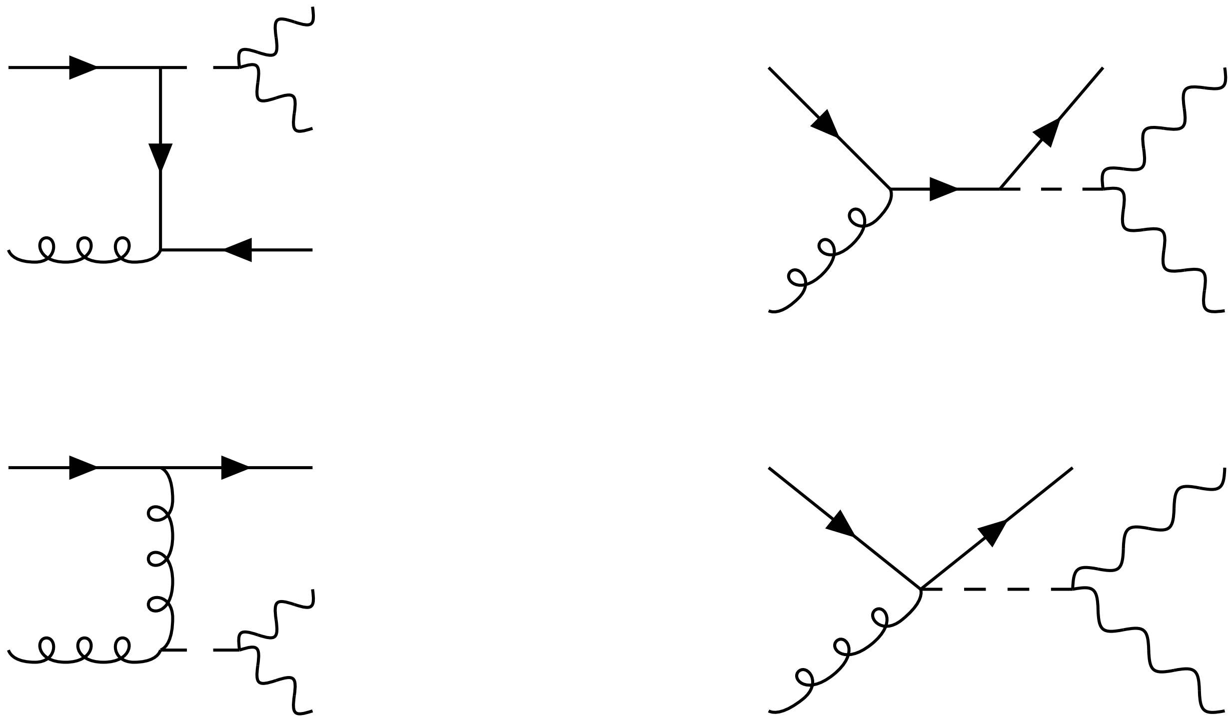
- SM gluon fusion via quark loop $\mathcal{O}\left(g_s^2 e_q^2 \kappa^2\right)$



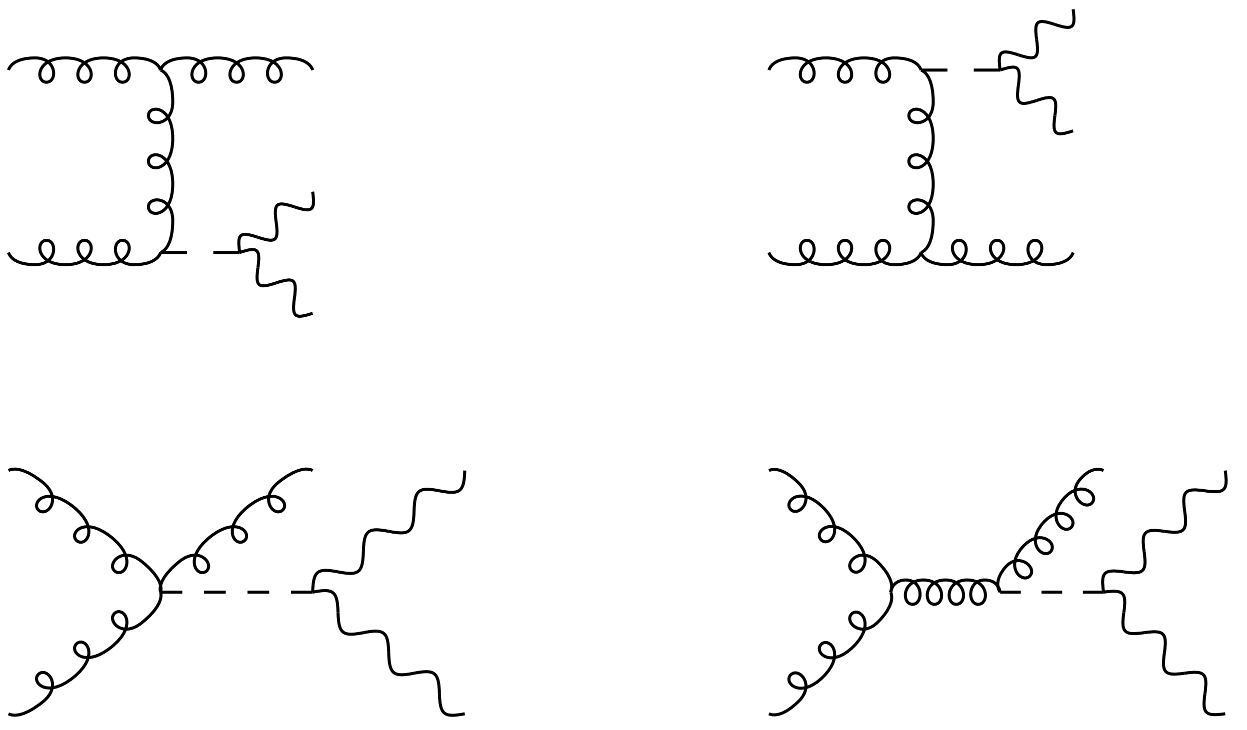
Real Contributions $q\,\bar{q} \longrightarrow \gamma\,\gamma\,g$



Real Contributions $q\ g \rightarrow q\ \gamma\ \gamma$



Real Contributions $g\, g \longrightarrow\ g\, \gamma\, \gamma$



Phase-space slicing method with *two cutoffs*

- Isolating 3-body phase space regions where soft and collinear singularities occur— impose arbitrary boundaries by introducing $small$ cut-off parameters δ_s, δ_c

		soft		hard		
		$0 \le E_g \le \delta_s \frac{\sqrt{s_{ab}}}{2}$		$E_g > \delta_s \frac{\sqrt{s_{ab}}}{2}$		
$d\sigma_{ab}^{real}$	=	$d\sigma_{ab}^{real}(\delta_s)$	+	$d\sigma_{ab}^{real}(\delta_s)$		
$d\sigma_{ab}^{real}$	=	$d\sigma_{ab}^{real}(\delta_s)$	+	$d\sigma_{ab}^{real}(\delta_s, \delta_c)$	+	$d\sigma_{ab,fin}^{real}(\delta_s, \delta_c)$
				$0 \le t_{ij} \le \delta_c s_{ab}$		$t_{ij} > \delta_c s_{ab}$
				hard collinear		hard non collinear

- Phase space integration on the mutually exclusive soft and collinear region are performed not on the full matrix element but in the leading pole approximation of soft and collinear region

- Now only the logarithms of the cut-off parameters are retained and all positive powers of the cut-off parameters are set to zero

Dependence on the cutoff parameters δ_s and δ_c

- Performing the phase space integrals in $4+\epsilon$ dimensions, the soft and collinear poles are exposed. Adding the virtual contributions, all double and single poles of soft (IR) origin automatically cancel

- Remaining collinear poles are then factorised in the parton distribution or fragmentation function as the case may be at some scale and some specific factorisation scheme

- Now we are left with

$$d\sigma_{ab}^{real} = d\sigma_{ab}^{real}(\delta_s) + d\sigma_{ab}^{real}(\delta_s, \delta_c) + d\sigma_{ab,fin}^{real}(\delta_s, \delta_c)$$

2-body PS 3-body PS

- 2-body part which depend explicitly on $\ln \delta_s$ and $\ln \delta_c$

- 3-body part which when integrated over the phase space using Monte Carlo technique, have an implicit dependence on the same logarithms with opposite signs

- Physical cross sections are hence independent of these arbitrary cut-off δ_s, δ_c

Virtual Corrections

- For diphoton production including gravity there are no UV singularities—
  - electromagnetic coupling does not receive QCD corrections
  - KK modes couple to SM energy momentum tensor which is a conserved quantity
- Performing the loop integrals the virtual contributions

$$
\begin{aligned}
d\sigma^V &= a_s(\mu_R^2)\, dx_1\, dx_2\, \mathcal{K}(\epsilon, \mu_R^2, s) \\
&\quad \Big\{ C_F \Big[\Big(-\frac{16}{\epsilon^2} + \frac{12}{\epsilon} \Big)\, d\sigma^0_{q\bar{q}}(\epsilon) + d\sigma^{fin}_{q\bar{q}} \Big] \Phi_{q\bar{q}}(x_1, x_2) \\
&\quad + C_A \Big[\Big\{ -\frac{16}{\epsilon^2} + \frac{4}{\epsilon}\frac{1}{C_A}\Big(\frac{11}{3}C_A - \frac{4}{3}n_f T_F\Big) \Big\}\, d\sigma^0_{gg}(\epsilon) + d\sigma^{fin}_{gg} \Big] \Phi_{gg}(x_1, x_2) \Big\}
\end{aligned}
$$

$$
\mathcal{K} = \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{s}{4\pi\mu_R^2} \right)^{\frac{\epsilon}{2}} \qquad a_s(\mu_R^2) = \frac{\alpha_s(\mu_R^2)}{4\pi}
$$

- SM gluon fusion diagram via quark loop would interfere with the LO gravity mediated diagram, but this is a finite contribution
- SM gluon fusion contribution, though at $\mathcal{O}(\alpha_s^2)$ is comparable to LO for small diphoton invariant mass, but falls of rapidly for larger invariant mass

Real Emission: Leading pole approximation (Soft region)

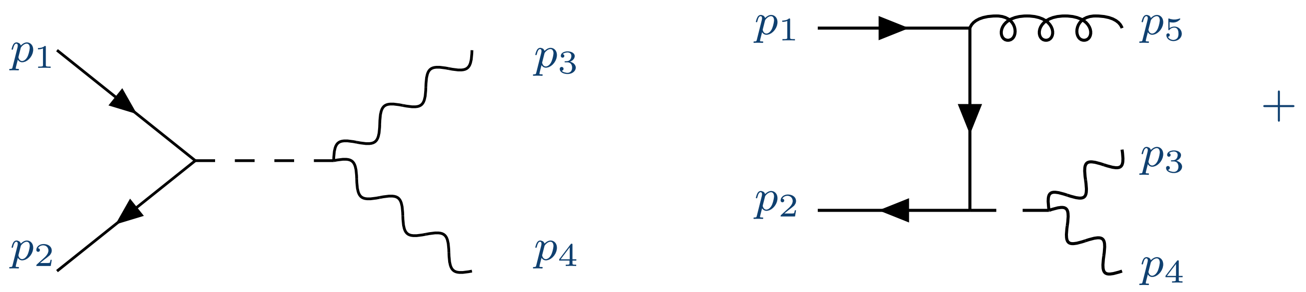
- Soft gluon limit $p_5 \longrightarrow 0$
  - Matrix element $$M_3^{soft} = -g_s \mu^{-\epsilon/2} \epsilon_\sigma(p_5) T_{ij}^a \left(\frac{p_2^\sigma}{p_2 \cdot p_5} - \frac{p_1^\sigma}{p_1 \cdot p_5} \right) M_2$$
  - Phase Space $$d\Gamma_3^{soft} = d\Gamma_2 \left(\frac{4\pi}{s_{12}} \right)^{-\epsilon/2} \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \frac{1}{2(2\pi)^2} d\mathcal{S}$$
- Performing both E_5 and angular integral over the Eikonal current

$$d\mathcal{S} \frac{2p_1.p_2}{p_1.p_5 \; p_2.p_5} = \frac{8}{\epsilon} \left(\frac{1}{\epsilon} + \ln \delta_s + \frac{\epsilon}{2} \ln^2 \delta_s \right)$$

$$d\sigma^{soft} = a_s(\mu_R^2) dx_1 dx_2 \mathcal{K}(\epsilon, \mu_R^2, s) \left(\frac{16}{\epsilon^2} + \frac{16 \ln \delta_s}{\epsilon} + 8 \ln^2 \delta_s \right) \left[C_F d\sigma^0_{q\bar{q}}(x_1, x_2, \epsilon) \Phi_{q\bar{q}}(x_1, x_2) + C_A d\sigma^0_{gg}(x_1, x_2, \epsilon) \Phi_{gg}(x_1, x_2) \right]$$

- Pole of order 2 corresponds to soft and colinear gluons and cancels with virtual contributions, while the ϵ^{-1} pole with coefficient $\ln \delta_s$ still remains

Real Emission: Leading pole approximation (Collinear region)



- Matrix element in the collinear limit $\quad p_t \to 0 \quad p_1 - p_5 \simeq z p_1$

$$\left| M_3^{col}(q(p_1)\,\bar{q}(p_2) \to \gamma\gamma g) \right|^2 = -\frac{2}{z t_{15}} P_{qq}(z,\epsilon) \left| M_2(q(zp_1)\bar{q}(p_2) \to \gamma\gamma g) \right|^2 g_s^2 \mu_R^{-\epsilon}$$

- Phase space in the collinear limit

$$d\Gamma_3 = d\Gamma_2 \frac{(4\pi)^{-\epsilon/2}}{16\pi^2 \Gamma(1+\epsilon/2)} dz dt_{15} (-(1-z)t_{15})^{\epsilon/2}$$

- Performing dt_{15} integral in the limit $\quad 0 < -t_{15} < \delta_c s_{12}$

$$d\sigma_{col} = a_s(\mu_R^2) \mathcal{K}(\epsilon, \mu_R^2, s_{12}) \; d\hat{\sigma}^0_{q\bar{q}}(s_{12}, t_{13}, t_{14}) \; f_{\frac{q}{P_1}}(\frac{x_1}{z}) dx_1 \; f_{\frac{\bar{q}}{P_2}}(x_2) dx_2$$
$$\frac{1}{\epsilon} P_{qq}(z,\epsilon) \left(\delta_c \frac{1-z}{z} \right)^{\frac{\epsilon}{2}} \frac{dz}{z}$$

Hard Collinear region

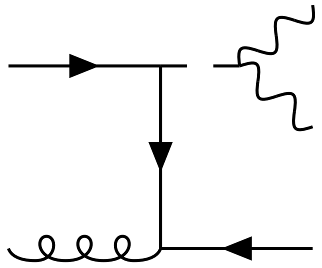
- $z \longrightarrow 1$ is the soft region. Hard region $E_5 > \delta_s \frac{\sqrt{s_{12}}}{2}$ translates to $0 < z < 1 - \delta_s$ for process where soft singularities exist, otherwise there is no restriction on z

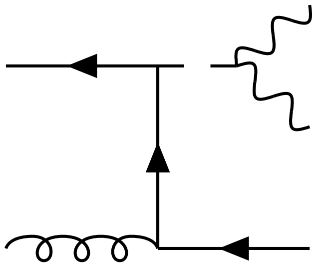
$$
\begin{aligned}
d\sigma_{HC} &= \frac{a_s(\mu_R^2)}{\epsilon}\mathcal{K}(\epsilon, \mu_R^2, s)dx_1 dx_2 \Bigg[d\hat{\sigma}_0^{q\bar{q}}(x_1, x_2, \epsilon) \\
& \quad \Big\{ \int_{x_1}^{1-\delta_s} \frac{dz}{z}\mathcal{H}(z,\epsilon,\delta_c)P_{qq}(z,\epsilon)f_{q_i}(x_1/z)f_{\bar{q}_i}(x_2) \\
& \quad + \int_{x_2}^{1-\delta_s} \frac{dz}{z}\mathcal{H}(z,\epsilon,\delta_c)P_{qq}(z,\epsilon)f_{q_i}(x_1)f_{\bar{q}_i}(x_2/z) + x_1 \leftrightarrow x_2 \Big\}_{q\bar{q}}
\end{aligned}
$$

$$
\mathcal{H}(z,\epsilon,\delta_c) = \left(\delta_c \frac{1-z}{z}\right)^{\epsilon/2}
$$

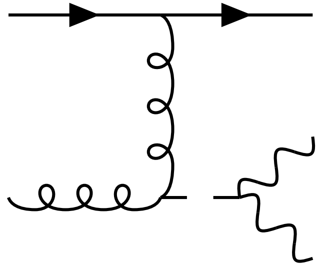
Hard Collinear region

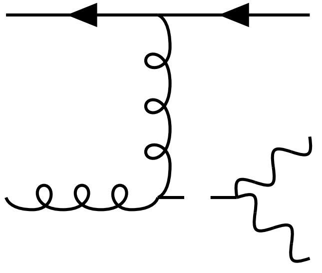
$$+ d\hat{\sigma}_0^{q\overline{q}} \int_{x_2}^{1} \frac{dz}{z} \mathcal{H}(z,\epsilon,\delta_c) \Big\{ P_{qg}(z,\epsilon) f_{q_i}(x_1) f_g(x_2/z) + P_{qg}(z,\epsilon) f_{\overline{q}_i}(x_1) f_g(x_2/z) + x_1 \leftrightarrow x_2 \Big\}_{qg}$$





$$+ d\hat{\sigma}_0^{gg} \int_{x_1}^{1} \frac{dz}{z} \mathcal{H}(z,\epsilon,\delta_c) \Big\{ P_{gq}(z,\epsilon) f_{q_i}(x_1/z) f_g(x_2) + P_{gq}(z,\epsilon) f_{\overline{q}_i}(x_1/z) f_g(x_2) + x_1 \leftrightarrow x_2 \Big\}_{qg}$$





- Particle emitted in the final state is a fermion— no soft singularities

Hard Collinear region

$$+ d\hat{\sigma}_0^{gg}(x_1, x_2, \epsilon) \int_{x_1}^{1-\delta_s} \frac{dz}{z} \mathcal{H}(z, \epsilon, \delta_c) P_{gg}(z, \epsilon) \Big\{ f_g(x_1/z) f_g(x_2) + x_1 \leftrightarrow x_2 \Big\}_{gg} \Bigg]$$

Renormalised parton distributions $\overline{MS}$ scheme

$$f_q(x) = f_q(x,\mu_F) - \frac{a_s(\mu_R^2)}{\epsilon}\frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)}\left(\frac{\mu_F^2}{4\pi\mu_R^2}\right)^{\frac{\epsilon}{2}}\int_x^1\frac{dz}{z}\Big[P_{qq}(z)f_q(\frac{x}{z}) + P_{qg}(z)f_g(\frac{x}{z})\Big]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{a_s(\mu_R^2)}{\epsilon}\mathcal{K}(\epsilon,\mu_R^2,s)\left(\frac{\mu_F^2}{s}\right)^{\frac{\epsilon}{2}} \int_x^1\frac{dz}{z}\Big[P_{gg}(z)f_g(x/z) + P_{gq}(z)\big(f_q(x/z) + f_{\overline{q}}(x/z)\big)\Big]$$

- Counter terms to cancel the collinear singularities are obtained by substituting the renormalised PDFs in LO cross section— mass factorisation

$$d\sigma_0 = dx_1\, dx_2\, d\hat{\sigma}_0^{q\overline{q}}(x_1,x_2,\epsilon)\sum_i\Big[f_{q_i}(x_1)f_{\overline{q}_i}(x_2) + f_{\overline{q}_i}(x_1)f_{q_i}(x_2)\Big]$$
$$+ \quad dx_1\, dx_2\, d\hat{\sigma}_0^{gg}(x_1,x_2,\epsilon)f_g(x_1)f_g(x_2)$$

Cancellation of colinear singularities

- $d\sigma^{HC}$ for $d\hat{\sigma}_0^{q\overline{q}}(x_1, x_2, \epsilon)$ hard process to $\mathcal{O}(a_s)$

$$\frac{1}{\epsilon}\mathcal{K}(\epsilon, \mu_R^2, s) f_q(x_1) \int_{x_2}^{1-\delta_s} \frac{dz}{z}\mathcal{H}(z, \epsilon, \delta_c) P_{qq}(z, \epsilon) f_{\overline{q}}(x_2/z)$$

- Corresponding counter term

$$-\frac{1}{\epsilon}\mathcal{K}(\epsilon, \mu_R^2, s) f_q(x_1, \mu_F) \left(\frac{\mu_F^2}{s}\right)^{\frac{\epsilon}{2}} \int_{x_2}^{1} P_{qq}(z) f_{\overline{q}}(x_2/z)$$

- Cancellation of collinear singularities in the hard collinear region $d\sigma^{HC}$ are not complete when the counter terms $d\sigma^{CT}$ is added— as the phase space is separated into soft and hard regions

$$\frac{1}{2} f_q(x_1, \mu_F) \int_{x_2}^{1-\delta_s} \frac{dz}{z} \left\{ P_{qq}(z) \ln\left[\delta_c \frac{1-z}{z} \frac{s}{\mu_F^2}\right] - P'_{qq}(z) \right\} f_{\overline{q}}(x_2/z)$$
$$+ \left[-\frac{1}{\epsilon} + \frac{1}{2} \ln\left(\frac{s}{\mu_F^2}\right)\right] f_q(x_1, \mu_F) \int_{1-\delta_s}^{1} \frac{dz}{z} P_{qq}(z) f_{\overline{q}}(x_2/z)$$

Cancellation of colinear singularities

$$
\begin{aligned}
d\sigma^{HC+CT} &= a_s(\mu_R^2)\, dx_1\, dx_2\, \mathcal{K}(\epsilon, \mu_R^2, s) \\
& d\hat{\sigma}^0_{ab} \Big\{ \frac{1}{2} \Big(f_a(x_1, \mu_F)\, \widetilde{f}_b(x_2, \mu_F) + \widetilde{f}_a(x_1, \mu_F)\, f_b(x_2, \mu_F) \Big) \\
& +2 \Big(-\frac{1}{\epsilon} + \frac{1}{2} \ln \frac{s}{\mu_F^2} \Big) A_{a\to b+c}\, f_a(x_1, \mu_F)\, f_b(x_2, \mu_F) + x_1 \leftrightarrow x_2 \Big\} \\
\widetilde{f}_q(x_1, \mu_F) &= \int_x^{1-\delta_s} \frac{dz}{z} \widetilde{P}_{qq}(z) f_q\Big(\frac{x}{z}, \mu_F\Big) + \int_x^1 \frac{dz}{z} \widetilde{P}_{qg}(z) f_g(x/z, \mu_F) \\
\widetilde{f}_g(x_1, \mu_F) &= \int_x^1 \frac{dz}{z} \widetilde{P}_{gq}(z) f_q\Big(\frac{x}{z}, \mu_F\Big) + \int_x^{1-\delta_s} \frac{dz}{z} \widetilde{P}_{gg}(z) f_g(x/z, \mu_F)
\end{aligned}
$$

$$
A_{a\to b+c} \equiv \int_{1-\delta_s}^1 \frac{dz}{z} P_{ab}(z) = \begin{pmatrix} \underbrace{8C_F \ln \delta_s} + 6C_F & 0 \\ 0 & \frac{22}{3}C_A - \frac{8}{3} n_f T_F + \underbrace{8C_A \ln \delta_s} \end{pmatrix}
$$

- All $+$ive powers of $\delta_s \to 0$

Cancellation of colinear singularities

• Now all poles of soft and colinear origin automatically cancel and is left with a $finite$ 2-body process explicitly dependent on $\ln \delta_s$ and $\ln \delta_c$

$$d\sigma^{2-body}(\delta_s, \delta_c) \quad = \quad d\sigma^{virt} + d\sigma^{real}(\delta_s) + d\sigma^{HC+CT}(\delta_s, \delta_c)$$

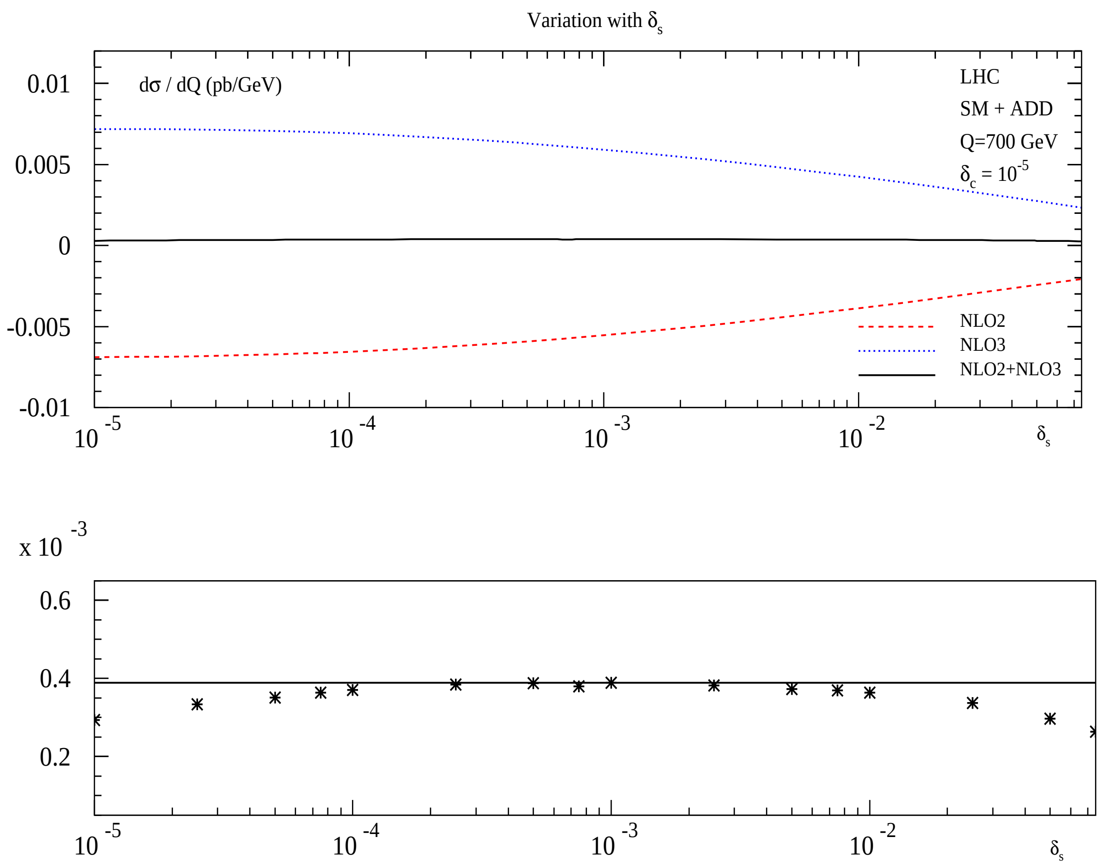
• This when added to the finite hard non-collinear 3-body contribution and on performing the phase space integration using a Mote Carlo techniques have implicit dependence on the same logarithms with opposite signs

$$d\sigma \quad = \quad d\sigma^{2-body}(\delta_s, \delta_c) + d\sigma^{3-body}(\delta_s, \delta_c)$$

• For a reasonable range where δ_s and δ_c are small, the results are stable, providing a check on the calculation

• The combined analytic and Monte Carlo method is flexible enough to accommodate various experimental cuts and compute different observables that are infrared and colinear safe

Stability plot for $d\sigma/dQ$ (SM+ADD)



- Numerical results least sensitive to slicing parameters over a wide range
- Choose a particular value for $\delta_{s,c}$ (stable region) for numerical predictions

Di-photon signal

- Prompt photons:
  - Direct: Both photons originating from the hard partonic interaction
  - Fragmentation: At least one photon produced in the hadronisation of a parton
- Fragmentation photon would be accompanied by hadronic activity in its vicinity
- Final state quark-photon collinear singularity appears in the calculation of the subprocess $gq \rightarrow q\gamma\gamma$.
- These singularities can be factorised and absorbed into the fragmentation functions $D_{\gamma/a}(z, \mu_F)$ where $a = q, \overline{q}, g$, to all orders in α_s — additional non perturbative input

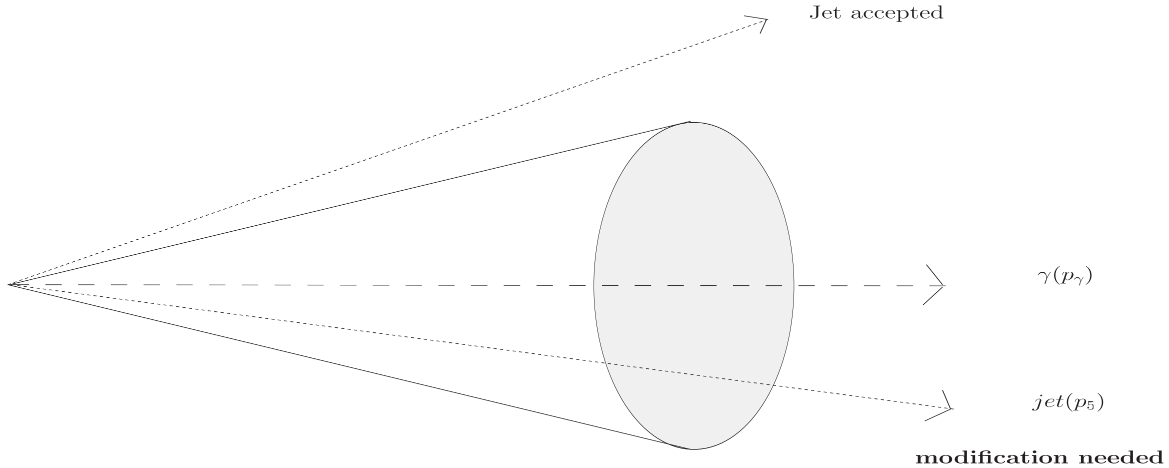
Binoth *et.al* arXiv:hep-ph/9911340

- We adopt an alternate smooth cone isolation criterion proposed by Frixione which ensures that the fragmentation contribution are suppressed with out affecting the cancellation of any of the singularities discussed earlier.

Frixione arXiv:hep-ph/9801442

Frixione's algorithm for the isolation of photons

- Method to define an isolated photon is to draw a circle of radius r_0 in the (η, ϕ) plane, centered on the photon candidate



- Demanding no hadronic activity in the region $r < r_0$ would not only remove the fragmentation contribution but also gluons from that region of *phase space*— event not IR safe

- Fragmentation mechanism is a collinear phenomenon, to eliminate its contribution— sufficient to veto only collinear configurations

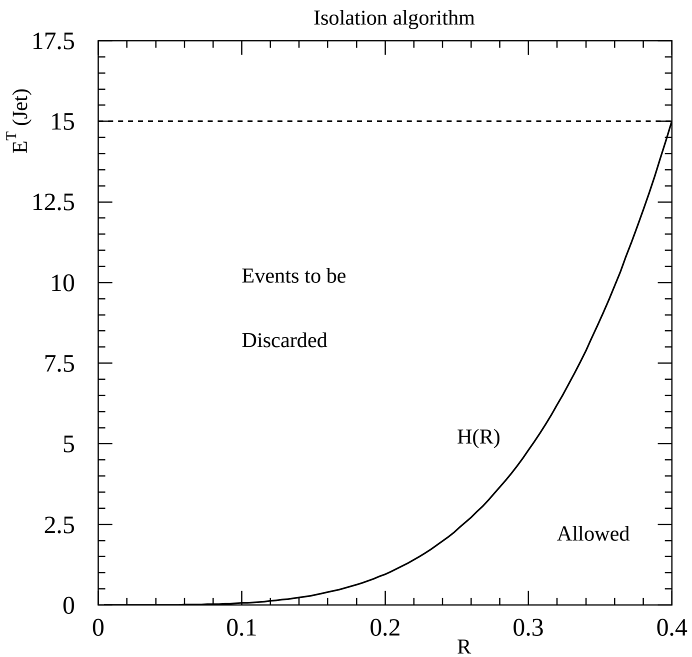
Smooth cone isolation prescription

• Define a continuous set of circles with $r < r_0$ and demand total transverse energy of hadronic activity permitted inside r, $E_T(r)$ decreases to zero as $r \to 0$

• $\sum_i E_{T,i} \leq E_T^{iso} \left(\frac{1-\cos(r)}{1-\cos(r_0)}\right)^n$

• Energy of parton emitted exactly collinear to the photon must vanish

• Contribution of fragmentation is restricted to $D_{\frac{\gamma}{q,g}}(z)\Big|_{z=1} = 0$

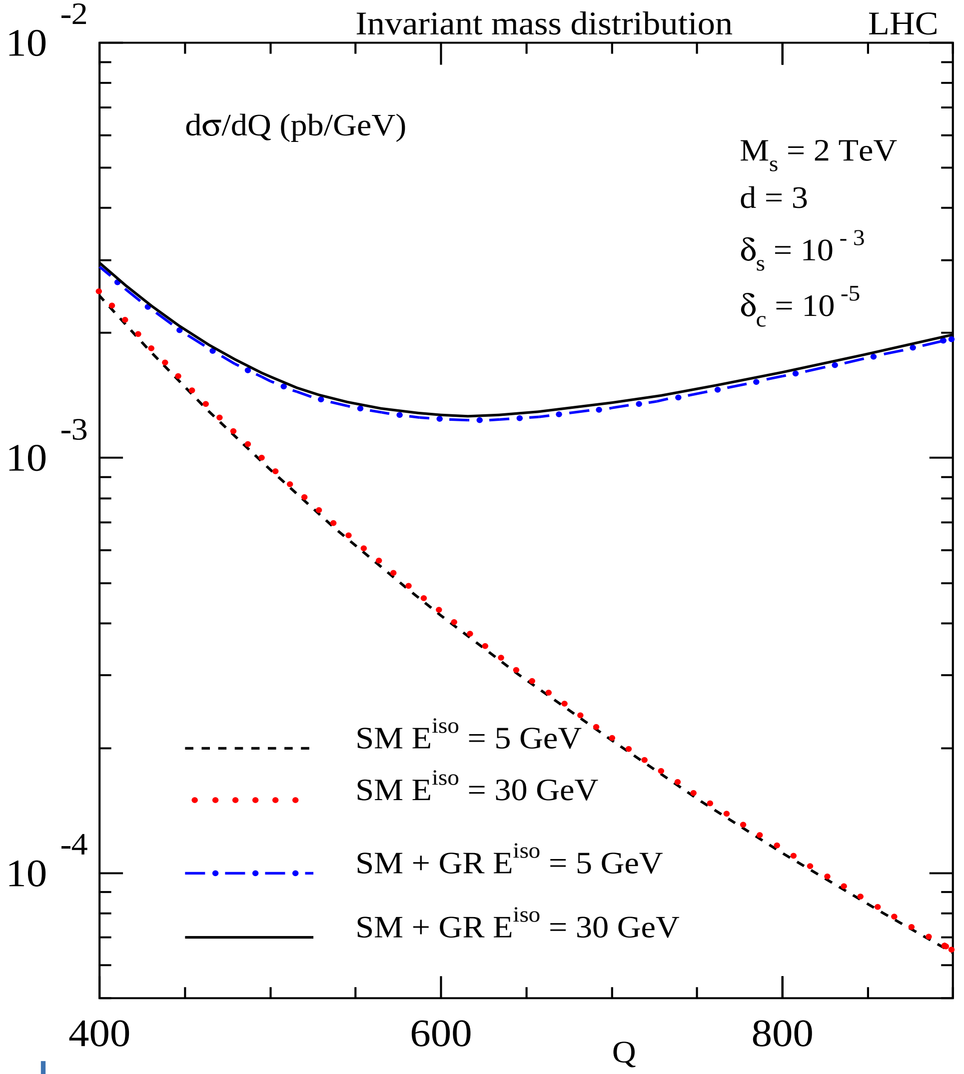


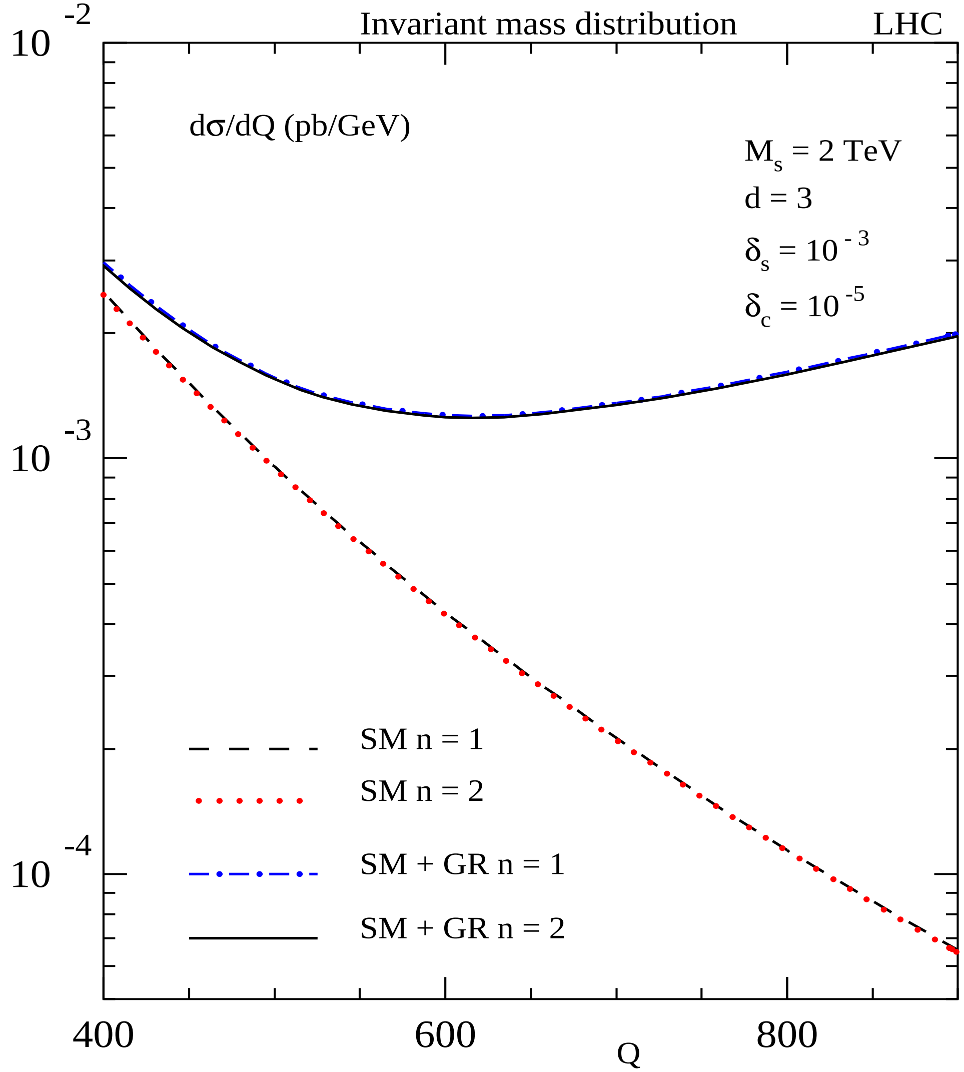
• No region of phase space is forbidden to radiation and at the same time has the virtue of entirely suppressing the poorly known non perturbative fragmentation contribution

Dependence of photon isolation criteria E_T^{iso} and n

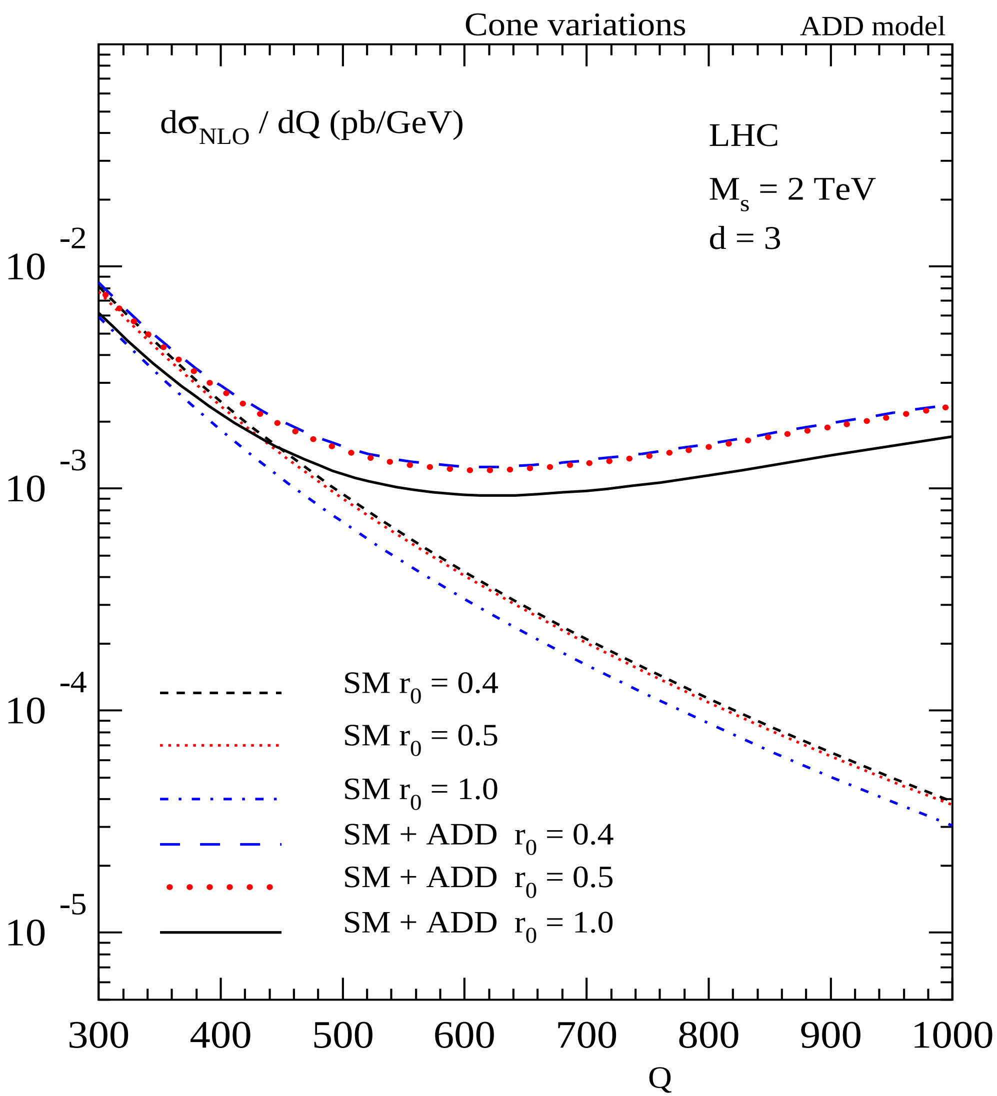
- ADD

Default choice $E_T^{iso} = 15$ GeV, $n = 2$, $r_0 = 0.4$





Effects of varying the cone size r_0



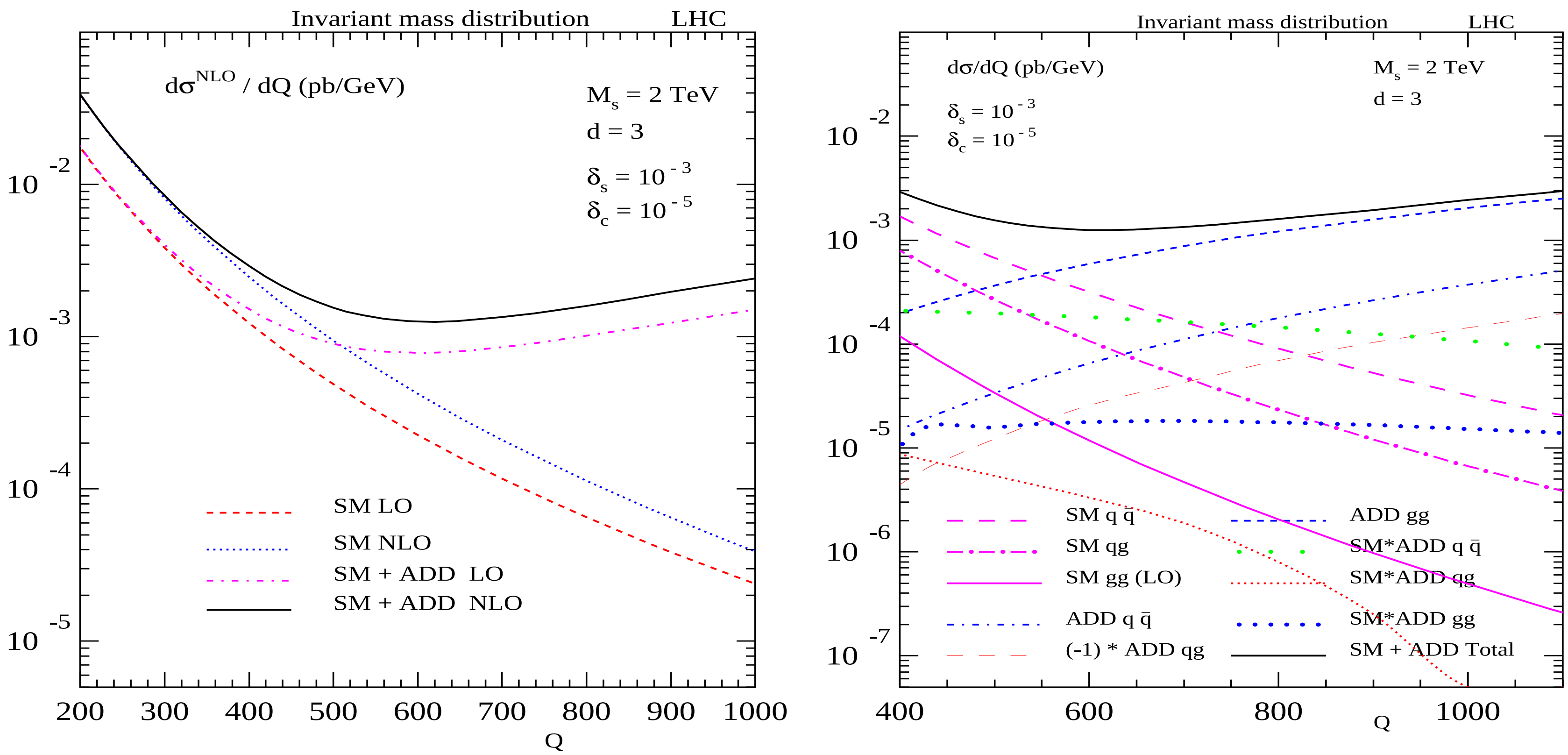
Numerical Results

- Phase space slicing parameters $\delta_s = 10^{-3}$ and $\delta_c = 10^{-5}$
- Photon isolation criteria $E_T^{iso} = 15$ GeV, $n = 2$, $r_0 = 0.4$
- Parton Distribution Functions:
  - LO CTEQ6L
  - NLO CTEQ6M

$n_f = 5$ light quark flavours and $\mu_F = \mu_R = Q$

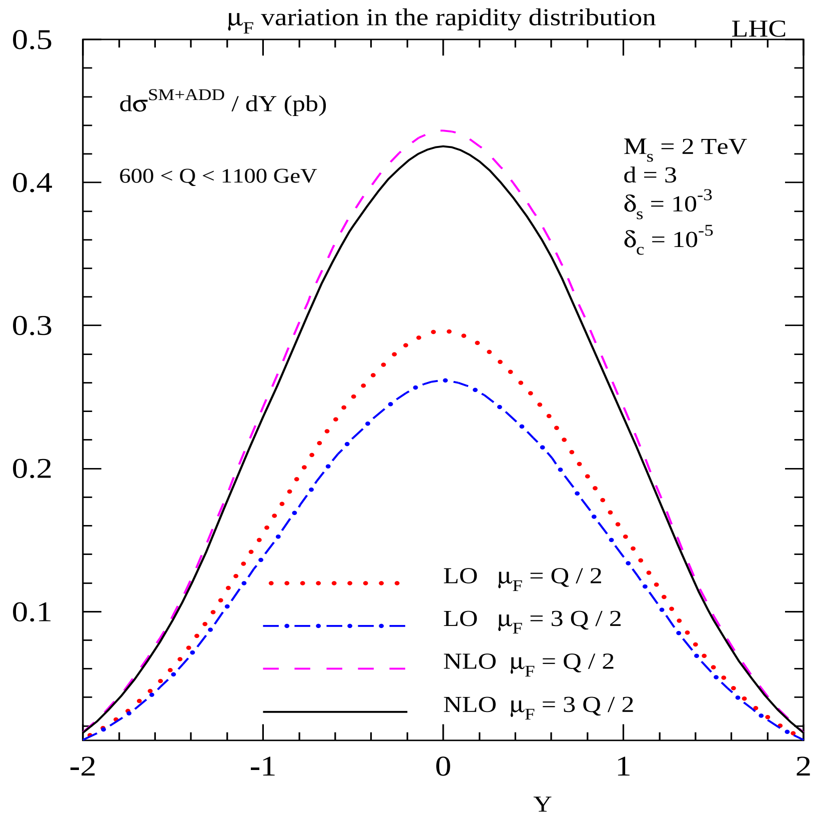
- ADD parameters $M_s = 2$ TeV, $d = 3$
- RS parameters $M_1 = 1.5$ TeV, $c_0 = 0.01$
- Kinematical cuts: (ATLAS & CMS)
  - $p_T^\gamma > 40(25)$ GeV for harder (softer) photons
  - $|y_\gamma| < 2.5$ for each photon
  - $r_{\gamma\gamma} = 0.4$ minimum separation between two photons in (η, ϕ) plain

Invariant mass distribution of the diphoton $d\sigma/dQ$ (ADD)



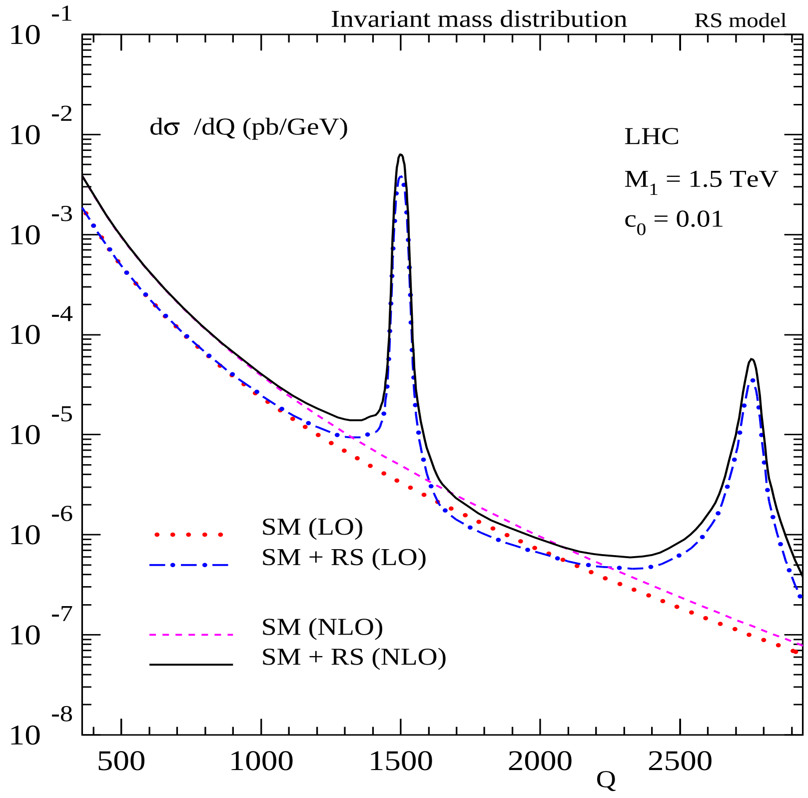
• SM gg-fusion process through quark loop ($\mathcal{O}(\alpha_s^2)$), is comparable to LO in the lower invariant mass Q region but falls of rapidly in the region of interest to large extra dim models

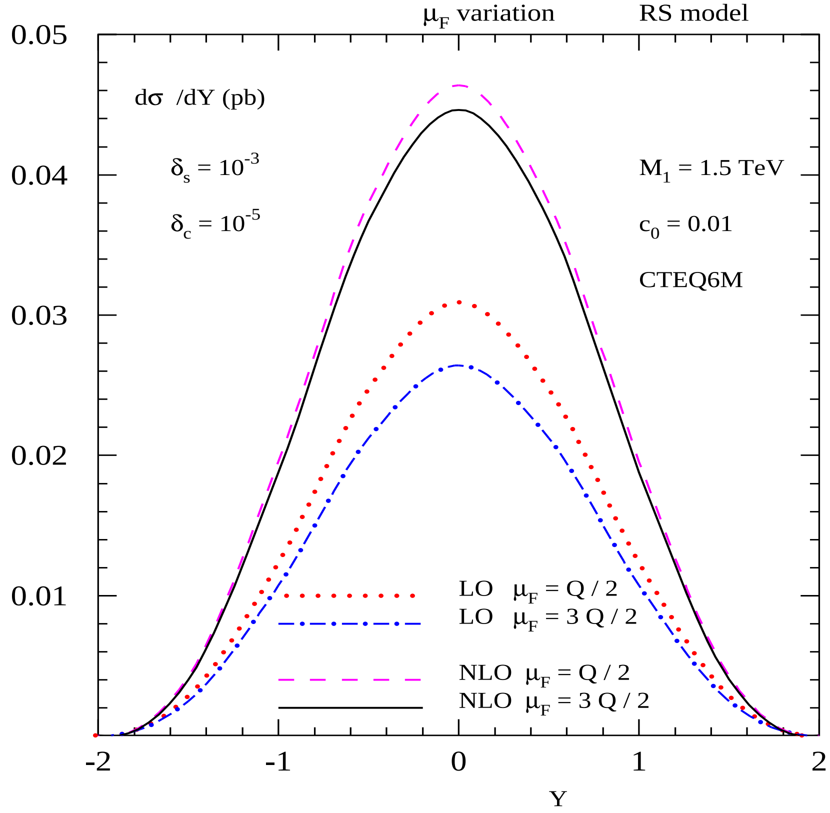
Factoriasation scale dependence of $d\sigma/dY$



- NLO results show significant improvement on the factorisation scale uncertainty entering thorough the PDFs at LO

Invariant mass distribution of the diphoton $d\sigma/dQ$ (RS)





Summary

- **NLO QCD corrections to production of direct photon pair at hadron collider in the context of extra dimension scenarios** $viz.$ **ADD and RS**

- **We use the semi analytical phase space slicing method to deal with all the soft and collinear singularities and the finite part is integrated numerically imposing the appropriate experiment cuts**

- **Various distributions** $viz.$ Q**,** Y**,** $\cos\theta$ **to NLO have been studied for ADD & RS models**

- **Theoretical uncertainties gets significantly reduced in going from LO to NLO**

- **Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders investigated**